論 文

[2115] A Time-Dependent Uniaxial Constitutive Model of Concrete as Composite Structural Material

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1. INTRODUCTION

It is well known that the seismic load on a structure subjected to an earthquake is extremely complicated because not only the seismic force but also the loading speed changes with time. In finite element method analysis of reinforced concrete structures under such loads with complex histories, it is necessary to have a constitutive model which can give the stress for arbitrary strain-time history. At present there are several models which can be used for cyclic loadings of constant loading speed or monotonic loadings of varied loading speeds separately (1)-(4), but no model suitable to seismic analysis is available. In this paper the time-dependent constitutive relation of concrete is simulated by a so-called "elasto-viscoplastic fracture model" composed of elastic-plastic-viscoplastic bar elements with different elastic, plastic and viscoplastic properties.

2. MODELING PROCEDURES

In this paper the uniaxial constitutive relation of concrete under arbitrary loading histories is proposed to be simulated by the behavior of a structure composed of elasto-viscoplastic bar elements as shown in Fig.1. In order to establish this model the properties of each element has to be known for a given strain-time history.

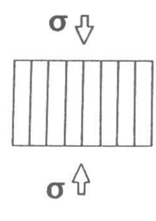

Fig.1 Concrete model

2.1 CONSTITUTIVE RELATION OF ELEMENTS

In the selection of the constitutive relations of individual elements, which use the same forms of equations but different coefficients for each element, it is advisable to use functions which are not only relatively simple but can also represent some basic characteristics of concrete. As shown in Fig.2, each element of the model is composed of three interrelated components, namely, elastic component, plastic component and viscoplastic component in order to account for the corresponding

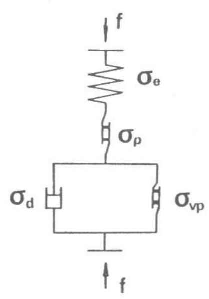

Fig.2 Element model

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strains. The characteristics of each component is explained first. In this paper, the stresses and strains used are normalized by the maximum stress and the corresponding strain respectively with plus for compression.

The elastic component of an element generates the elastic strain of the element only. Assuming the initial elastic modulus of the elastic component being E, the stress of the elastic component is

$$\sigma_e = E \cdot \varepsilon_e \tag{1}$$

As external load increases, plastic strain of concrete will develop even if load speed is very high. The plastic strain depends not only on the stress σ_e applied to this element or the elastic strain according to Eq.(1) but also on viscoplastic strain ε_{vp} because plastic strain is affected by the compact of concrete due to viscoplastic strain. In this model plastic strain is presumed as follows

$$\varepsilon_p/\varepsilon_{e1}=(Exp(a/(1.0-\varepsilon_e/\varepsilon_{e1})^b)-Exp(a)-\varepsilon_{vp}/(\varepsilon_{e1}+c\cdot\varepsilon_{vp}))_{max} \tag{2}$$

in which a, b and c are material coefficients. The symbol ε_{e1} is the maximum compressive elastic strain that this element can reach. The notation $(\ )_{max}$ means the maximum value in loading history. For tensile loading it is assumed that time independent plastic deformation does not occur.

In an individual element the time dependent strain is included in a viscoplastic component which consists of a dash pot and a strain hardening slider as shown in Fig.2.

$$d\varepsilon_{vp}/dt = \gamma \cdot \sigma_d/E \tag{3}$$

in which ε_{vp} is viscoplastic strain, γ is supposed to depend on the stress on the dash pot σ_d and a fluid parameter of concrete γ_o,

$$\gamma = \gamma_o(\sigma_d/(E\cdot\varepsilon_{e1}))^d/(1-(\sigma_d/(E\cdot\varepsilon_{e1}))^e) \tag{4}$$

in which b, e are constants. These formula of the dash pot can also be used for tensile viscoplastic strain ε_{vpt} with the maximum tensile elastic strain ε_{e1t} in place of ε_{e1}. As viscoplastic strain increases the yielding stress of the slider will also become greater due to strain hardening. During the loading process, the yielding stress of compressive strain hardening is given by

$$\sigma_{yc} = E\cdot c_{rp1}\cdot\varepsilon_{e1}\cdot(1-g/Ln(\varepsilon_{vp}/\varepsilon_{e1}+Exp(g))) \tag{5}$$

