論 文

[2146] CRACK PROPAGATION ANALYSIS BY FEM AND BEM USING THE TENSILE SOFTENING MODEL

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1. INTRODUCTION

Due to the fact that classical engineering design criteria are inadequate to yield fracture prevention measures, fracture mechanics is increasingly gaining importance in the study on the failure of concrete structures. The analysis of crack propagation (initiation and control), in particular, attracts attention due to its critical role for the safety of concrete structures. The propagation of cracks is related to the fracture toughness of concrete which has to do with the softening, i.e, the existence of a descending branch in the stress-deformation diagram [1] . This concept is utilized in this paper which aims at analyzing crack propagation in four-point bending tests of center-notched beams using the tensile softening model. The softening of concrete is due to the damage within the fracture process zone which grows with crack propagation. In a tension test, the energy absorption of concrete is mainly associated with the tensile softening curve. Here, FEM and BEM are employed in the analysis of the Load-CMOD(crack mouth opening displacement) variation as the crack propagates along the center-line of the beam.

2. TENSILE SOFTENING MODEL

The mechanical behavior of most engineering materials under uniaxial tension consists of the following four stages:linear elastic, strain-hardening, perfectly plastic and strain-softening. For a concrete specimen, the linear elastic range is immediately followed by strain-softening, and the intermediate stages do not appear [2] . The fracture mechanism of concrete is completely

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defined by the elastic $\sigma-\varepsilon$ curve and the tensile softening $\sigma-\omega$ curve. The $\sigma-\omega$ curve of concrete is assumed to be a material property independent of the size of the specimen and suitably describes the tensile fracture zone [1] . The importance of the $\sigma-\omega$ curve stems from the fact that the area under the curve is the fracture energy G_f which is a material property of concrete. The $\sigma-\omega$ curve is usually approximated by the linear or bi-linear relations. In this paper, the linear relation with σ_t as the maximum tensile strength is adopted as shown in Fig.1, where

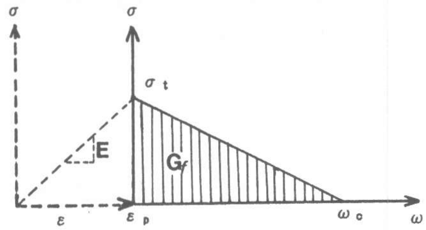

Fig.1 Linear $\sigma-\varepsilon$ and $\sigma-\omega$ curves

$$\varepsilon_p = \sigma_t/E \quad (1)$$
$$G_f = G_c = K_{IC}^2/E \quad (2)$$
$$\omega_c = 2G_f/\sigma_t \quad (3)$$

with the K_{IC} values determined based on P_{AE} which corresponds to the load level at which both total AE counts and the CMOD values acceleratedly increase [3] . As described in Eq.(2), note that G_f is approximated by the fracture energy G_c which is obtained based on the fracture toughness of concrete K_{IC}.

3. EXPERIMENT

Two kinds of concrete were used in the experiment: AE(air entrained) and SFR (steel fiber reinforced) concrete. Steel fibers of 30mm length were mixed into the SFR specimens at 1% volume ratio. Vinsol 70 was added by 0.05% of the cement weight to both AE and SFR mixes. The mix proportions, the compressive strength σ_c and Young's modulus E are summarized in Table 1.

Table 1. Mix proportions and mechanical properties

Specimen Type	Unit Weight(kg/m³)				Compressive Strength σ_c(kgf/cm²)	Young's Modulous E x10⁵kgf/cm²
	Water	Cement	Sand	Gravel		
AE	169	375	695	1156	451	2.48
SFR	169	375	684	1138	529	2.66

These are the average results of three cylindrical specimens of 10cm diameter and 20cm height. Substituting σ_t and G_f into Eq.(3), the ω_c values were obtained. Shown in Table 2 are the constants of the softening curves.

Table 2. Constants of the softening curves

Specimen	σ_t(kgf/cm²)	G_f(kgf/m)	ω_c($\times10^{-2}$mm)
AE	38.6	1.26	0.653
SFR	50.7	2.48	0.977

Four-point bending tests were performed on a center-notched beam of dimensions 10 cmx10cmx40cm as shown in Fig. 2. The notch was made by inserting a plastic plate of 1mm thickness into the mould. Beams with 2cm, 3cm, 4cm and 5 cm notch depths were tested.

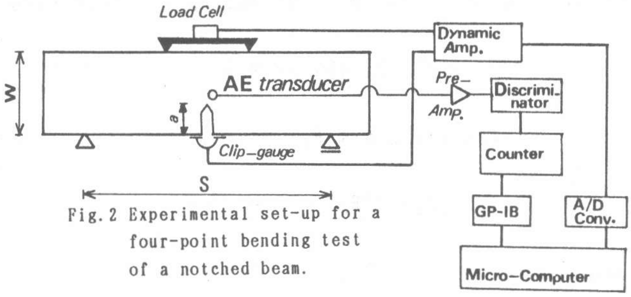

Fig. 2 Experimental set-up for a four-point bending test of a notched beam.

