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論 文

[2214] Macro Model for Confinement Effectiveness of Lateral Ties in Columns

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1. INTRODUCTION

It is widely accepted that the provision of lateral reinforcement plays a major role in enhancing the axial compressive behavior of the concrete core, which can be categorized in aspects of strength and ductility. The prediction of the afore mentioned capacity increase has received much attention in research specially during the past two decades. Several empirical or semi-analytical models have been proposed [1,5,8,9,10,11], based on experimental studies on scaled down confined concrete columns as well as nearly full size specimens. The reinforcement arrangements adopted in larger scale specimens are reflections of general detailing practices, whereas, the smaller scale specimens are with simpler arrangements restricted by size. Also in many of the researches the important issue of the effect of cover concrete has been addressed in several marginally different ways. Many of these recently produced models are based on a specific set of data in a specific range of detailing and are proved to produce very good predictions within these ranges. Saatcioglu et al.[9] has attempted a unified analytical model by fixing the coefficients on significant parameters based on results of several larger scale experimental studies and is proven to produce good correlations within that domain.

However it is seen that when these models are applied to test results of different scale as well as detailing, the predictions are not very convincing. The reason for this discrepancy could be attributed to improper portrayal of significant parameters in the analytical relations. Therefore a specific need arises for a method to identify the sensitivity of the governing parameters within their logical physical ranges. One such method is FEM analysis based on generalized micro-mechanical models for concrete, which was adopted in this study for the afore mentioned purpose.

Here, it is prudent to briefly mention the significant, material as well as geometrical and detailing parameters governing the confinement phenomena in laterally reinforced concrete compression members;

1. Amount of lateral reinforcement expressed in terms of the ratio of the volume of lateral reinforcement to the same of confined concrete core measured within center lines of the peripheral tie or hoop (p).
2. Longitudinal distribution of the confining action, expressed as the ratio of center to center spacing of lateral reinforcement to the least core dimension (s/d).
3. Shape of the confined core which is an indicator of the lateral distribution of the confining action.
4. Strength of the confining agent, which could be considered as the yield strength when a normal steel with a flat yield plateau is used (f_y).
5. Unconfined strength of concrete, in a specimen of the same geometry as the confined core (f_{co}).
6. Geometrical distribution of lateral reinforcement at a sectional level and the presence of longitudinal reinforcement (if any), which is also an indicator of the lateral distribution of confining action (C^*).
7. Flexural rigidity of the tie arms which once more is an indicator of the lateral distribution of confining action (F_r).

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In order to evaluate the confining action under above mentioned factors, arising simultaneously in most of the practical cases of columns, certain degree of de-coupling should be done with the aid of micro mechanical approach to arrive at the basic macro models for confinement action. Through this study it was attempted to identify the relation between seven main factors and the rate of applied straining.

2. CONCEPTUAL MODEL FOR CONFINEMENT

Though confinement pressures vary across and along a confined core, the average of these over the whole volume, which could be referred to as volumetrically averaged confinement is a quantity representive of the level of confinement. In terms of concrete lateral confining stresses in two orthogonal directions x and y (where z is parallel to core axis, fig. 3), spatially averaged lateral confinement can be expressed as in eq.1. Based on equilibrium between concrete and the confining agent, in terms of stress generated in the agent (σ_s), the same quantity can be given through eq.2. The values V_c and V_s are volume domains of concrete and steel respectively.

$$\sigma_v = \frac{1}{V_c}\int_{V_c}\frac{\sigma_{c,xx}+\sigma_{c,yy}}{2}dV \tag{1}$$

$$\sigma_v = -\frac{1}{2V_c}\int_{V_s}\sigma_s dV \tag{2}$$

This is an important relation since, through this the spatially averaged confinement is equated to the spatially averaged confining agent stress over its domain. Peak of this value,which reflects the maximum level of confinement, will be attained when all provided confining agent comes to yield, at which level eq.2 tends to eq.3. Based on this reasoning, ratio of actual confinement achieved, to the maximum denoted by eq.3 can be utilized to indicate the mechanically defined confinement effectiveness of a particular detailing as depicted by eq.4, which will be referred to as confinement effectiveness coefficient (α). Here, p is the volumetric lateral reinforcement ratio and f_y is the yield strength of confining agent.

