論 文

[2142] AN ANALYTICAL MODEL OF FATIGUE CRACK GROWTH IN CONCRETE

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1. INTRODUCTION

Recently a remarkable progress has been accomplished in the application of fracture mechanics to concrete [1]. The existence of fracture process zone is identified and governing mechanisms are clarified. Although analytical prediction of crack growth phenomena under static loading has been enabled, mechanism of fatigue crack growth under cyclic loading has not been studied.

In the present study an analytical model of fatigue crack growth in concrete is proposed in order to identify its governing mechanism. Consideration is based on the tension-softening behavior of concrete under cyclic load. Low-cycle fatigue crack growth tests are carried out where the distribution of crack opening displacement is measured by laser speckle method. Experimental results are compared with analytical predictions by the proposed model to discuss the mechanism of fatigue crack growth.

2. MECHANISM OF FATIGUE CRACK GROWTH IN CONCRETE

It has been clarified that the static crack growth in concrete is governed by the existence of fracture process zone; see Fig. 1 [2]. The microcracking and bridging are major mechanisms in fracture process zone. The bridging zone, which is a part of extended macrocrack with stress transmitted by aggregate, is modelled by a Dugdale-Barenblatt-type model with a tension-softening curve. The tension-softening curve presents the relationship between opening displacement and transmitted stress after the peak stress in uniaxial tension test. It represents the material property on static crack growth.

It is straight forward to expect that the knowledge on static crack growth is extended to fatigue crack growth with proper modification and the tension-softening behavior under cyclic tensile loading contains material characteristics for fatigue crack growth. The post-peak cyclic behavior of concrete in tension is investigated by Reinhardt et al. [3]; see Fig. 2 for a typical result. It is seen from Fig. 2 that cyclic tension-softening behavior is complex and includes many characteristics one of which may be the mechanism responsible for fatigue crack growth.

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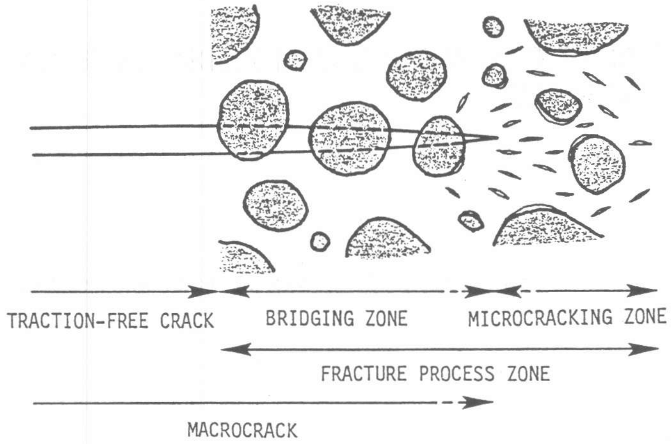


FIG. 1 Schematic illustration of fracture process zone.

One of the noticeable features in cyclic behavior is the residual crack opening displacement. At a point which undergoes tension-softening during loading, there exists remaining opening displacement at an unloaded state. This works to resist the elastic constraint which acts to close the crack faces resulting in compressive stress at the point and tensile stress at the crack tip. This may be the mechanism of fatigue crack growth. However, Shin [4] showed from theoretical examination that this residual crack opening displacement can not be the source of fatigue crack growth.

Another feature is the material behavior in reloading. The opening displacement and transmitted stress in reloading process are almost proportional with a reduced slope. It should be noticed that the maximum stress in reloading is decreased to about 85% of the stress before the unloading. In this paper, this degradation in reloading process is considered to be the source of fatigue crack growth, and its analytical model is developed. No attention is paid to unloading process. The mechanism of fatigue crack growth is discussed through the comparison of analytical prediction with experimental results.