in which g and c_{rp1} are coefficients, $E\cdot c_{rp1}\cdot\varepsilon_{e1}$ is maximum yielding stress that can be gained through viscoplastic compressive strain hardening of the slider. In the case of tensile viscoplastic strain hardening, the yielding stress σ_{yt} can be obtained by the same strain hardening rule as Eq.5, but with ε_{e1t} replacing ε_{e1} and different values of coefficients. The slider only moves when tensile stress or compressive stress is greater than corresponding yielding stress. The stresses of these components in an element should satisfy equilibrium. Hence, the stress of an element is

$$\sigma_e = E\cdot\varepsilon_e = \sigma_{vp} + \sigma_d \tag{6}$$

in which, σ_{vp} is the stress of viscoplastic hardening slider

$$\sigma_{vp} = \begin{cases} \sigma_e, & \text{when } \sigma_{yt}<\sigma_e<\sigma_{yc} \\ \sigma_{yt}, & \text{when } \sigma_{yt}>\sigma_e \\ \sigma_{yc}, & \text{when } \sigma_e>\sigma_{yc} \end{cases} \quad (7)$$

hence,

$$d\varepsilon_{vp}/dt = \gamma \cdot (\sigma_e - \sigma_{vp})/E \quad (8)$$

The total strain of the element should be equal to the sum of elastic, plastic, visco-plastic strains and crack strain ε_c

$$\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_{vp} + \varepsilon_{vpt} + \varepsilon_c \quad (9)$$

During unloading of the present model some of the elements may be in tension because all the elements have different stresses due to their different properties. If tensile strain of an element is greater than its maximum tensile strain ε_{elt}, the element will be cracked and can not bear tensile load later, but can still bear compression when the micro cracks closing because a part of the surface will contact. There will be variation of contact area and crack strain of a cracked element if elastic strain changes due to compression. Relation of contact area A_c and crack strain was chosen as

$$(A_c - A_{min})/(1 - A_{min}) = R_c(2 - R_c) \quad (10)$$

$$R_c = 1 - \varepsilon_c/\varepsilon_c^{max} \quad (11)$$

$$A_{min} = 1/(3\varepsilon_c^{max}/\varepsilon_{el}+1) \quad (12)$$

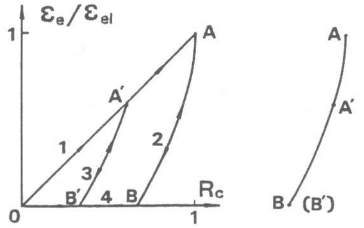

Fig.3 Elastic strain versus change of crack strain

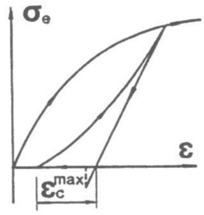

Fig.4 Stress versus strain of an element

where, ε_c^{max} is the maximum crack strain ever underwent. A_{min} is the contact area when reversing to compressive loading. R_c represents the recovery of crack strain. The relation of crack strain to elastic strain is given for loading, unloading and reloading respectively. If compression is applied without reverse from where crack strain is at its maximum value,

$$\varepsilon_e/\varepsilon_{el} = R_c \quad (13)$$

as shown in Fig.3 by path 1, i.e. the envelope. For unloading from point A(path 2 in Fig.3) the crack will not open to its original width when elastic strain ε_e is 0. The residual crack strain can be represented by $(R_c)_o$ (point B in Fig.3). From point A to point B a parabolic function is assumed

$$R_c = -(1-(R_c)_o)(\varepsilon_e/\varepsilon_{el})^2 + 2(1-(R_c)_o)(\varepsilon_e/\varepsilon_{el}) + (R_c)_o \quad (14)$$