The load was applied through the load cell and the CMOD was measured by a clip gauge. The AE events were detected by an AE sensor attached at a location of 1cm from the notch tip. These events were monitored through a pre-amplifier, a discriminator and measured by a counter before being stored in a micro-computer. Upon reaching the maximum value, the load was reduced step by step and the Load-CMOD curve was obtained using a plotter.

4. ANALYTICAL MODEL

The analyses were performed using both BEM [4] and FEM [5]. In the case of BEM, the linear element was considered. The BEM and FEM models for the 4cm notched right-half portion of the beam are shown in Figs. 3 and 4, respectively, where the same mesh of discretization was adopted all along the ligament and notch. It was assumed that the cohesive forces which exist between the elements are acting only at nodal points. The crack propagates only when the stress at the crack tip is equal to σ_t. The analysis of the crack

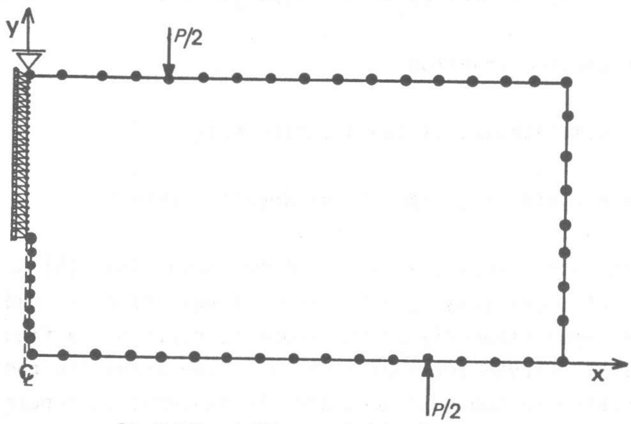

Fig. 3 BEM model (4cm notched beam)

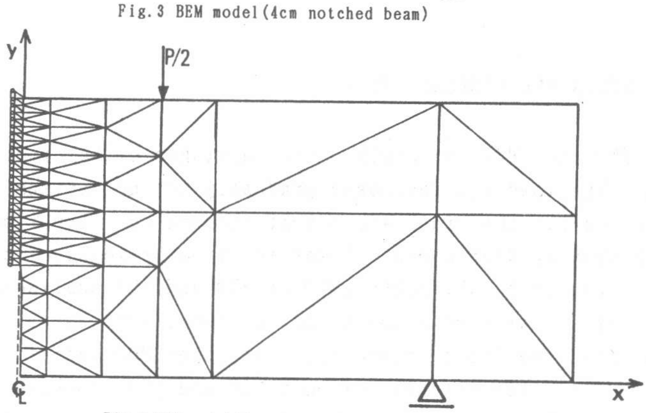

Fig. 4 FEM model (4cm notched beam)

propagation was done by considering the following system of three equations:

$$u_i - \sum_j AK_{ij}F_j - (BK_i)P = 0 \qquad (4)$$

$$(2\sigma_t/\omega_o)u_i + F_i = \sigma_t \qquad (5)$$

$$\sum_j CI_jF_j + (CR)P = -\sigma_t \qquad (6)$$

where

CR = cohesive traction at the crack tip due to a unit applied load

BK_i = displacement at the departed node i due to a unit applied load

CI_j = cohesive traction at the crack tip due to a unit cohesive force applied at the departed nodes j successively

AK_{ij} = displacement at the departed node i due to a unit cohesive force applied at the departed nodes j successively

P = applied traction

u_i= displacement at the departed node i

F_i= cohesive traction at the departed node i

At each step, the solution of equations (4), (5) and (6) after a sufficient number of iterations yields the values of P, u_i and F_i. Then P and F_i are applied simultaneously to the model to perform the final analysis of tractions and displacements for that step. Next the crack tip node is departed, the crack is advanced to the next node and the analysis is repeated.

5. RESULTS AND DISCUSSIONS

The load P/2 was applied at a distance of 5cm from the center-line of the beam. The load was increased gradually to the maximum after which the load was decreased but the crack width kept increasing. Experimentally, the CMOD(crack mouth opening displacement) was measured by means of a clip gauge attached to the notch mouth, while the CMOD by FEM and BEM analyses was obtained as double the horizontal displacement of the lower left corner node of the half-beam model analyzed due to symmetry. The Load-CMOD variation was analyzed for 2cm, 3cm, 4cm and 5cm notches of both AE and SFR specimens. In both FEM and BEM analyses, the crack propagated along the center-line of the beam by 2.5mm every step to make good comparisons between the experimental and analytical results.

The Load-CMOD variation for 2cm notched beams of AE and SFR concretes are shown in Figs.5 and 6, respectively. In the case of 2cm and 3cm notched beams, the analytical results obtained by FEM and BEM are pretty close up to the peak load after which the load obtained by FEM decreases more rapidly than that obtained by BEM for the same values of CMOD.

