コンクリート工学年次論文報告集 11-2 1989

論 文

[2111] Fundamental Study on the Application of Fracture Mechanics to the Crack Propagation Analysis of Thermal Stress Fields of Concrete

Denzil Lokuliyana* and Tada-aki Tanabe**

1.INTRODUCTION

In the case of mass concrete structures, there happen to be generated major cracks during even construction, and sometimes at much later age of concrete (i.e. 10 years and 20 years after the completion of construction). These cracks are mainly due to thermal heat which is produced by hydration of fresh concrete. In the design of these type of structures thermal cracking is taken into account in predicting both the ultimate load capacity and behavior in service conditions since width, spacing and length of cracks have major influence on structural performance. Most of the exsisting design codes are based on tensile strength approach and not on fracture mechanics. But it is understood that the structures should be designed according to fracture mechanics when fracture is considered.

However, fracture obviously cannot form if the tensile strength (peak stress) is not reached. So the strength criterion is a necessary condition of fracture, but not a suffcient one. For the fracture to form, the maximum principal stress must also reduce to zero and for this to happen the energy characterizing the complete stress–strain curve must be supplied. Thus the strength criterion indicates only whether the fracture can initiates, while the energy criterion indicates whether the fracture can actually form or propagate. In this study cracking is modeled in continuous or smeared manner, and fracture is treated as a propagation of smeared crack band through concrete[1].

With this fundamental idea, the crack propagation analysis is done and its applications to real structures were discussed.

2.BASIC FRACTURE CRITERION FOR HETEROGENEOUS MATERIALS

In this analysis, heterogeneous materials are approximated by an equivalent homogeneous continuum having the size equal to the size of several times of the maximum aggregate size[2]. Hence in finite element analysis the element size is not taken as smaller than the size of this equivalent homogeneous continuum. The crack propagation is treated as a band of parallel, densely distributed microcracks having blunt crack front since the actual crack path in concrete is highly tortuous. Therefore, this unknown crack band path is assumed to propagate to the element in which principal tensile stress is the largest and sharing a side with the crack front element in finite element analysis.

The constitutive equation for the strain–softening zone(fracture process zone) is simply modeled by changing isotropic elastic moduli matrix to an orthotropic matrix, by reducing the material stiffness in the direction normal to the crack in the band.

Cartesian co-ordinates X , Y and Z are introduced with the cracks being assumed to be normal to X axis. Assumed strains are small or linearized and the normal stress and strain components can be written as

---

* Graduate Student, Department of Civil Engineering, Nagoya University

** Professor, Department of Civil Engineering, Nagoya University

$$\{\sigma\} = \{\sigma_x, \sigma_y, \sigma_z\}^T \quad and \quad \{\epsilon\} = \{\epsilon_x, \epsilon_y, \epsilon_z\}^T \tag{2.1}$$

where T is the transpose. Since in the geomaterials cracks usually occures in principal stress directions, principal components of stresses and strains are taken into account.

The elastic stress–strain relation for the normal components may then be written as

$$\{\sigma\} = [D_{uc}]\{\epsilon\} \tag{2.2}$$

where $[D_{uc}]$ is the stiffness matrix for uncracked material. When the elastic material is intersected by continuously parallel cracks normal to X axis, the stiffness matrix reduced to an orthotropic one, $[D_{fr}]$ [3] and stress-strain relation then can be written as

$$\{\sigma\} = [D_{fr}]\{\epsilon\} \tag{2.3}$$

This matrix $[D_{fr}]$ gives stiffness matrix for fully cracked material derived by assuming ϵ_x of the material between the cracks is unrelated to ϵ_y and ϵ_z and that the stress normal to the cracks must be zero and assuming that the material between cracks behaves as an uncracked elastic material in a plain stress state. This is a simplification because the material between the cracks becomes damaged by discontinuous microcracks. Then the compliance matrix for partial discontinuous microcracks takes the form

$$[C_{cr}(\mu)] = \begin{bmatrix} C_{11}\mu^{-1} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ sym. & & C_{33} \end{bmatrix} \tag{2.4}$$

where μ[2] is the cracking parameter which increases the strain ϵ_x and dicreases σ_x . Then the stiffness matrix for partially cracked concrete takes the form of

$$[D_{cr}(\mu)] = [C_{cr}(\mu)]^{-1} \tag{2.5}$$

Taking limit of μ to zero, the stiffness matrix for fully cracked material can be obtained as

$$[D_{fr}] = \lim_{\mu \to 0} [C_{cr}(\mu)]^{-1} \tag{2.6}$$

3.ENERGY CONSIDERATION FOR FRACTURE PROPAGATION

3.1 Fracture Energy

The fracture energy is defined here as the energy consumed by crack formation per unit area of the crack plane(Fig. 3.1).