If the unloading and reloading path is not on path 2, the relation between elastic strain and crack strain is given by shifting path 2 as shown in Fig.3. From above equations, for a given strain or stress of an element, all the strain components and stress can be solved. Fig.4 illustrates the stress of an element corresponding to its strain during loading, unloading, broken and reloading performed instantaneously. Assuming the initial area of an element is a unity, the load F carried by this element is

$$F = A_e \cdot E \cdot \varepsilon_e \tag{15}$$

in which A_e is effective area less than a unity. In experiments, fracturing of concrete corresponding to micro crack growth can be observed as load increasing. In this model the fracturing is taken into consideration by reducing the area effective in carrying load. It is assumed that the part of area destroyed due to compressive fracturing is irrecoverable even if compression becomes smaller later. Two factors which cause the compressive damage of concrete are considered here. One is the damage of elastic component due to elastic strain, i.e. element stress, and another one is the damage of viscoplastic strain hardening slider due to viscoplastic strain developed with time. They are defined by $K_e(\varepsilon_{em})$, a function of the maximum elastic strain experienced by the element ε_{em} and $K_{vp}(\varepsilon_{vp})$, a function of compressive viscoplastic strain ε_{vp}. The effective area can be obtained from the two damage functions and the contact area in Eq.10

$$\begin{aligned} A_e &= K_e(\varepsilon_{em}) \cdot K_{vp}(\varepsilon_{vp}) \cdot A_c \\ &= \mathrm{Exp}(-c(\mathrm{Exp}(a/(1-\varepsilon_e/\varepsilon_{e1})^b)-\mathrm{Exp}(a))^d) \cdot \mathrm{Exp}(-c'(\varepsilon_{vp})^{d'}) \cdot A_c \end{aligned} \tag{16}$$

2.2 ELEMENT PROPERTIES AS DENSITY FUNCTIONS OF ELASTIC LIMITS

After establishing the constitutive model of individual elements, the problem remained is how to find the composition of elements, i.e. the parameter E, ε_{e1} and ε_{e1t} of each element. In this paper elements are divided according to their maximum compressive elastic strain ε_{e1} of elastic components. By this way elastic moduli of the elastic components and their maximum tensile strains are defined as functions of ε_{e1}(for clarity x is used instead of ε_{e1}, afterwards). The total initial elastic modulus of the elastic components of those elements whose maximum elastic compressive strains are within $(x_1, x_1+\triangle x)$ can be obtained from a density function f(x)

$$\triangle E = \int_{x_1}^{x_1+\triangle x} E_o \cdot f(x) \cdot dx \tag{17}$$

where: E_o is the initial stiffness of concrete composite and

$$\int_0^{\infty} f(x)dx = 1 \tag{18}$$

The constitutive relation can be written as,

$$\sigma(\varepsilon) = \int_0^{\infty} A_e \cdot E_o \cdot \varepsilon_e \cdot f(x)dx \tag{19}$$

where ε is the strain of the model. A_e, ε_e are functions of ε and loading history. In order to get better results both on envelope and on unloading-reloading loops, elastic moduli E_{nu} of very small and instant unloading from the envelope are also used in solving for f(x). We can have these elastic moduli from the following equation

$$E_{nu}(\varepsilon) = \int_0^{\infty} A_e \cdot E_o \cdot f(x) \cdot dx \tag{20}$$

From experiments or available models we can find the expected values of Eqs.19,20 $\sigma'(\varepsilon)$ and $E'_{nu}(\varepsilon)$ under high load speed. The density function f(x) can be so selected as to make the weighted square-root error between $\sigma(\varepsilon)$, $E_{nu}(\varepsilon)$ computed from this model and $\sigma'(\varepsilon)$, $E'_{nu}(\varepsilon)$ the smallest

$$E_{rror}^2 = \int_0^{\varepsilon} (\rho_1(\varepsilon) \cdot (\sigma'(\varepsilon)-\sigma(\varepsilon))^2 + \rho_2(\varepsilon) \cdot (E'_{nu}(\varepsilon)-E_{nu}(\varepsilon))^2)d\varepsilon \tag{21}$$

in which $\rho_1(\varepsilon)$ and $\rho_2(\varepsilon)$ are weighting functions. We can choose the base functions for f(x), for example as polynomials, and then find the coefficients. In order to solve the problem numerically, we rewrite Eqs.(19),(20) in discrete form as

$$\sigma(\varepsilon) = \sum_{i=1}^{n} A_e \cdot E_i \cdot (\varepsilon_e)_i \tag{22}$$

$$E_{nu}(\varepsilon) = \sum_{i=1}^{n} A_e \cdot E_i \tag{23}$$

By minimizing the error function Eq.(21) we can obtain E_i (i=1,,,n), which can be used as the elastic moduli of the elastic components of discrete elements. The maximum tensile strain of the elastic components can be related to their corresponding maximum compressive strain by a function. A simple one is a linear function of x: $\varepsilon_{eit} = c \cdot x$, where the constant c can be obtained from unloading-reloading loops. The coefficients in the constitutive relation of elements can also be changed in order to obtain better results.

