論 文

[2110] Analysis of a Prestressed Concrete Plates with Partially Bonded Tendons

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1. INTRODUCTION :

Some recent developments in the construction technique of unbonded post-tensioning members have given impetus to its use in the construction work. It has also been reported that the use of tendons coated with a bituminous rust inhibitor, and wrapped with paper, can help in producing low cost P.C. elements. The performance of the resulting unbonded P.C. elements can be judged, if it is compared with that of the traditional bonded P.C. elements.

Figure.(1.1) shows that the bonded one doesn't deflect as much under a specific load as the unbonded one does. Behavior of partially bonded members lies between the two extreme cases.

Although there are differences among bonded, partially bonded, and unbonded members, conventional methods of analysis usually treat the different types of structures in the same manner. This is due to the complications and difficulties which arise in the calculation of the change of tendon forces in the steel. In this study, by combining the standard plate element for concrete portion and modified line element of steel tendons which span from one anchorage to the other, an analysis method is developed for partially bonded prestressed concrete structures, which enables to obtain the behavior of plate considering the bondage factor in the analysis.

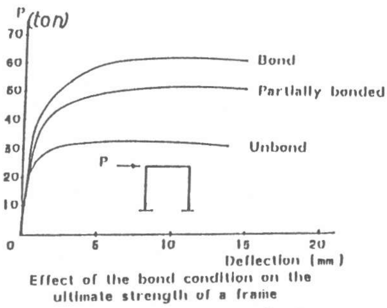

Fig.(1.1) [1]

2. THEORETICAL COSIDERATION:

2.1 Concrete Stiffness Matrix Analysis and Descretization by F.E.M.

Pecknold's 20 degrees-of-freedom isoparametric rectangular flat plate element has been used in the present analysis[4] . Using the principle of virtual work, the load-displacement relationship for concrete is written as follow,

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$$\begin{Bmatrix} Fv_1 \\ Fv_2 \end{Bmatrix} = \begin{bmatrix} \int_V B_1^T D B_1 \, dv & \int_V z B_1^T D B_2 \, dv \\ \int_V z B_2^T D B_1 \, dv & \int_V z^2 B_2^T D B_2 \, dv \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} \tag{2.1}$$

Where, Fv_1, v_1 : Inplane force and nodal displacement vectors.
Fv_2, v_2 : Bending force and nodal displacement vectors.
B_1 , B_2 : Inplane and plate bending strain matrices.
z : distance from the middle surface.

The plate is divided into layers, as shown in Fig.(2.1). Each layer may have different material properties, but these properties are assumed to be constant over the layer thickness . The integration extends over the thickness of each layer, as the material properties vary discretely through the thickness of plate.

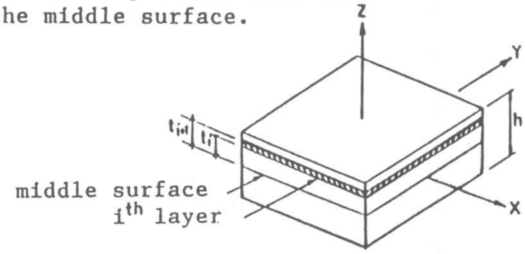


Fig.(2.1)- Layered Element

The load-displacement relation of concrete for layered elements is given from Eq.(2.1) as,

$$\begin{Bmatrix} Fv_1 \\ Fv_2 \end{Bmatrix} = \begin{bmatrix} \sum_i (t_{i+1}-t_i) \int_S B_1^T D B_1 \, ds & \sum_i 1/2(t_{i+1}^2-t_i^2) \int_S B_1^T D B_2 \, ds \\ \sum_i 1/2(t_{i+1}^2-t_i^2) \int_S B_2^T D B_1 \, ds & \sum_i 1/3(t_{i+1}^3-t_i^3) \int_S B_2^T D B_2 \, ds \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} \tag{2.2}$$

Where t_{i+1} and t_i are the distances from the middle surface to top and bottom of i layer, as shown in Fig(2.1).

2.2 Prestressed Tendon Stiffness Matrix Formulation:

Strain energy of tendons varies according to the extent of bondage between concrete and tendon. If perfect bond and no sliding is occured, the change of tendon strain due to external load will exactly follows the change of strain in the concrete fibre at the same level. On the other extreme case of a perfectly unbonded member, the magnitude of strain of tendon will be same throughout its entire length, i.e.the change of strain in the perfectly unbonded tendon will be,

$$\Delta\varepsilon_{sa} = \frac{1}{L}\int_0^L \Delta\varepsilon_{cs} dx \tag{2.3}$$

where L denotes the total length of tendon.