By considering the bi-linear stress strain relation, the energy released per unit volume when concrete gets fully cracked can be written as

$$W_f = \int_0^{\epsilon_0} \sigma_x d\epsilon_x = \frac{1}{2} f_t^2 (C_{11} - C'_{11}) \tag{3.1}$$

Then the fracture energy can be calculated as,

$$G_f = w_c W_f \tag{3.2}$$

where, w_c is the width of the crack band.

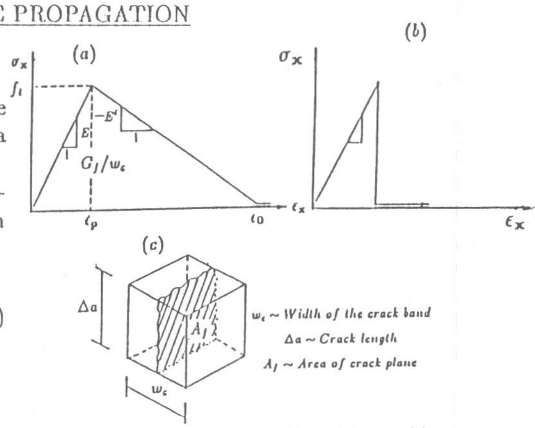

Fig. 3.1. (a). Energy released by cracking.
(b). Sudden stress drop.
(c). Crack plane.

The value of the fracture energy is approximately predicted from the elementary characteristics of concrete for the optimum fits of various fracture test data by various researchers and, recently CEB

(Comite Euro–International du Beton) code draft sugested the appropiate G_f values. As an example Bazant's formula [2] was adopted and is defined as,

$$G_f = 0.1697(f_t + 8.95){f_t}^2 d_a/E \tag{3.3}$$

In which, f_t is the tensile strength in kgf/cm^2, d_a is the maximum aggregate size in mm, E is the elastic modulus in kgf/cm^2 and G_f is the fracture energy in $kgf - cm/cm^2$. At the same time, it is discovered [2] that the optimum width for the crack band is $3d_a$. Then from equations (3.1) and (3.2) the condition for the compliance C_{11}^t to be negative becomes,

$$C_{11}^t = C_{11} - \frac{2G_f}{w_c f_t^2} < 0 \tag{3.4}$$

$$w_c < \frac{2G_f}{C_{11} f_t^2} = \frac{2G_f E}{f_t^2} \tag{3.5}$$

This gives the limiting value for the width of the crack band front. From equation (3.5), to preserve the value of fracture energy the width of the fracture front can be changed by varying the softening compliance and/or tensile strength. Hence for large structures for larger values of finite element width a sudden stress drop with equivalent strength[2] can be used.

4.FORMULATIONS GOVERNING THERMAL STRESS PROBLEMS

4.1 Change of Energy due to Crack Propagation

There are several methods to calculate the amount of energy released when crack propagates. One of these method is that calculating the difference between total energy contained in all elements of the structure before and after crack advance. But, this method gives inconveniences in initial stress problems. Therefore, the method which is tractable for the thermal stress problems due to hydration heat is formulated and here utilizing the Rice's [5] work. Consider a continuous body is externally loaded by surface traction T_i^0 on its boundary S_T as shown in Fig.4.1(a).

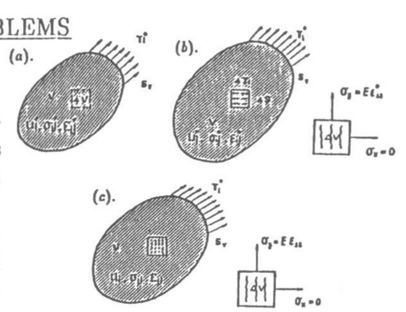

Fig. 4.1. (a). State before cracking.
(b). Intermediate state.
(c). State after cracking.