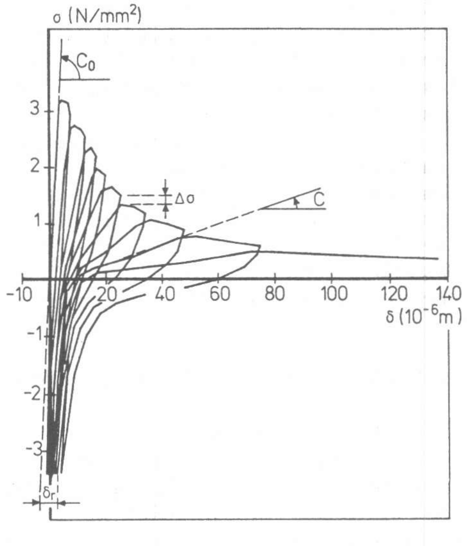


FIG. 2 Stress-deformation relation under cyclic tensile load [3].

3. ANALYTICAL MODEL OF FATIGUE CRACK GROWTH

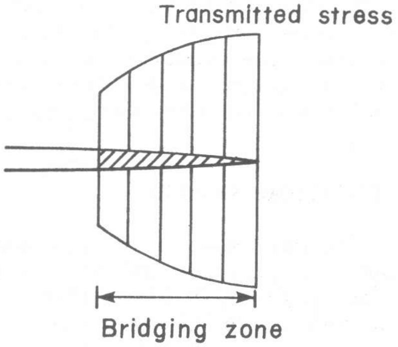
Dugdale-Barenblatt-type model of bridging zone is modified for fatigue crack growth. Bridging zone is modelled as an extended part of a semi-infinite crack along which stresses are transmitted due to bridging by aggregate.

First, we consider the initial loading; see Fig. 3a. Crack opening displacement and transmitted stress satisfies a tension-softening relation. Here a linear tension-softening relation AB in Fig. 4 is assumed for simplicity. Stress is bounded at the end of bridging zone. The problem is reduced to integral equations for transmitted stress [5]. By solving them numerically, the relationship between the applied load and bridging zone length and distribution of transmitted stress (and accordingly crack opening displacement) are obtained.

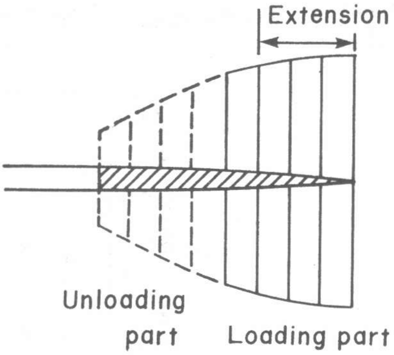
Next, we consider the unloading and reloading. The crack is extended during reloading whose amount is to be determined; see Fig. 3b. Along some part of bridging zone (loading part), the same tension softening curve AB is satisfied. Along the other part of the bridging zone (unloading part), the transmitted stress is assumed to be proportional to the crack opening displacement: for example, if point C in Fig. 4 is the state before unloading the proportional relation OD is satisfied during reloading. (Effect of unloading process is ignored.) The stress at point E is set to be $\alpha\sigma_0$, where σ_0 is the stress before unloading at point C and α is a constant called degradation parameter. Different position in the unloading part satisfies proportional relation with different slope depending on the stress state before unloading. The degradation parameter α is assumed to be constant independent of the stress before unloading.

With those conditions and the condition that the stress is bounded at the tip of the bridging zone, the problem is formulated and solved numerically. The unloading part in which the proportional relation is satisfied is determined so that the solution along OD does not exceed point D. An iterative solution procedure is employed to satisfy this condition.

As a solution the amount of crack extension and new distribution of transmitted stress are obtained.



(a) Initial loading state



(b) Reloading state

FIG. 3 Model of fatigue crack growth.

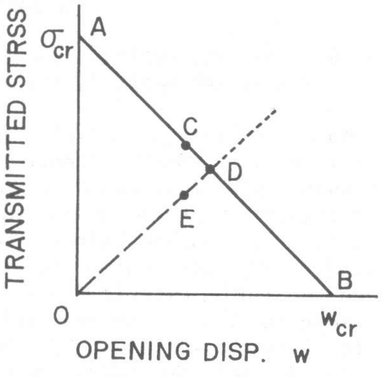


FIG. 4 Material behavior in loading part and unloading part.

