論 文

[2176] Constitutive Equation and Yield Condition of Concrete Loaded at Early Age

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1. INTRODUCTION

Hardened concrete properties are already well investigated. A great amount of concrete research has, to a great extent, been devoted to the properties of the fresh concrete as well. On the contrary, the intermediate stage, when the newly placed concrete has just started to solidify and a certain strength can be developed, has not been examined thoroughly. Concrete is, however, very often exposed to loads at very early ages in the construction processes. Therefore, it is important to investigate the mechanical properties of concrete at early ages. For early age concrete, age or time is one major factor. Time may be, however, definitely not the best parameter when trying to define early age. It is probably more correct to define early age by a degree of hydration or a level to which some property such as strength has developed.

In the present study, a material model is proposed in aim to express the behavior of concrete in the early stage. In the early stage, the behavior of concrete is rather viscous than elastic. Therefore, for the case of material modeling, the viscous as well as elastic effect is to be taken into consideration for a lower stress level. At a higher stress level, the young concrete gradually approaches to a yield value, after which the plastic property comes into effect along with the viscous effect. To accomodate that yielding behavior, a yield criterion must be set up. Therefore, the total behavior of the young concrete consists of three parts. Two (elastic and viscous) parts are active below a threshold value and all the three (elastic, viscous and plastic) parts are to be included above the threshold in the formation of the constitutive matrix. In the following text, the constitutive law and a yield condition applicable to express continuously the behaviors of both viscoelastic and viscoplastic states are proposed. In this analysis, concrete is considered homogeneous.

2. ESTIMATION OF THE MATERIAL CONSTANTS

For the case of early age concrete, almost all the material properties, e.g., the modulus of elasticity, E , the compressive strength, f'_c, the cohesion, c , are dependent on

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the age of concrete [1]. The quantitative and comprehensive relationships of the properties of early age concrete with the age effect are scarce. In this study, the constants are estimated from the test data by Byfors [2]. Early age concrete can not be tested before age of 3-6 hours in general and hence the properties are calculated from the data after that age. From the plots of compressive strength vs. time and elastic modulus vs. compressive strength, the elastic constants are estimated and these material constants have been expressed for various range of ages, t_a, and are shown in Table 1. For the constants of the rheological part of the constitutive equation, the values are estimated from the plots of rheological constants against age by Okamoto [3].

Table 1: Age-Dependence of Material Properties

1. Compressive Strength f'_c (Kgf/cm^2)	
$f'_c = 0.0$	$t_a < 4$ hours
$f'_c = 11.2\ Log_2 t_a - 20.4$	$4 \leq t_a < 8$ hours
$f'_c = 112.1\ Log_2 t_a - 326.2$	$8 \leq t_a < 16$ hours
$f'_c = 209.1\ Log_2 t_a - 713.6$	$16 \leq t_a < 24$ hours
$f'_c = 61.7\ Log_2 t_a - 38.4$	$t_a \geq 24$ hours
2. Modulus of Elasticity E (Kgf/cm^2)	
$Log_{10}E = 0.288\ t_a + 0.088$	$t_a < 2$ hours
$Log_{10}E = 0.500\ t_a - 0.033$	$2 \leq t_a < 4$ hours
$Log_{10}E = 30.203\ f'_c + 0.905$	$1< f'_c \leq 10 Kgf/cm^2$
$Log_{10}E = 0.792\ f'_c + 36.493$	$10< f'_c \leq 100 Kgf/cm^2$
$Log_{10}E = 0.034\ f'_c + 44.138$	$f'_c > 100 Kgf/cm^2$
3. Cohesion c (Kgf/cm^2)	
$Log_{10}c = 0.43$	$t_a \leq 10$ hours
$Log_{10}c = 0.0625\ t_a - 0.32$	$10< t_a \leq 24$ hours
4. Rheological Constants E_1 (Kgf/cm^2), η $(Kgf.s/cm^2)$	
For $t_a \leq 3.0$ hours	For $t_a \geq 3.0$ hours
$Log_{10}E_1 = 0.644 t_a - 0.111$	$Log_{10}E_1 = 0.355 t_a + 0.755$
$Log_{10}\eta_1 = 0.889 t_a + 1.111$	$Log_{10}\eta_1 = 0.155 t_a + 3.311$
$Log_{10}\eta_0 = 0.783 t_a + 3.739$	$Log_{10}\eta_0 = 0.108 t_a + 5.760$

Note : t_a stands for the age of concrete.