Let U_i^0, ϵ_{ij}^0 and σ_{ij}^0 denotes the displacements, strains and stresses in the state just before cracking (Fig.4.1(a)). Now let's consider that cracking occures in volume ΔV under constant load T_i^0 creating a new traction free surface ΔS (Fig.4.1(c)) . Let U_i, ϵ_{ij} and σ_{ij} denote the new values of displacements, strains and stresses for this cracked state. Then the total potential energy for the state just before cracking Π_{uc} is,

$$\Pi_{uc} = \frac{1}{2}\int_V \sigma_{ij}^0 \epsilon_{ij}^0 dv - \int_{S_T} T_i^0 U_i^0 ds \tag{4.1}$$

while for the state after cracking Π_{cr} is

$$\Pi_{cr} = \Pi_{uc} + \Delta\Pi = \frac{1}{2}\int_{V-\Delta V} \sigma_{ij}\epsilon_{ij} dv + \frac{1}{2}\int_{\Delta V} E{\epsilon_{22}}^2 dv - \int_{S_T} T_i^0 U_i ds \tag{4.2}$$

(Repeated indices imply summation over $i, j = 1, 2, 3$)

Since the body is assumed to be elastic, $\Delta\Pi$ is independent with path followed from uncracked state to cracked state. Now let's consider an intermediate state (Fig.4.1(b)) in which the volume ΔV of the material is removed from the body, but surface traction $\Delta T_i = {\sigma_{ij}}^0 {n_j}^0$ are applied to the newly formed surface ΔS in order to preserve initial state of deformation within the remaining part of the body. Now the total potential energy for this intermediate state is then,

$$\Pi_{im} = \frac{1}{2}\int_{V-\Delta V} {\sigma_{ij}}^0 {\epsilon_{ij}}^0 dv + \frac{1}{2}\int_{\Delta V} E{{\epsilon_{22}}^0}^2 dv - \int_{S_T} {T_i}^0 {U_i}^0 ds \tag{4.3}$$

Then from equations (4.1) and (4.3) the change of potential energy for the first transition (i.e. state before cracking to intermediate state) becomes,

$$\Delta W_1 = -\frac{1}{2}\int_{\Delta V} (\sigma_{ij}{}^0\epsilon_{ij}{}^0 - E\epsilon_{22}{}^{0^2})dv \tag{4.4}$$

Subsequantly, the traction T_i, applied to the surface ΔS, is released to zero to reach the final cracked state. Applying the hypothesis of conservation of energy to the second transition, it can be equalized the total potential energy change within the total body to the work done by the traction $T_i{}^0$ on S_T and T_i on ΔS, i.e.,

$$\frac{1}{2}\int_{V-\Delta V} (\sigma_{ij}\epsilon_{ij} - \sigma_{ij}{}^0\epsilon_{ij}{}^0)dv + \frac{1}{2}\int_{\Delta V} (E\epsilon_{22}{}^2 - E\epsilon_{22}^{0\;2})dv = \int_{S_T} T_i^0(U_i - U_i^0)ds + \frac{1}{2}\int_{\Delta S} \Delta T_i(U_i - U_i^0)ds \tag{4.5}$$

Then, from the equations (4.2) and (4.5)

$$\Delta\Pi = -\frac{1}{2}\int_{\Delta V} (\sigma_{ij}{}^0\epsilon_{ij}{}^0 - E\epsilon_{22}{}^{0^2})dv + \frac{1}{2}\int_{\Delta S} \Delta T_i(U_i - U_i^0)ds \tag{4.6}$$

Since the total change in potential energy is path independent,

$$\Delta\Pi = \Delta\Pi_{1^{st}transition} + \Delta\Pi_{2^{nd}transition} \tag{4.7}$$

Now comparing equations (4.4),(4.6) and (4.7) the 1^{st} part of the equation (4.6) becomes the energy released during 1^{st} transition and the 2^{nd} part becomes for the 2^{nd} transition, i.e. ,

$$\Delta W_1 = -\frac{1}{2}\int_{\Delta V} (\sigma_{ij}{}^0\epsilon_{ij}{}^0 - E\epsilon_{22}{}^{0^2})dv \tag{4.8}$$

$$\Delta W_2 = \frac{1}{2}\int_{\Delta S} \Delta T_i(U_i - U_i^0)ds \tag{4.9}$$

and the total energy released during cracking now can be expressed as

$$\Delta\Pi = \Delta W_1 + \Delta W_2 \tag{4.10}$$

The force $\int_{\Delta S} \Delta T_i ds$ can be calculated as the surface traction forces which must be applied on concrete within volume ΔV in order to equilibriate the stress changes $\Delta\sigma_{ij}$ due to creation of crack upon passing from the initial to the intermediate state. These changes of stresses are

$$\begin{aligned} \Delta\sigma_{11} &= \sigma_{11}^0 \\ \Delta\sigma_{22} &= \sigma_{22}^0 - E\epsilon_{22}^0 = \frac{E}{1-\nu^2}\{\epsilon_{22}^0 + \nu(\epsilon_{33}^0 + \epsilon_{11}^0)\} - E\epsilon_{22}^0 \end{aligned} \tag{4.11}$$

For initial strain problems due to temperature change, the relevant effective stresses and strains should be substituted to the above equations.