For the following cycles, similar procedure is repeated. The slope of proportional relation is assumed to be reduced only when the point reaches the tension-softening relation AB in Fig. 4, which implies the damage progress. The reduction in slope of proportional relation is not considered in the present model if a point along a proportional relation remains to be below the tension-softening envelope under the cyclic load.

4. ANALYTICAL RESULTS

Typical results of the model presented in the previous section are shown in Fig. 5 for indicated values of normalized applied stress intensity factor $K_{IA}/\sqrt{E'\sigma_{cr}w_{cr}/2}$. (The stress variation parameter $\lambda = 0$ corresponds to a very large specimen [5].) The results are normalized as shown in the figure: $\ell_{bc0} = 0.366E'w_{cr}/\sigma_{cr}$ is the critical bridging zone length at maximum load $K_{IA}/\sqrt{E'\sigma_{cr}w_{cr}/2} = 1$ in static fracture for $\lambda = 0$ with σ_{cr} and w_{cr} being the tensile strength and the critical crack opening displacement, respectively; $E'=E/(1-\nu^2)$. ($w_{cr} = 25\mu m$ fits various tension-softening curves; $\ell_{bc0}=8.27cm$ for $\sigma_{cr}=3.39MPa$, $w_{cr}=25\mu m$, $E=2.94x10^4MPa$, and $\nu=0.2$).

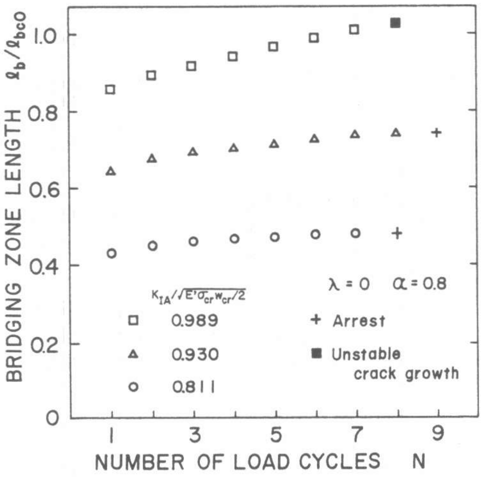


FIG. 5 Growth of fatigue crack with increasing load cycles for different values of applied stress intensity factor.

When the load is small, the amount of initial crack extension to the initial crack length remains small. The amount of crack extension decreases with increasing loading cycles, and crack growth stops after several cycles. With increasing load the crack growth rate increases and the number of cycles before the crack growth arrest increases. When the load is large and close to the static maximum load, the crack growth becomes unstable (static fracture) after several cycles.

Results for different values of degradation parameter α are shown in Fig. 6 for $\lambda = 0$ and $K_{IA}/\sqrt{E'\sigma_{cr}w_{cr}/2} = 0.93$. The fatigue crack growth rate is strongly dependent on the value of α. With decreasing value of α, the crack growth rate and the number of cycles before the crack growth arrest increase.

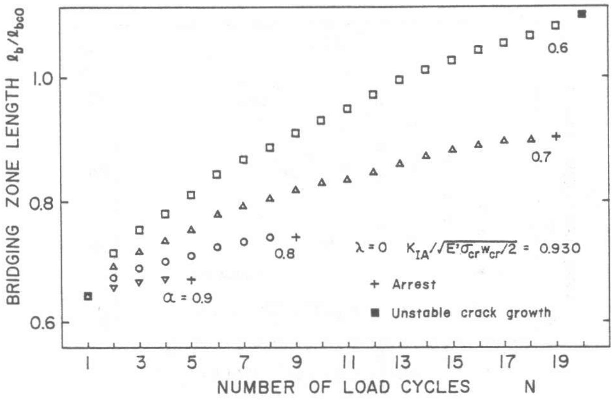


FIG. 6 Fatigue crack growth behavior with different values of α.