For the case between the two extremes, i.e. perfectly bonded and perfectly unbonded, if we assume a sliding coefficient,Ks, for the distribution of strain, the change of strain as shown in Fig.(2.2) in a tendon at any section can be expressed in the following way [2]:

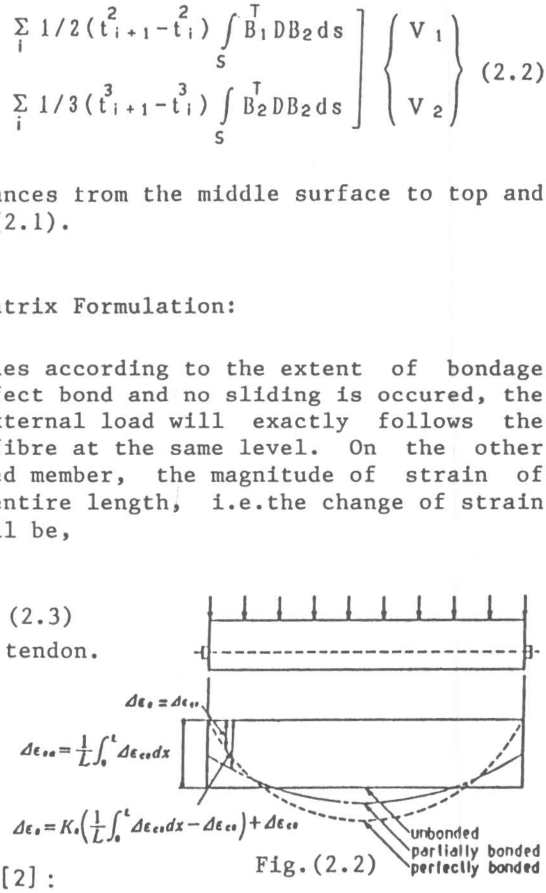


Fig.(2.2)

$$\Delta\varepsilon_s = K_s\left(\frac{1}{L}\int_0^L \Delta\varepsilon_{cs}dx - \Delta\varepsilon_{cs}\right) + \Delta\varepsilon_{cs} \tag{2.4}$$

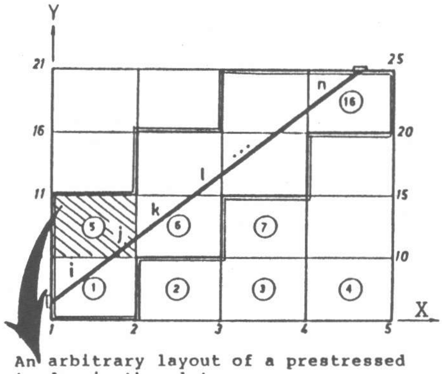
When Ks=0, Eq.(2.4) corresponds to the fully bonded condition, while Ks=1 corresponds to the unbonded condition.

Therefore the strain in the steel tendon at any point inside the element i is

$$\varepsilon_{si} = K_s\left(\frac{1}{L}\int_0^L \varepsilon_{csi}\, dx - \varepsilon_{csi}\right) + \varepsilon_{csi} \tag{2.5}$$

An arbitrary layout of P.C. tendon is shown in Fig.(2.3.a).

In Eq.(2.5), $\varepsilon_{csi}=C_i D_i$ is the concrete strain in element i refering to local coordinate system X´-Y´ (Fig.2.3.c), and C_i is the matrix relating the strain and element nodal displacement vector D_i . The integration is performed from 0 to L, the total length of a specific tendon. Equation (2.5) can be written as,



An arbitrary layout of a prestressed tendon in the plate

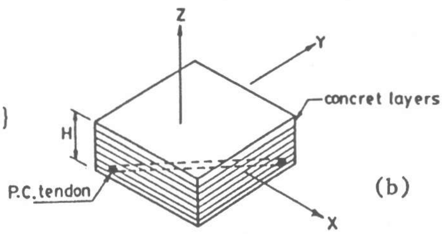
(a)

$$\varepsilon_{si} = \left[k_s/L \int_0^{L_1} C_1 ds_1,\ k_s/L \int_{L_1}^{L_2} C_2 ds_2, \ldots, \right.$$

$$\left. k_s/L \int_{L_{i-1}}^{L_i} C_i ds_i + (1-k_s)C_i, \ldots, k_s/L \int_{L_{n-1}}^{L_n} C_n ds \right]\{DS\} \tag{2.6}$$

$$i=1,\ldots,n$$

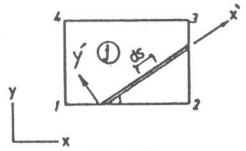
where, n is the number of elements which contain the specific tendon.



