論 文

[2171] MODIFIED MICROPLANE MODEL FOR CONCRETE STRESS -STRAIN RELATIONSHIP

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ABSTRACT

The microscopic behavior of concrete is investigated. Concrete is idealized to have plural types of particles, aggregate and mortar, in contrast to the other research works in which concrete was assumed to have single type of particles. The main objective of this paper is to study the nonuniform strain distribution along all microplanes and the nonhomogeneity of the microstructure. The results are compared with the available macroscopic data to obtain the range of the values for the parameters corresponding to the nonuniform of strains and the nonhomogenous microstructure.

1. INTRODUCTION

During the last two decades, a tremendous progress in the development of constitutive models has been made to investigate the characteristics of concrete. The macroscopic models, are mostly used. Despite of the initial significant success, these models, however, have gradually shown some limitations. Also, Micromechanical models have been also analyzed for different kinds of materials. In the microplane model proposed by Bazant(2), concrete was assumed to have one kind of microplane which exists at the contact surfaces between aggregates. In that model, strain distribution was assumed to be uniform and the microstructure was assumed to be homogenous. In the author's previous work(3), although two kinds of microplanes were considered (i.e. the contact surfaces between aggregates together and the contact surfaces between aggregate and mortar), the strain distribution was assumed to be uniform and the microstructure was assumed to be homogenous. In the present study, both the nonuniformity of strain distribution and the nonhomogeneity of the microstructure are considered.

2. THEORETICAL CONSIDERATIONS

2.1 AVERAGE STRESS TENSOR FROM AVERAGES OF CONTACT FORCES

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If in a region v there is a stress state σ_{ij} which is in equilibrium but otherwise may be arbitrarily distributed, the average stress $\bar{\sigma}_{ij}$ is defined[1] in Eq.(1..a). Using the divergence theorem and the equilibrium condition (i.e. $\sigma_{ij,j} = 0$), the volume average integral in Eq.(1..a) can be written in the form of Eq.(1..b).

$$\bar{\sigma}_{ij} = \frac{1}{v}\int_v \sigma_{ij} dv ...(1..a) \qquad \bar{\sigma}_{ij} = \frac{1}{v}\sum_{m=1}^{n} f_i^m x_j^m ...(1..b) \tag{1}$$

where n is the number of contacts per particle, both x_j and f_i are the contact vector and the contact force at mth contact point, respectively as shown in Fig.1. Using the definition in Eq.(1) for the idealized particles in concrete (see Fig.2) and by using the average volume again, the mean stress on any volume can be defined by summing the stresses on all particles within the volume as follows:

$$\bar{\sigma}_{ij} = \frac{1}{V^A}\left[\sum_{C_1} f_i^{c1} l_j^{c1} + \sum_{C_2} f_i^{c2} l_j^{c1}\right] + \frac{1}{V^M}\sum_{c_2} f_i^{c2} l_j^{c2} \tag{2}$$

where C_1 and C_2 are the total number of contacts between aggregates alone and between aggregate and mortar particles respectively, V^A and V^M are the total volume of aggregate and mortar particles, respectively. In addition, To consider the relative frequency of contacts with different orientations, a function $E(\theta)$ is defined such that $CE(\theta)\Delta(\theta)$ is the number of contacts with normals between θ and $\theta + \Delta\theta$. This function satisfies that $\int_0^{2\pi} E(\theta)d\theta = 1$. If the contacts are grouped within a finite number of orientational intervals, then, the grouped averages $\bar{fl}(\theta)$ in Eq.(2) can be calculated. Since $\bar{l}_j(\theta) = \bar{l}n_j(\theta)$, and considering that $\bar{l}_1$ and $\bar{l}_2$ are the averages of the radii of aggregates and mortar particles respectively, for a large number of contacts and very small orientational intervals $\Delta\theta$, Eq.(2) can be rewritten in the form:

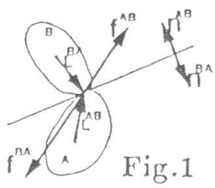

Fig.1
Contact Force, Contact Normal and Contact Vector

$$\bar{\sigma}_{ij} = \frac{C_1\bar{l}_1}{V^A}\int_0^{2\pi} E(\theta)\bar{f}_i^{c1}(\theta)n_j(\theta)d\theta + \frac{C_2\bar{l}_1}{V^A}\int_0^{2\pi} E(\theta)\bar{f}_i^{c2}(\theta)n_j(\theta)d\theta + \frac{C_2\bar{l}_2}{V^M}\int_0^{2\pi} E(\theta)\bar{f}_i^{c2}(\theta)n_j(\theta)d\theta \tag{3}$$