5. COMPARISON WITH OBSERVATION BY LASER SPECKLE METHOD

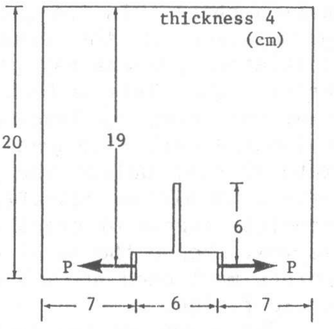


FIG. 7 Fatigue test specimen.

Fatigue tests are carried out for a concrete specimen shown in Fig. 7; Mix proportion is W:C:FA:CA = 0.45:1:2:3.5, f_c=26.1 MPa, $f_t=\sigma_{cr}$=3.39MPa. Constant cyclic load is applied through wedge-loading device. P_{max}= 2470N = 252kgf which corresponds to K_{IA}= 0.869MPa$\sqrt{m}$ = 2.80kgf mm$^{-3/2}$ is about 80% of the maximum load for static fracture. P_{min} is set to be almost zero. To measure the crack length at each load cycle crack opening displacement is measured by laser speckle method [6]. Crack opening displacement is obtained as the difference between displacements at two close points just above and below the final crack path. A large value of maximum load (low-cycle fatigue test) is chosen so that the crack extension is detected by laser speckle method.

Figure 8 shows the measured crack opening displacement at each load cycle. From this result, crack lengths are evaluated and plotted in Fig. 9 together with the analytical prediction. (The stress variation parameter λ is determined to be 0.3 for this specimen [5].) It can be concluded that the agreement between the observation and analytical prediction is satisfactory if one takes into account of the simplifications in modelling, the measurement error for very small crack opening displacement shown in Fig. 8 and the rough way of estimating crack extension length from results in Fig. 8.

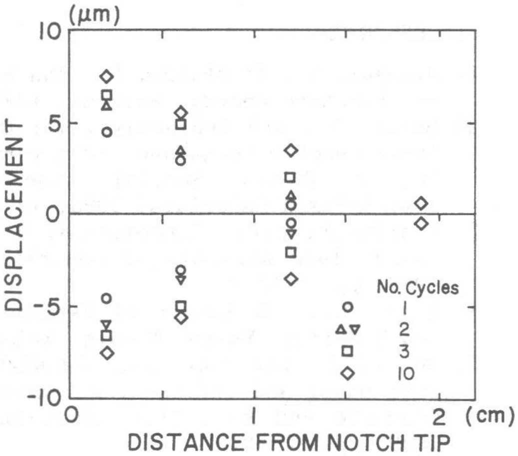


FIG. 8 Crack opening displacement.

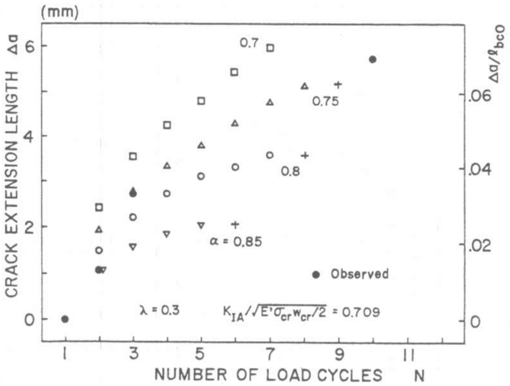


FIG. 9 Observed and predicted crack extension length.

6. CONCLUDING REMARKS

In general, fatigue process consists of initiation phase and propagation phase. In the present study, an existing crack is assumed and only propagation of the crack is considered. Even in this situation, the initiation process may play some roles since stress is cycled ahead of the crack tip. This effect is related to the cyclic behavior in pre-peak nonlinear stage of tensile test. In the present model cyclic effect in the unloading part (the proportional relation with reduced slope) is not taken into account unless the point reaches the tension-softening curve. This effect should be clarified and included in the model. The present model predicts arrest of crack extension after several cycles when the load level is low. Hence the model can not be applied to high-cycle fatigue. The two points mentioned above seem to be closely related to the behavior in high-cycle fatigue.

The study on the mechanism of fatigue crack growth has just started. Extensive studies from both theoretical and experimental point of view must be continued to fully understand fatigue phenomena in concrete.

7. REFERENCES

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