$$\sigma_v = -\frac{1}{2}pf_y \tag{3}$$

$$\alpha = \frac{\sigma_v}{\frac{1}{2}pf_y} \tag{4}$$

Further, if the stress distribution of the confining agent can be experimentally evaluated, the spacial averaged confinement can be computed. It is crucial to predict confinement effectiveness represented by α when macro design equations for laterally confined RC columns are constructed. Through systematic parametric studies using micro models, eq.5 is proposed.

$$\alpha = \frac{1}{1+K^*+K(s/d)^3} \tag{5}$$

The value K^* is the governing factor when s/d becomes zero (ie. lateral casing), which represents lateral stress uniformity and material properties and will be termed "limit factor". The Value K, termed "uniformity factor", accounts for distribution of lateral stresses, when reinforcement is placed discretely. This factor is sensitive to the shape arrangement and flexural stiffness of lateral reinforcement. Derivations of these factors will be discussed through following sections.

3. MICRO APPROACH BASED ON NON-LINEAR FEM

Three dimensional non-linear finite element analysis was conducted to evaluate the sensitivity of the significant variables on the confinement effectiveness. The micro-mechanical constitutive equations and the development of the non-linear solving technique is reported elsewhere [3,4].

In idealization of the confined concrete members, circular and square sections were considered, with lateral reinforcement in the form of continuous casings as well as at discrete spacings. These covered the range from uniform confinement, to highly non-uniform confinement indicating non-uniform micro stress over the volume of the column. The confining agent (assumed to be steel) was idealized both as truss elements as well as beam elements. For the circular sections the idealization criteria has no influence, while for square sections it has a considerable bearing. Both these types of modelling were adopted since rectilinear lateral ties were assumed by many previous researchers to provide confinement through axial action only, the flexural contribution being negligible. It is logically assumed that the most non-uniformly confined case is a square section with steel only providing confinement at the four corners and placed discretely, while the most uniformly confined case is a circular section confined by a steel casing, in the domain of axi-symmetric sections. This reasoning was applied in establishing the relations for the expression of confinement effectiveness.

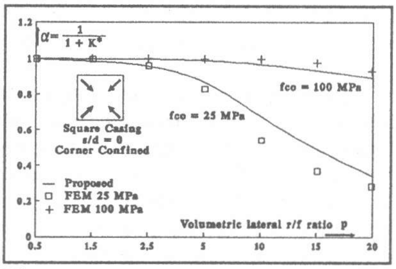

Figure 1 Variation of α with p and $\underline{f_{co}}$

The proposed model was qualitatively structured on the trends observed through the FEM analysis. Variation of confinement effectiveness coefficient (α) with lateral reinforcement ratio (p) for casing confined square core under different concrete strength levels are given through fig.1. The difficulty in experimentally obtaining the pure confinement action parted by a steel casing without the effect of longitudinal restraint parted by the casing was overcome by using FEM results in developing the relation, eq.6, for basic form of limit factor (K^*), with steel modelled as truss elements. Increase in the parameter K^* indicates a reduction in the confinement effectiveness. The variation of α with spacing ratio (s/d) under two extreme cases of circular and square sections with steel modelled as truss elements is depicted through fig. 2 against proposed relation, which indicates the sensitivity of the uniformity factor (K).

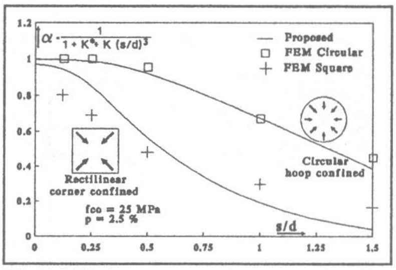

Figure 2 Variation of α with spacing ratio(s/d)

$$(K^*)_{sq.casing} = 3[100p/f_{co}]^2 \qquad (6)$$

It should be noted that in reality, for a given type of tie arrangement, the bar size (ϕ), s/d and p can not be independently varied. Therefore, only the coupled effect of flexural stiffness of the bar and spacing effect for a given amount of reinforcement or the coupled effect of flexural stiffness and amount of reinforcement at a given spacing can be seen.