The orientational distribution function $E(\theta)$ of the microplanes is assumed to be as follows

$$E(\theta) = \frac{1}{2\pi}[1 + Acos^2\theta - Asin^2\theta] \tag{4}$$

where A is a parameter whose range will be discussed later

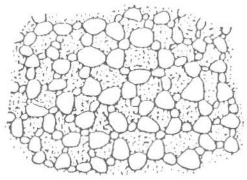

Fig.2
Idealized Distribution of Particles in Concrete

2.2 AVERAGE STRAIN AND THE NONUNIFORM STRAIN DISTRIBUTION

In a similar manner to the average stress, the average strain is defined as follows:

$$\bar{\varepsilon}_{ij} = \frac{1}{v}\int_v \varepsilon_{ij} dv \tag{5}$$

where ε_{ij} is the state of strain in the compatible condition. Using this definition for aggregate and mortar particles and by using the average volume, the average strain of aggregate and mortar systems are:

$$\bar{\varepsilon}_{ij}^{AA} = \frac{1}{V^A}\sum_a \bar{\varepsilon}_{ij}^{a} v^a \ , \qquad \bar{\varepsilon}_{ij}^{MM} = \frac{1}{V^M}\sum_m \bar{\varepsilon}_{ij}^{m} v^m \tag{6}$$

where V^A and V^M are the total volume of aggregate and mortar particles, respectively. Also by using the average volume, the total average strain ($\bar{\varepsilon}_{ij}$) and the average strain for aggregate-mortar system ($\bar{\varepsilon}_{ij}^{AM}$) are obtained to be as follow:

$$\bar{\varepsilon}_{ij} = \bar{\varepsilon}_{ij}^{AM} = \frac{1}{V}[\bar{\varepsilon}_{ij}^{AA} V^A + \bar{\varepsilon}_{ij}^{MM} V^M] \tag{7}$$

In addition, the relation between the average strain of aggregate system ($\bar{\varepsilon}_{ij}^{AA}$) and the average strain of aggregate-mortar system ($\bar{\varepsilon}_{ij}^{AM}$) will be assumed to be as follows:

$$\bar{\varepsilon}_{ij}^{AA} = K \ \ \bar{\varepsilon}_{ij}^{AM} \tag{8}$$

where K is a parameter whose range will also be discussed later.

2.3 RELATION BETWEEN AVERAGE CONTACT FORCES AND STRAIN TENSOR

Neglecting the possible rotation between particles , the average contact force is linked with the contact displacement using the linear contact law as follow:

$$f_n^c = k_n(\frac{\Delta l_n^c}{l}) \ , \qquad f_s^c = k_s(\frac{\Delta l_t^c}{l}) \tag{9}$$

where $(\frac{\Delta l_n^c}{l})$ and $(\frac{\Delta l_t^c}{l})$ are the relative normal and tangential displacements at the contact, k_n and k_s refer to the normal and shear stiffnesses of contact, f_n and f_s are the components of contact force and l is the distance between particle centers in contact. Eq.(3) for the average stress tensor contains only the averages of forces of same orientations. If $\bar{\delta}_n^c(\theta)$ and $\bar{\delta}_t^c(\theta)$ are the average normal and shear displacements of groups of contacts having the same orientations, Eq.(9) can take the form:

$$\bar{f}_n^c(\theta) = k_n \bar{\delta}_n^c(\theta) \ , \qquad \bar{f}_s^c(\theta) = k_s \bar{\delta}_t^c(\theta) \tag{10}$$

Here, the normal and shear microstrain ($\varepsilon_n, \varepsilon_t$), which govern the progressive cracking and failure of microstructure, are assumed to be equal to the resolved components of macroscopic strains. It can be expected that the averaged normal and shear displacements of groups of contacts with similar orientations can be written as follow:

$$\bar{\delta}_n^{aa}(\theta) = \frac{\Delta \bar{l}_n^{aa}(\theta)}{l} = \bar{\varepsilon}_{ij}^{AA} n_i n_j \ , \qquad \bar{\delta}_t^{aa}(\theta) = \frac{\Delta \bar{l}_t^{aa}(\theta)}{l} = \bar{\varepsilon}_{ij}^{AA} t_i nj \tag{11}$$

$$\bar{\delta}_n^{am}(\theta) = \frac{\Delta \bar{l}_n^{am}(\theta)}{l} = \bar{\varepsilon}_{ij}^{AM} n_i n_j \ , \qquad \bar{\delta}_t^{am}(\theta) = \frac{\Delta \bar{l}_t^{am}(\theta)}{l} = \bar{\varepsilon}_{ij}^{AM} t_i n_j \tag{12}$$

where n and t are the direction cosines of the unit normal and unit tangent of the microplane (i.e. $n = (cos\theta, sin\theta)$ and $t = (-sin\theta, cos\theta)$)