From the above equations we can calculate stress of concrete under any strain-time history by giving proper values to those coefficients.

3. MODEL PREDICTIONS

Based on the constitutive relation of Maekawa model(1) as an envelope of instant loading, we managed to find the density function f(x) and the coefficients needed in the elastic and plastic components. The coefficients of viscoplastic components are adjusted to make the results satisfactory. Using those coefficients we computed some results considering the effects of loading speeds to simulate several phenomena observed in experiments. Fig.5 shows the constitutive relation obtained for a instantaneous loading by using the above proposed model. It can be seen that the envelopes from present model and Maekawa model are in good agreement. The instantaneous elastic moduli for the two unloading-reloading loops also coincide with the experimental results reported(2),(3). Fig.6 is calculated from the present model for a loading history with varied loading speed. It can be observed that because the loading speed is not infinite, viscoplastic strain develops and causes the unloading curve steeper than the corresponding ones of instant unloading in Fig.5. At points C and F, although the stress of the model is zero, some of the elements have been cracked, some are in tension and the others are in compression because all of them have different values of ε_{ei} and ε_{eit}. The tensions will produce tensile viscoplastic strain, which relaxes the residual stress in the model and allows the strain recover to point D and F, respectively. This phenomenon is often observed in experiments. In Fig. 7 it is shown that the stresses are increased to certain levels instantly, kept as constants for the same time to allow viscoplastic strain to develop, and then increased instantly again. Paths 1, 2 and 3 demonstrated that if the levels at which the stresses were kept were relatively low, the stress-strain curves obtained for the later instant loadings would approach the stress-strain curve of instant loading from the very beginning. The stress-strain relation of the loading with infinite slow speed is also shown in this figure. Fig.8 shows an experimental result of a test sample under a cyclic loading in Reference 2 and the analytical result computed by using the present model. From this figure we can see that this model simulated this experimental result very well before the sample is near to be damaged.

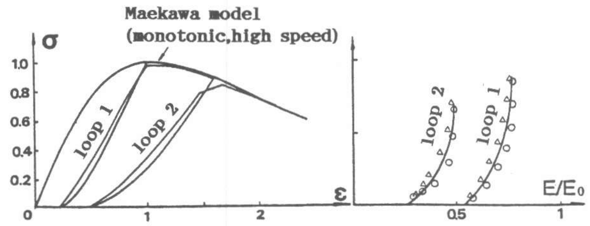

Fig.5 Instantaneous loading

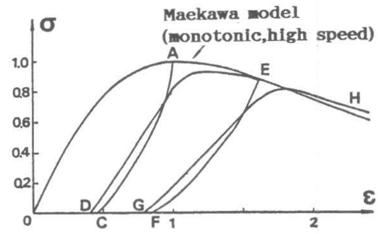

Fig.6 Loading of varied speed

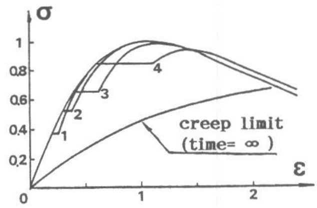

Fig.7 Time-dependent strain

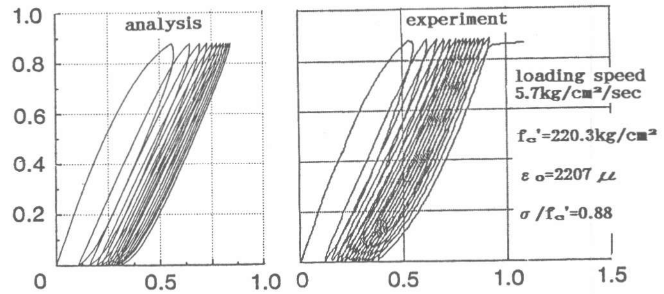

Fig.8 Response to cyclic loading

4. CONCLUSION

From this study it can be concluded that concrete behaves as a composite structural material during unloading and reloading, which causes the nonlinearity of unloading-reloading loops. The model proposed here based on the above concept can simulate the constitutive relation of concrete with good accuracy, but it needs simplifying in order to apply it to numerical analysis of structures.

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