4. EXPERIMENTAL STUDY

To understand the effects of amount of reinforcement, spacing, and the effect of flexural stiffness of the ties on confinement effectiveness, a series of experiments were conducted. The square specimens were provided with perfectly butt welded square lateral ties. Butt welding with nearly same type of filler and grinding down to original bar diameter at the weld was necessary to have homogeneous properties all round the tie and to have the tie in one plane. The outer dimensions of the ties were selected with 3mm reduction from the column lateral dimensions (clearance for strain gauges), resulting in a virtual elimination of cover. No longitudinal reinforcement was provided. All columns were horizontally cast to circumvent any weak zones at the top end and to generate moulded loading ends. Uniform concrete properties were obtained by the use of HPC (High Performance Concrete). Concentric monotonic loading was applied at a quasi-static rate of 2-5 $\mu\epsilon$/secon 200, 2000 ton universal testing machines beyond peak load capacity. The columns were instrumented for measurement of axial strains by "Π gauges" mounted on integrally cast steel rods and overall displacements of the machine plattens were measured by LVDTs as a cross check beyond peak. To evaluate the stress distribution in lateral steel, the central lateral tie was instrumented with 5mm strain gauges on inner and outer fibers along tie arms which was very necessary to obtain the stress distribution in the tie. All instruments were monitored through an A/D converter-scanner at 80 msec per channel. Fig. 3 illustrates instrumentation, setup and specifications for experiments.

The cover concrete was intentionally eliminated for the purpose of experimental verification for modelling. But, observations indicated spalling of surface layer concrete between ties in some cases at the ultimate. By observation of the spalling which was small, it was assumed that the core to be bounded by the center lines of ties. The results of experimental investigation given in condensed form through Table 1 was used in conjunction with FEM results to calibrate the model for confinement effectiveness coefficient (α). From these results in fig. 4, it was clearly identified that the flexural contribution of tie arms were considerable as the diameter increased.

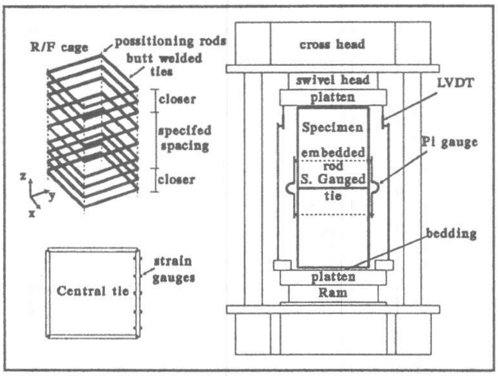

Figure 3 Experimental setup and instrumentation

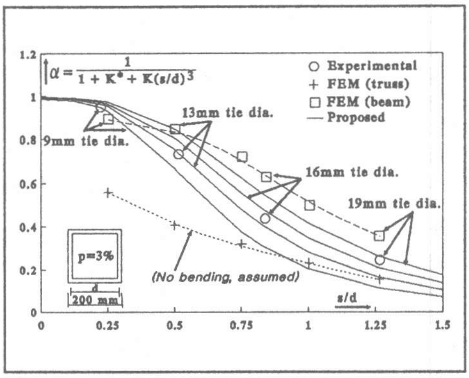

Figure 4 Experimental and FEM results of α

Table I Experimental, proposed model, results and specifications

Spec. No.	Size sq. mm	ϕ mm	s mm	s/d	p %	f_y MPa	f_{co} MPa	α Exp.	α Mod	f_{cc} Exp. MPa	f_{cc} Mod MPa
1	200	13.0	192	1.04	1.5	328	35.2	0.27	0.26	37.7	37.3
2	200	9.0	42	0.22	3.2	335	35.6	0.94	0.95	46.3	47.8
3	200	13.0	94	0.51	3.0	328	35.6	0.74	0.74	43.9	45.0
4	200	15.9	150	0.83	2.9	314	35.6	0.46	0.48	42.6	42.0
5	200	18.8	225	1.26	2.8	316	35.6	0.25	0.26	39.9	39.2
6	200	18.7	104	0.58	5.9	312	35.6	0.79	0.77	53.4	53.9
7	150	9.0	43	0.31	4.3	335	38.0	0.89	0.91	53.0	53.4
8	400	24.8	119	0.32	4.4	304	36.3	0.74	0.90	51.4	50.9

Since the confinement action is a combined effect of axial and flexural actions of ties, the ratio of elastic flexural stiffness to axial stiffness, (eq.7), was introduced as a term in the expressions of *K* and *K**. Since in circular sections effect of flexural stiffness becomes negligible, and hoop action dominates, a limiting value of F_r ($= 15$ *for* $\phi/d = 1/5$), is assumed to represent circular sections. This limiting value can be explained when a square hoop is considered, which span reduces such that it assumes a circular profile due to code specified minimum curvatures (r > = 2.5 x ϕ) at the corners. Through this reasoning, for simple square and circular ties the factors *K* and *K** are expressed in eq. 8 and eq. 9.

$$\frac{(av.flex.stiffness)}{(av.axial stiffness)} \propto [\phi/L]^2 \quad \Rightarrow \quad [F_r]_{simple} = (1+350[\phi/d]^2) \tag{7}$$

$$[K]_{simple} = \frac{7}{1+350[\phi/d]^2} \tag{8}$$

$$[K^*]_{simple} = \frac{3[100p/f_{co}]^2}{1+350[\phi/d]^2} \tag{9}$$

Here, ϕ is the diameter of tie bar and *L* is the span of a tie arm.