2.4 STRESS - STRAIN RELATIONSHIP

If the average normal and tangential contact forces in Eq.(10) are combined with Eqs.(11) and (12) and the resulting value is introduced into Eq.(3) and considering the results of Eqs.(7), and (8), the following incremental stress strain relationship can be obtained:

$$d\bar{\sigma}_{ij} = D_{ijkl} \quad d\bar{\varepsilon}_{kl} \tag{13}$$

where $D_{ijkl} = \eta_1 \int_0^{2\pi} (k_n a_{ijkl} + k_s b_{ijkl})^{c_1} \bar{E}(\theta) k d\theta + \eta_2 \int_0^{2\pi} (k_n a_{ijkl} + k_s b_{ijkl})^{c_2} \bar{E}(\theta) d\theta$

$a_{ijkl} = n_i n_j n_k n_l$, $\quad b_{ijkl} = t_i n_j t_k n_l$

$\eta_1 = C_1 \bar{l}_1 / 2\pi V^A$, $\quad \eta_2 = C_2 \bar{l}_1 / 2\pi V^A + C_2 \bar{l}_2 / 2\pi V^M$

$\bar{E}(\theta) = 2\pi E(\theta) = 1 + A cos^2\theta - A sin^2\theta$

c_1 refers to the contact between aggregate-aggregate, while c_2 refers to the contact between aggregate-mortar particles, respectively.

2.5 NORMAL and SHEAR STIFFNESSES OF THE MICROPLANE (i.e. k_n and k_s)

Normal and shear stiffnesses of the microplanes are taken as follow:

$$k_n = \frac{d\sigma_n}{d\varepsilon_n} , \qquad k_s = \frac{d\tau_{nt}}{d\varepsilon_{nt}} \tag{14}$$

where σ_n and ε_n are the normal stress and the normal strain on the microplane, τ_{nt} and ε_{nt} are the shear stress and the shear strain on the microplane. Here, we made an assumption that k_s is considered to be linear with k_n (i.e. $k_s = \lambda k_n$)

2.6 NORMAL STRESS AND NORMAL STRAIN RELATIONSHIP ON THE MICROPLANE

Since the normal microstress and the normal microstrain relationship governs the progressive cracking on the microplane, and since our aim is to describe the damage of the microstructure, at which σ_n reduces to zero, the following expressions are assumed if $\varepsilon_n \geq 0$:

$$\sigma_n^{aa} = E_1 \varepsilon_n^{aa} e^{-k_t(\varepsilon_n^{aa})^p} , \qquad \sigma_n^{am} = E_2 \varepsilon_n^{am} e^{-k_t(\varepsilon_n^{am})^p} \tag{15}$$

and for $\varepsilon_n \leq 0$

$$\sigma_n^{aa} = E_1 \varepsilon_n^{aa} e^{k_c(\varepsilon_n^{aa})^{p_1}} , \qquad \sigma_n^{am} = E_2 \varepsilon_n^{am} e^{k_c(\varepsilon_n^{am})^{p_1}} \tag{16}$$

where E_1, E_2, k_t, k_n, p, and p_1 are positive constants.

3. POISSON'S RATIO AND THE INITIAL MACROSTIFFNESS AND MICROSTIFFNESS RELATIONSHIP

To check the value of Poisson's ratio, the case of uniaxial strain is considered (i.e. $\bar{\varepsilon}_x = 0$ and $\bar{\varepsilon}_y \neq 0$). Assuming that for small strains $\sigma_n^{aa} = E_1 \varepsilon_n^{aa}$ and $\sigma_n^{am} = E_2 \varepsilon_n^{am}$, using Eqs.(13), and (14), the following results can be obtained:

$$\nu = \frac{1-\lambda}{4-2A} , \qquad E = \frac{\pi}{4}(3 + \lambda - 2A)(\eta_1 E_1 K + \eta_2 E_2) \tag{17}$$

Since the elastic Poisson's ratio of concrete is usually taken around 0.20, from Eq.(17) it can be easily found that $\lambda = 0.20 + 0.40A$.