4.2 Energy Criterion for Crack Propagation

When the maximum principal tensile stress reaches the tensile strength, discontinuous microcracks initiation is occured. But, still it is unknown whether the cracks can actually propagate to the next element or not. This can be decided by the energy concept.

The total energy released by cracking is already explained in the previous section. It has two components,

1. The elastic energy released by cracking (ΔW_1)
2. The energy supplied by unloading of concrete between the cracks (ΔW_2).

Now the energy, externally supplied to the structure to extend the crack band by length Δa is written as

$$\Delta U = G_f \Delta a - (\Delta W_1 + \Delta W_2) \tag{4.12}$$

When $\Delta U > 0$, then no crack extension occures, without supplying external energy to the structure, and so the crack band is stable and propagation will not occures. If $\Delta U < 0$, no external energy is requred to be supplied to the structure, and so the crack band is unstable and, propagation occures in a dynamic manner. If $\Delta U = 0$, no energy needs to be supplied and none is released, and the crack band may propagate in a static manner.

5.ACCURACY OF THE METHOD

A centrally loaded, simply suppored plain concrete beam (Fig. 5.1) was analyzed for crack propagation. Bilinear stress–strain relation for the fracture process zone having the slope of $E^t = -4.3 \times 10^4 kgf/cm^2$ [2] for strain softening curve is used.

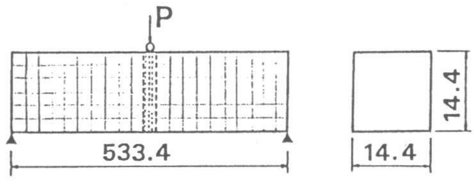

Fig. 5.1. Simply supported centrally loaded plain concrete beam having center crack at the peak load.

The value for the fracture energy was calculated from equation (3.3) and, finite element analysis was done in the two dimensional stress field. When the applied load is reached the value of $640kgf$ cracks are initiated at the center of the bottom fiber of the beam. The value of $20kgf/cm^2$ was used as the tensile strength of concrete. From the criterion for fracture propagation, it was found that at the same load cracks were propagated up to the top fiber and beam failed suddenly.

The value of peak load obtained was deviated 5% from the value obtained from simple bending theory. From this simple analysis it was understood that the energy criterion works for fracture propagation.

Next, another plain concrete notched beam (Fig.5.2) was analyzed for fracture energy. Comparision was done with the experimental work [4] to determine the fracture energy of both fiber reinforced concrete and plain concrete structures. The value obtained from the experiment [4] for the fracture energy was $76N/m$ and the value used in the analysis obtained from eq.(3.3) was $80N/m$.The load deflection curve of the experiment is shown with the calculated fracture values.

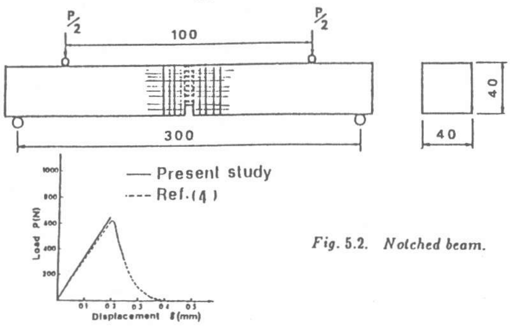

Fig. 5.2. Notched beam.

The structure was analyzed using finite element method and the fracture propagation was decided by the fracture criterion. The value obtained for the peak load at which the notched beam suddenly collapsed was agreed with the experimental results of [4].

6.ESTIMATION OF CRACK PROPAGATION IN MASSIVE CONCRETE

A massive concrete structure shown in Fig.(6.1) was analyzed for crack propagation using the idea of fracture energy and the crack band concept. A block of hardened concrete ($size 15.5 \times 5.0 \times 1.5m$) is carrying a block of fresh concrete ($size 15.5 \times 1.0 \times 1.5m$.) The bottom of the hardened concrete block is assumed to be rollar supported and the interface between fresh and hardened concrete blocks is assumed to be rigidly connected for the simplycity in the analysis.