3. MATERIAL MODELING

In an effort to model the elasto-viscoplastic material characteristics of the concrete at an early age, the total strain is seperated into the elastic strain represented by a linear elastic compliance, the viscous or time-dependent strain represented by a rheological model and the plastic strain characterized by a strain rate which is zero when stresses are below a certain threshold (yield) value and exhibits a finite strain rate only when the threshold is exceeded. Therefore, the total strain rate can generally be defined as a combination of these three parts as follows.

$$\dot{\epsilon}_{ij} = \dot{\epsilon}^e_{ij} + \dot{\epsilon}^v_{ij} + \dot{\epsilon}^p_{ij} \tag{1}$$

where $\epsilon_{ij}, \epsilon_{ij}^e, \epsilon_{ij}^v, \epsilon_{ij}^p$ respectively stand for the total strain, the elastic strain, the viscous strain and the plastic strain. The superimposed dot indicates the time derivative.

For the constitutive equation of the early age concrete in the low stress range, a viscoelastic model can be used to calculate the time-dependent strain component. For this purpose, the four-element viscoelastic model is used in this study. It is a series combination of a Maxwell model and a Voigt model as shown schematically in Fig.1 and the strain is devided into the instantaneous strain and the viscous strain. Then, the relationship between stress and strain is given by the following equation.

$$\dot{\sigma}_{ij} = D_{ijkl}(\dot{\epsilon}_{kl}^e + \dot{\epsilon}_{kl}^v) \tag{2}$$

Fig.1 Schematic Representation of Material Model

where D_{ijkl} is the viscoelastic tangential modulus. The elastic and the viscous portions of the strain are expressed respectively as

$$\epsilon_{ij}^e = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \tag{3}$$

$$\epsilon_{ij}^v = \frac{\sigma_{ij}}{E_1}\left[1 - \exp\left\{-\frac{E_1}{\eta_1}t\right\}\right] + \frac{\sigma_{ij}}{\eta_0}t \tag{4}$$

where

E : Instantaneous modulus of elasticity
ν : Poisson's ratio
δ_{ij} : Kronecker's delta
E_1 : Delayed modulus of elasticity
η_0 : Relaxation coefficient of viscosity
η_1 : Delayed coefficient of viscosity
t : Time elapsed after loading

For defining the plastic part of strain, let $F(\sigma)$ be the yield function with $F < 0$ denoting the viscoelastic region. To define the relationship between the stress and the strain using the classical plasticity theory, the idea of a plastic potential defined as $Q(\sigma)$ is used assuming that the rate of plastic strain vector is normal to the surface of the plastic potential. Then, for a nonzero plastic strain rate in the high stress range, we can get the following relationship.

$$\dot{\epsilon}_{ij}^p = \lambda\frac{\partial Q}{\partial \sigma_{ij}} \tag{5}$$

where λ is related to the magnitude of the plastic work, and is a scalar factor of proportionality which must always be positive when plastic flow occurs in order to assure the irreversible nature of plastic deformation. Now, assuming the associated flow rule, $Q \equiv F$, the plastic strain rate is given as follows.
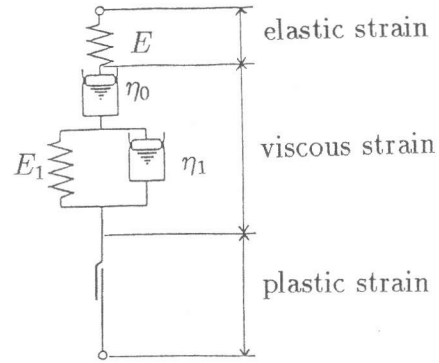
$$\dot{\epsilon}_{ij}^{p} = \lambda \frac{\partial F}{\partial \sigma_{ij}} \tag{6}$$

Then, the incremental stress-strain relationship in the plastic range becomes as follows.

$$\dot{\sigma}_{ij} = D_{ijkl} \left(\dot{\epsilon}_{kl} - \lambda \frac{\partial F}{\partial \sigma_{kl}} \right) \tag{7}$$

Now, for the plastic flow to continue, the state of stress must remain on the yield surface. Using the criterion for plastic loading, the following equation must hold.

$$\dot{f} = \frac{\partial F}{\partial \sigma_{ij}} \dot{\sigma}_{ij} = 0 \tag{8}$$

Solving for the proportionality constant λ, we can get the expression for λ as follows.

$$\lambda = \frac{\frac{\partial F}{\partial \sigma_{ij}} D_{ijkl}}{\frac{\partial F}{\partial \sigma_{mn}} D_{mnpq} \frac{\partial F}{\partial \sigma_{pq}}} \dot{\epsilon}_{kl} \tag{9}$$

Substituting Eq.(9) into Eq.(7), the following incremental stress-strain relationship is obtained.