4. NUMERICAL IMPLEMENTATION AND COMPARISON OF TEST DATA

4.1 UNIAXIAL TENSION

The values of the parameters taken in the previous work(3) (i.e. p, p_1, k_c, and k_t) are used here. The values are $p = 1.5$, $p_1 = 2.0$, $k_c = 20.0 * 10^4$, and the values of $10^{-4}k_t$ for the concrete shown in Fig.3 from a to d are 29.5, 34.7, 24.0, and 13.3 respectively. Also, the values of $10^{-4}E$ (E is the initial macroscopic stiffness) are 15.5, 16.5, 17.5, and 15.25 kgf/cm^2 . Fig.3 shows the effect of the parameters corresponding to the nonhomogeneity of microstructure (i.e. parameter A) and the nonuniformity of strain distribution (i.e. parameter K).

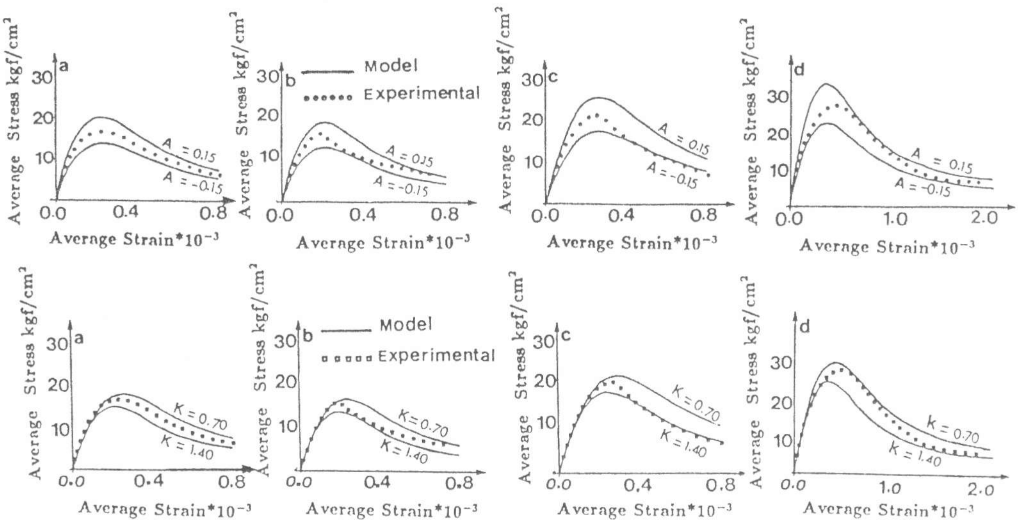

Fig.3 Effect of A and K Parameters (Uniaxial Tension)

4.2 UNIAXIAL COMPRESSION

The values of the parameters used are $p = 1.50$, $p_1 = 2.0$, $k_t = 26.0 * 10^4$ and the values of $10^{-4}k_c$ for concrete shown in Fig.4 from g to j are 18.8, 22.2, 18.8, and 19.6 and the values of $10^{-4}E$ (E is the initial macroscopic stiffness) are 20.65, 32.8, 29.5, and 50.2 kgf/cm^2.