(b)

General View

or, $$\varepsilon_{si} = [B_i]\ \{DS\} \tag{2.7}$$



(c)

Plan View

Prestressing steel tendon in an segment i

Fig.(2.3)

Where, $\{DS\}^T = \{D_1, D_2, \ldots, D_i, \ldots D_n\}$ is the nodal displacement vector for all the elements which contain the same steel tendon. From the principle of virtual work the element stiffness matrix of the tendon is,

$$K_{si} = A_s E_s \int_{L_{i-1}}^{L_i} B_i^T B_i\, ds \tag{2.8}$$

Therefore stiffness matrix for total length of the tendon is the sum of stiffness matrices of all elements containing the tendon:

$$K_s = \sum_{i=1}^{n} [K_{si}] \tag{2.9}$$

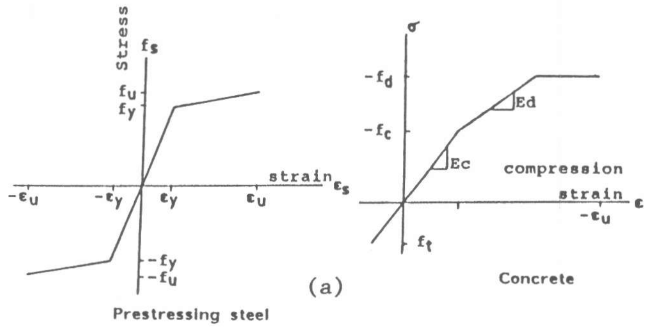
Finally, the general stiffness matrix of the structure is the sum of stiffness matrices for concrete and steel ,

$$[K] = [K_c] + \sum_{i=1}^{n} [K_{s_i}] \tag{2.10}$$

2.3 Crack Analysis :

The assumed uniaxial stress-strain curves for steel and concrete are shown in Fig.(2.4.a),[2],[4].

When the tensile strain in any critical section reaches the limit tensile strain, the section will start to crack. In this study, the behavior of the cracked members are analyzed by modifying the elasticity matrix D of the cracked layer as the crack progresses through the layers.



(a)

The assumed stress-strain relation for concrete with a crack oriented at an angle α counterclockwise with respect to the X- axis is,

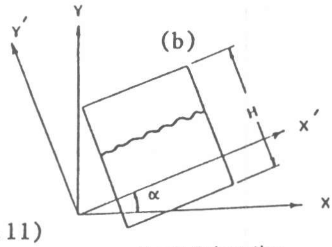


Fig.(2.4)

$$[D_{cr}] = [T_1]^{-1} \begin{bmatrix} E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \beta E/2(1+\nu) \end{bmatrix} [T_2] \quad (2.11)$$

$[T_1]$, $[T_2]$ are the transformation matrices for stress and strain vector as

$$\{\sigma'\} = [T_1]\{\sigma\} \quad \text{and} \quad \{\varepsilon'\} = [T_2]\{\varepsilon\}$$

β : Shear retension factor. (in the present study, β=0.4,[4])

3. NUMERICAL EXAMPLE :

To demonstrate the applicability of the method developed in the present study , two numerical examples are presented:

3.1 Numerical example I:

A two way square concrete slab is analyzed, the slab is simply supported on all the sides as shown in Fig.(3.1). The slab is 12 cm thick with overall dimensions of 200x200 cm, two steel tendons with cross sectional area 20 cm^2 have been provided . For the finite element analysis the slab is divided into sixteen equal square elements. Vertical loads are applied gradually and the load-deflection curve has been plotted for the following cases as shown in Fig.(3.2)

1- Rinforced concrete case, curve A
2- Perfectly bonded prestressed concrete case, curve B.
3- Partially bonded prestressed concrete case, curve C.

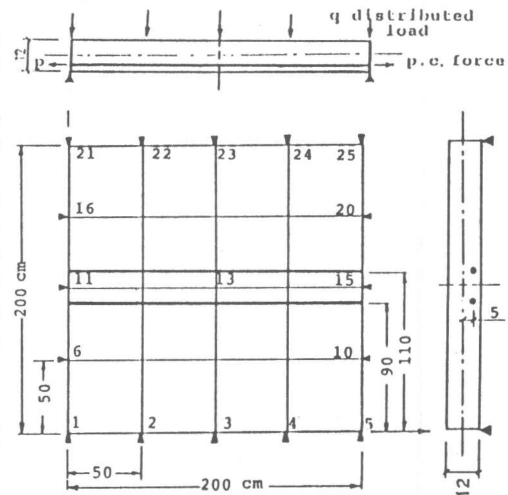
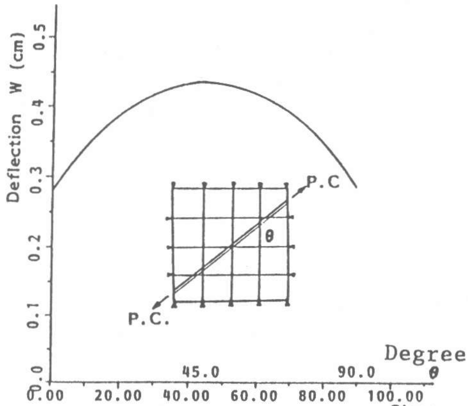