$$\dot{\sigma}_{ij} = \left[D_{ijkl} - \frac{D_{ijtu} \frac{\partial F}{\partial \sigma_{rs}} \frac{\partial F}{\partial \sigma_{tu}} D_{rskl}}{\frac{\partial F}{\partial \sigma_{mn}} D_{mnpq} \frac{\partial F}{\partial \sigma_{pq}}} \right] \dot{\epsilon}_{kl} \tag{10}$$

Eq.(10) can be written in a convenient form as

$$\dot{\sigma}_{ij} = (D_{ijkl} - S_{ijkl}) \dot{\epsilon}_{kl} \tag{11}$$

where

$$S_{ijkl} = \frac{D_{ijtu} \frac{\partial F}{\partial \sigma_{rs}} \frac{\partial F}{\partial \sigma_{tu}} D_{rskl}}{\frac{\partial F}{\partial \sigma_{mn}} D_{mnpq} \frac{\partial F}{\partial \sigma_{pq}}} \tag{12}$$

For the yield condition, $F(\sigma) = 0$ a Drucker-Prager type of yield function is used because concrete is a pressure-dependent material. The yield function of this model is expressed as follows.

$$F(I_1, J_2) = \alpha I_1 + \sqrt{J_2} - k \tag{13}$$

where I_1, J_2 are the stress invariants, i.e., $I_1 = \sigma_{kk}$, $J_2 = s_{ij} s_{ij}/2$, $s_{ij} = \sigma_{ij} - \delta_{ij} I_1/3$, and α, k are the positive material constants. For the known values of the cohesion, c, and the angle of internal friction, ϕ, of concrete, we can relate these material constants to each other as follows [4].

$$\alpha = \frac{\tan \phi}{\sqrt{9 + 12 \tan^2 \phi}} \tag{14}$$

$$k = \frac{3c}{\sqrt{9 + 12 \tan^2 \phi}} \tag{15}$$

4. NUMERICAL SIMULATION AND DISCUSSION

As an example of numerical simulation, a two-dimensional plane stress case subjected to imposed longitudinal strain is analyzed. Here, the value of ϕ is taken as 35.0 degrees and is kept constant through the analysis [5]. The first loading is done at the age of 2 hours and the load is increased step by step at the interval of 30 minutes (cf. Fig.2). In other cases of this analysis, the ages of loading are 4 and 5 hrs. At every stage of loading, the stress is calculated and at that stress level the viscous strain is evaluated. As the load increases step by step, the state of material is checked whether it is elastic or plastic. For the higher stress level, the plastic strain comes into effect with the viscous strain. Here, a parametric study is done using the age of loading as an influencing factor. The stress-strain relationship (cf. Fig.3) shows that the initial increase of stress is very small for small values of the age at loading, t'. However, as the age of concrete at loading increases, the stress develops at a faster rate. This indicates that the stiffness of early age concrete increases at an increasing rate until concrete hardens sufficiently. In Fig.3, the stress is normalized by the maximum stress attained during the given strain history. On the dependency of viscous strain on the age of concrete (cf. Fig.4), it is found that the rate of increase of the strain is somehow constant at the early stage but decreases in the subsequent stage. After the onset of yielding, its rate of increase gradually diminishes to zero due to the hardening of concrete. Regarding the plastic behavior, the rate of increase of plastic strain (cf. Fig.5) takes a constant value after yielding for various ages at loading, even though the yielding takes place at different times. For volumetric strain (cf. Fig.6), the amount of compaction increases initially without any considerable amount of stress increase. However, as the stress approaches its maximum value, the rate gradually decreases to zero, and then, the volume starts to increase due to the plastic dilatancy. Also, the stress-time relationship varies for different loading ages (cf. Fig.7).