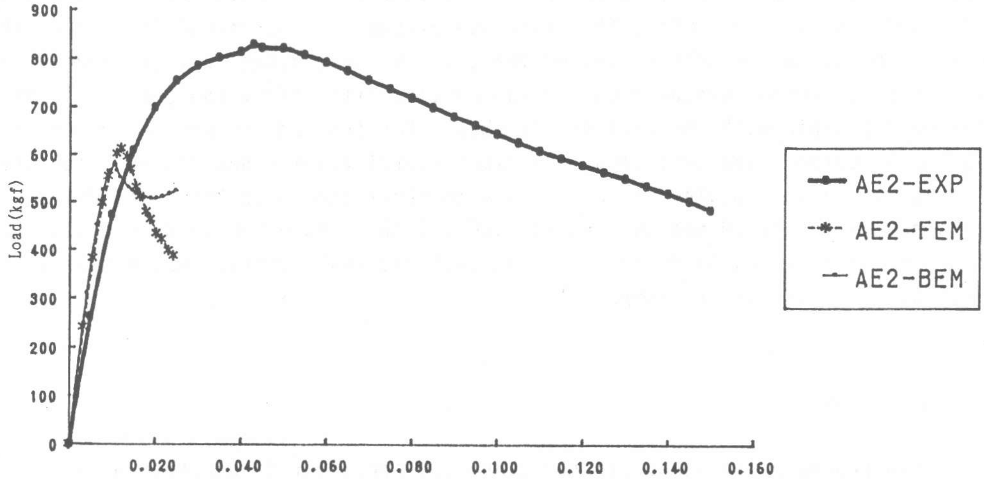

Fig.5 Load-CMOD variation (2cm notched beam-AE)

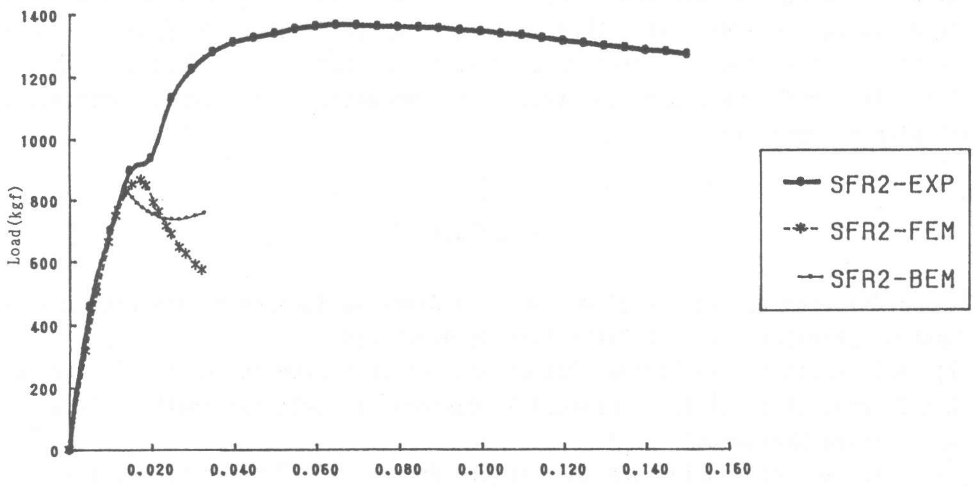

Fig.6 Load-CMOD variation (2cm notched beam-SFR)

As can be seen from Figs. 5 and 6, the analytical results are in good agreement with the experimental results up to the maximum load obtained by the analysis. The difference between the experimental and analytical results after that point may be attributed to the fact that the actual softening curve of concrete is nonlinear in contrast to the linear softening model adopted here. This may also explain the difference in the maximum loads obtained by the analysis and the experiment. It may be also due to the fact that AE(Acoustic Emission) is active only in the area where it is applied. After the maximum load obtained by the analysis, the predicted loads corresponding to bigger CMODs are lower than those obtained by the experiment. The analysis, i. e, the departing of nodes, was stopped after the crack has propagated by around 3cm from the notch tip along the center-line of the beam, because going further causes the analysis to become unstable due to the nonlinearity effect and the load starts increasing again with the increase in CMOD. For 4cm and 5cm notched beams, the agreement between the analytical and experimental results was not satisfactory.

The G_f values obtained from Eq. (2) were about one tenth the usual G_f values obtained by the RILEM method. We carried out the simulation using G_f values 10 times those computed by Eq. (2), but the analysis was unstable and the obtained results were not satisfactory.

6. CONCLUSION

This research was aimed at predicting the behaviour of concrete beams under crack propagation up to the maximum (peak) load assuming linear $\sigma-\omega$ softening curve and linear elastic $\sigma-\varepsilon$ curve. Determination of the peak load is not the goal of this paper. The next step in this research is to combine both the tensile softening model and LEFM (Linear Elastic Fracture Mechanics) in an attempt at analyzing the nonlinear behavior of concrete. This nonlinearity effect is very important in trying to understand the crack propagation mechanism of concrete.

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