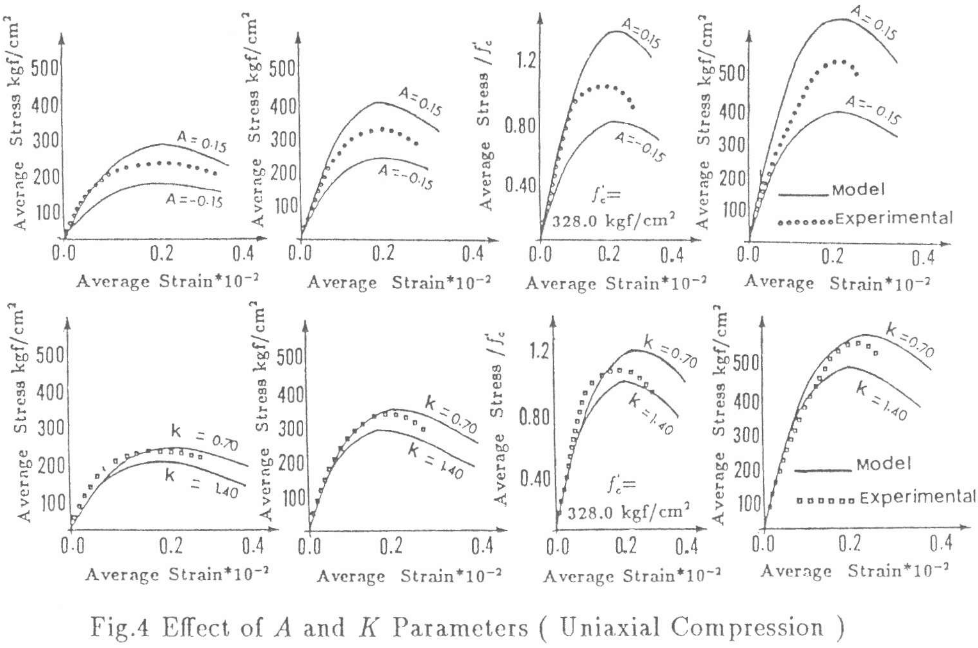

Fig.4 Effect of A and K Parameters (Uniaxial Compression)

4.3 BIAXIAL LOADING

The values of $p = 1.5$, $p_1 = 2.0$, $k_c = 13.8 * 10^4$, $k_t = 24.5 * 10^4$, and the initial macroscopic stiffness $E = 29.20 * 10^4$ are used. Fig.5 shows the effect of A and K parameters within the selected ranges.

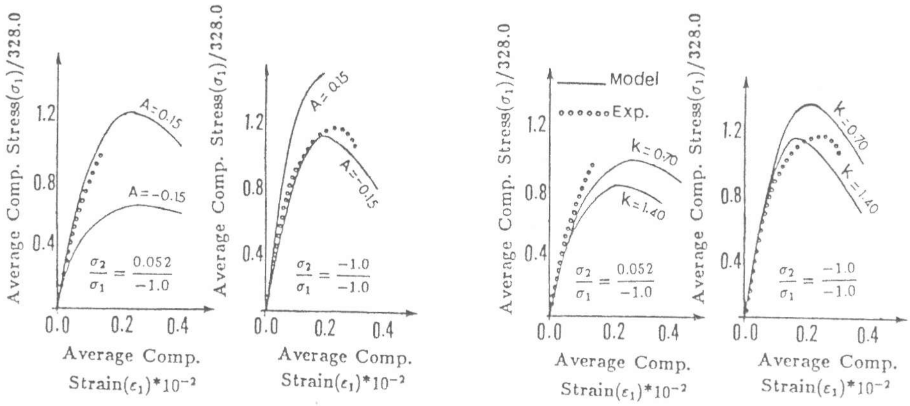

Fig.5 Effect of A and K Parameters (Biaxial Loading)

5. CONCLUSIONS

Both the nonhomogeneity of microstructure and the nonuniformity of strain distribution are investigated. Two parameters are considered, parameter A in Eq.(4) to study the nonhomogeneity of microstructure and parameter K in Eq.(8) to study the nonuniformity of strain distribution. It can be concluded as follows :

1. The macroscopic elastic Poisson's ratio is found to be influenced by both the ratio between normal and shear stiffnesses of microplane and the parameter corresponding to the nonhomogeneity of the microstructure (i.e. parameter A).

2. Through the proposed model, the relation between the initial macroscopic and microscopic stiffnesses is obtained. This relation is influenced by both parameters A and K.

3. A range from -0.15 to 0.15 is good enough for the parameter A, while a range from 0.70 to 1.40 is reasonable for the parameter K.

4. From the results, both the nonhomogeneity of the microstructure and the nonuniformity of strain distribution strongly exist.

REFERENCES

1) Bathurst, R.J. and Rothenburg, L.," Micromechanical Aspects of Isotropic Granular Assemblies with Linear Contact Interaction," Journal of Applied Mechanics, ASME, Vol.55, March,1988, pp.17-23.

2) Bazant, Z.P. and Oh, A. M.," Microplane Model for Progressive Fracture of Concrete and Rock ," Journal of Engineering Mechanics, ASCE, Vol.111, No.4, April, 1985, pp.559-582.

3) Farahat, A. M., Wu, Z. S., and Tanabe, T.," Development of A Microplane Model of Concrete with Plural Types of Granular Particles," JSCE, submitted.