The present model is applicable to normal concrete at an age of between 2 and 24 hrs. The mechanical behavior within that age range can be expressed continuously. The material constants, however, changes sharply at some points during the solidification and hardening or hydration process. This fact is reflected in the results shown in Figs.3-7.
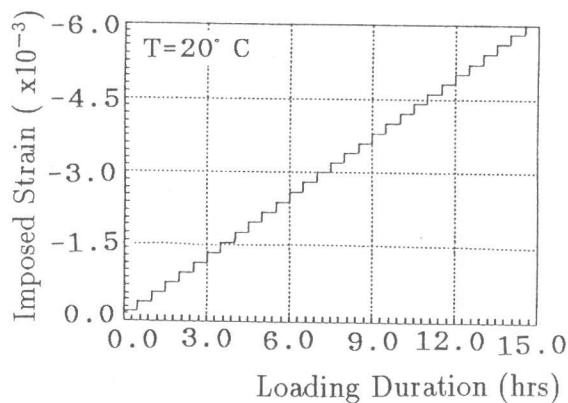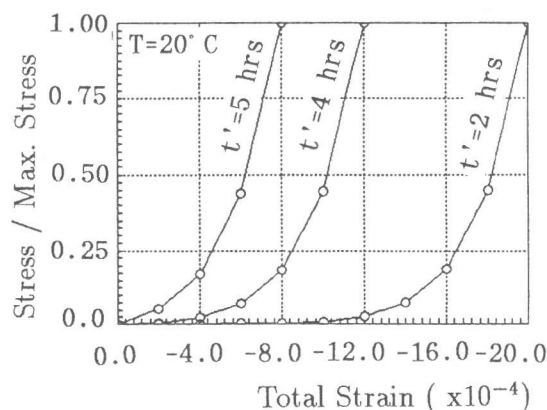

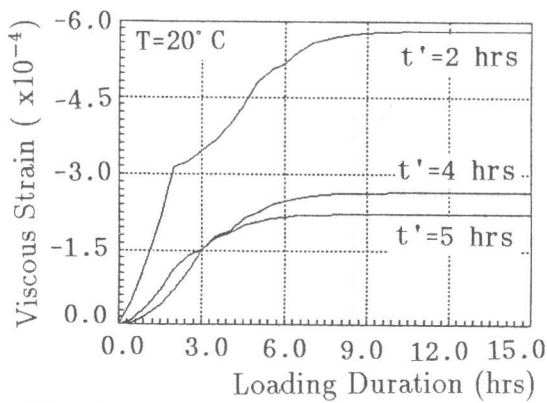Fig.2 Loading History

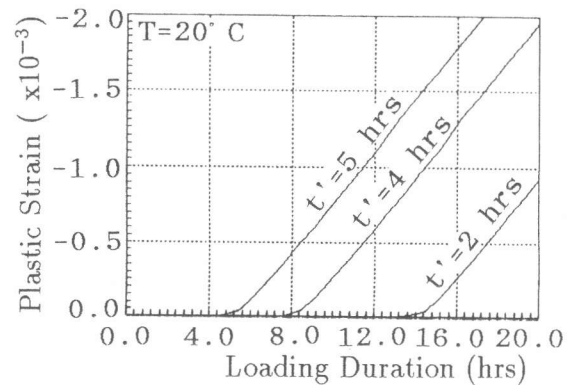Fig.3 Stress-Strain Relationship

Fig.4 Time-Variation of Viscous Strain
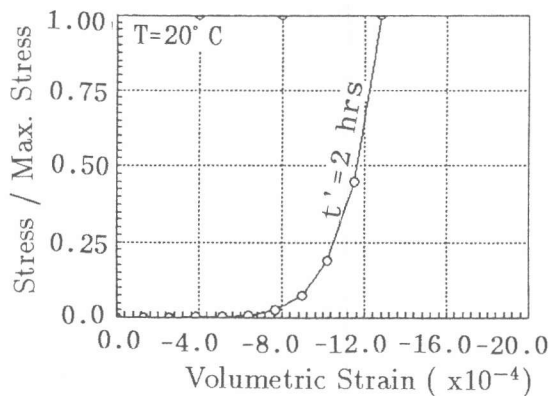

Fig.5 Time-Variation of Plastic Strain
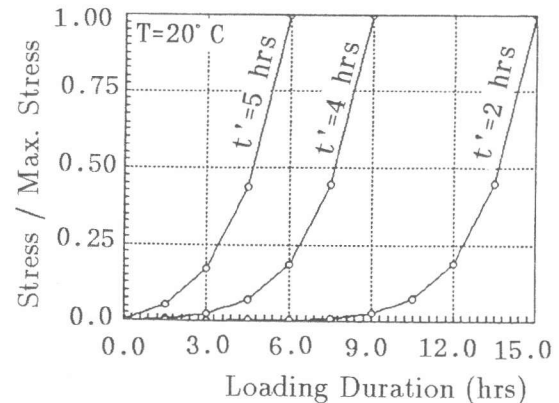

Fig.6 Stress-Volumetric Strain Relationship

Fig.7 Time-Variation of Stress

5. CONCLUSIONS

A constitutive relationship for early age concrete is modeled as a combination of viscoelastic and plastic material models. The age-dependence of the material constants are taken into account. Through the numerical simulation for an imposed strain history, the appropriateness of the present model is checked. The effect of other relevant parameters such as temperature and mix proportions can also be taken into consideration in this model.

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