Fig.(3.1)

Fig.(3.3) shows the inclination effect of tendon on the vertical deflection of this plate. An increase of 36% in deflection can be seen as the slope is changed from 0° to 45°.



Inclination Effect of the Tendon on the Vertical Deflection

Fig.(3.3)

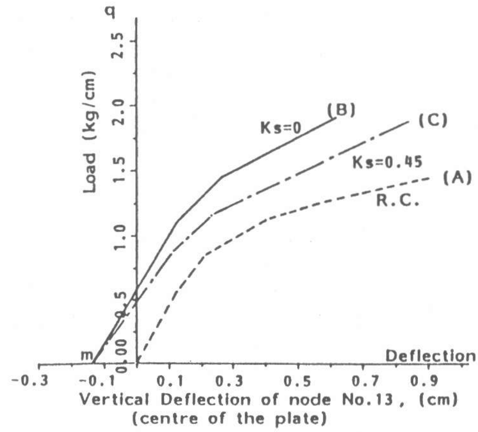


Fig.(3.2)

3.2 Numerical example II:

To show the accuracy of the result, A one way rectangular slab of overall dimensions 100x10 cm is analyzed, the thickness of slab is 7.0 cm. The slab is supported at its corners only and the layout of steel tendon is shown in Fig.(3.4). For F.E.M. calculation the slab is divided into 10 equal square elements. The results obtained by F.E.M. and by plate theories are compared and the differences are shown as follow :

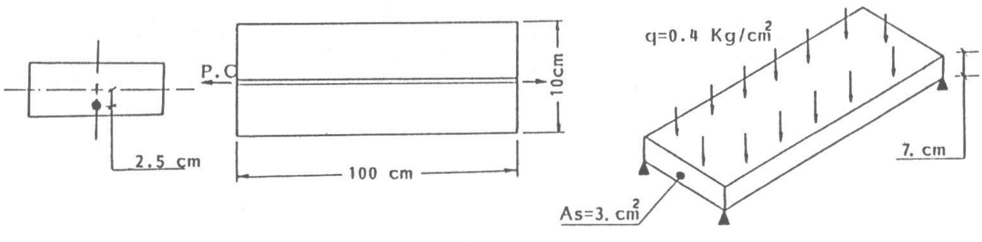


Fig.(3.4)

Central Deflection (w) By F.E.M. (cm)	Central Deflection (w) By plate theory (cm)	Difference %
Concrete: 0.0860791	0.0867694	0.81 %
Concrete + Steel(Ks=0.0) Perfectly bonded, 0.05891596	0.0608456	3.10 %

Concrete + Steel (ks=1.0), unbonded case, W = 0.064 cm

The difference in the central deflection between bonded and unbonded case is, 8.50%

In the case of a prestressed concrete, the applied tensile P.C.force is 10000 Kg. The deflection obtained by F.E.M. is 0.654 cm, and the one which obtained by plate theory is 0.669 cm. So the difference is 2.19%

3.3 Result and Discussion :

When we compare the curves B and C, it may be understood that the load conditions has little effect in the lower loading stage while it surely affect to substantial extent when the load intensity becomes large and bond stress between steel and concrete becomes large. It is considered that the real exising structure is maintaining bond condition like the category C and it is never in perfect bond condition. This is the point where the analysis method developed in this study may perform an important role in the analysis of existing prestressed concrete structures.

4. SUMMARY AND CONCLUSION

The analysis method of partially bonded prestressed concrete plate structures is developed using the F.E.M formulation. The bond condition between steel and concrete is expressed by the bond coefficient and compatibility of steel strain and concrete strain is maintained along the length between two anchorages of the steel even though the relative slip is allowed.

Using the developed method, several numerical examples are calculated. They showed the results which reflects clearly the bond effect, which is considered to be useful to advanced design of prestressed concrete structures.

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6. REFERENCES :

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