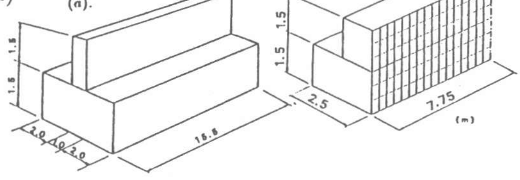

Fig. 6.1. (a). A block of fresh concrete casted on hardened concrete block.
(b). Two – dimensional Finite Element Mesh.

The thermal stress distribution was obtained for every 3 hours during 1*st* day, every 1/2 day up to 7*th* day and every 7*th* day up to 28*th* day after the fresh concrete block is casted. The analysis was done for two dimentional stress state. The change of Young's modulas was derived from [6] (Fig.6.2) for unit weight of concrete 300 kg/m^3.

From this analysis it was found that the centre of the body has compressive zone and the areas closer to the boundary has tension zone. On the 7*th* day after casting the maximum principal tensile stress is exceeded the tensile strength at a point 3m from both edges, and from the present theory it was found that the cracks penetrated to a depth of 1 m, and then the next crack was initiated at the center and penetrated up to 1.5 m depth and stopped (Fig.6.3). Cracking dates, penetration and crack spacing obtained was very similar to the one experimentally obtained.

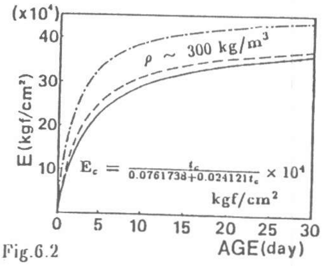

Fig.6.2

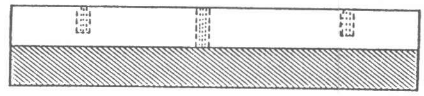
Fig.6.3

7.CONCLUSION

The crack propagation in mass concrete structures was analyzed using the energy criterion of fracture and smeared crack band concept.

It is known that the dissipated energy by the cracking can be calculated in two ways as follows.

i. Considering the total energy difference of the structure before and after the crack advanced.

ii. Taking the summation of the strain energy released by cracking and the work done by the traction on the cracked surface when it is released to zero.

The first method of calculating energy however, can not be applied to a structure with initial stresses due to hydration heat of cement. Hence the detailed derivation of formula for the second method is performed.

With these preparations, a simply supported plain concrete beam was analyzed and the path obtained was reasonable. The two-dimensional analysis done for thermal crack propagation in a mass concrete structure has given reasonable results. A further extension can be done to analyze massive concrete structures three-dimensionally.

ACKNOWLEDGEMENT

The author is pleased to acknowledge staffs and students in the Laboratory of Department of Civil Engineering, Nagoya University, for their earnest help in preparing this report.

REFERENCES

1. *Rashid, Y. R.* , Analysis of Prestressed Concrete Pressure Vessels, Nuclear Engineering and Design, Vol. 7, No. 4, pp. 334–344 (1980).
2. *Shi, G. C., and Ditommasho, A.* , "Fracture Mechanics of Concrete" - Ch. 1.
3. *Suidan, M.* , and *Schnobrich, W. C.* , Finite Element Analysis of Reinforced Concrete, Journal of the Structural Division, ASCE, Vol. 99, No. ST10, Proc. Paper 10081, pp. 2109–2122 (1973).
4. *Ohgishi, S., Ono, H.*, and *Itho, S.*, Fractire Toughness, J_{IC} and Fracture Energy G_F in the New Materials Fiber Reinforced Concrete, Published in International Workshop on "Fracture Toughness and Fracture Energy Test Methods for Concrete and Rock" Oct. 1988.
5. *Rice, J. R.*, "Mathamatical Analysis in the Mechanics of Fracture," Fracture, on Advanced Treatise, H. Liebowitz, ed., Vol. 2, Acadamic Press, New York, N. Y., 1968, pp. 191–250.
6. The Committee Report: Identification of Degree of External Restraint and the Proposed Method of Thermal Stress Calculations of Massive Concrete, JCI Committee on Thermal Stress of Massive Comcrete, Concrete Journal Vol. 23, No. 8, No. 9, No. 10, JCI 1985.