5. EXTENSION OF THE MODEL

The above described analytical form is applicable to simple layouts of ties. Better uniformity achieved when cross ties and or multiple hoops and longitudinal reinforcement is present can be easily incorporated into the model by modifying the uniformity factor K regarding microscopic stress distribution. The proposed modification factor is based on FEM analysis results and previously reported experimental results using more complex tie layouts and longitudinal reinforcement. This modification factor accounts for the lateral spacing(a) of longitudinal reinforcement or spacing(a) of corners of complex tie arrangements around the periphery of the member as a ratio a/t, where t is the significant core dimension parallel to a in eq 10 and illustrated in fig. 5. Also when complex tie arrangements are used, the evaluation of F_r should be done on a weighted average basis on all rectilinear tie arms (n). This is mathematically expressed through eq. 11. The final forms of the factors K and K^* is depicted through eq. 12. In these equations L_i is effective span of a tie arm, and 'ρ' is the longitudinal reinforcement ratio based on concrete core area.

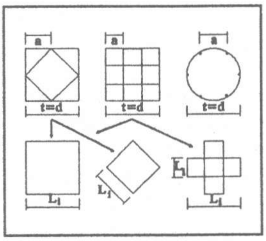

Figure 5 R/F configurations

$$C^* = \frac{1}{2}\frac{(1+a/t)}{(1+10\rho)} \tag{10}$$

$$F_r = (1+350[(\phi/L)^2]_e) = (1+350\frac{\sum_{i=1}^{n}(\phi_i/L_i)^2 L_i}{\sum_{i=1}^{n} L_i}) \qquad circ.\ L_i = 5\phi_i \tag{11}$$

$$K = \frac{7C^*}{F_r} \qquad\qquad K^* = \frac{3[100p/f_{co}]^2}{F_r} \tag{12}$$

This generalized macro model can be applied to isotropically confined concrete cores to evaluate the confinement effectiveness of a particular lateral confining arrangement.

6. STRENGTH OF CONFINED CORE

The above discussed confinement effectiveness coefficient is related to the strength enhancement of the confined core. As stated before, the confinement effectiveness coefficient α multiplied by the maximum available confining capacity of the agent $\frac{1}{2}pf_y$ would result in the spatially averaged confining stress applied to the core. For very closely spaced circular ties bordering near the conditions offered by a casing, the above value can be assumed to govern the increase of strength in the confined core. For other cases of non-circular sections with relatively distant placed ties (s/d= 0.20-1.5) the strength will also be a function of spacing (s'/d) (s' is the clear spacing between lateral reinforcement), and uniformity of lateral stress distribution (K). Through FEM analysis a critical confining stress (σ_m) governing the strength (eq.13) was derived.

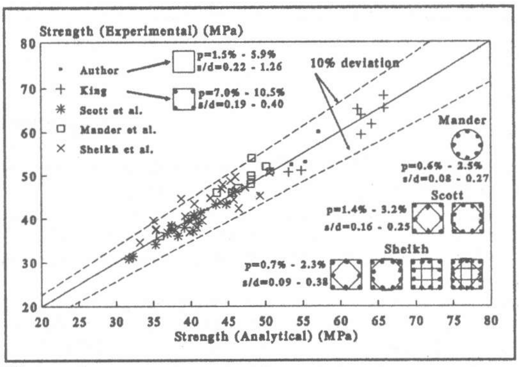

Figure 6 Prediction of experimental confined strength

$$\sigma_m = \frac{[\alpha.\frac{1}{2}pf_y]}{1+0.6K(s'/d)^{\frac{1}{2}}} \tag{13}$$

This critical sectionally average confining stress can be closely considered as an equivalent tri-axial confinement, which enables experimentally obtained relations between strength and confining stress to be utilized. Though linear relations between tri-axial confinement and strength gain have been proposed, it had been observed [9] that for low confining stresses proportional strength gains are higher. Since lower critical confining stresses are prevailing in passive confined columns in practice, relation in eq. 14 is proposed for the strength of confined concrete (f_{cc}). The increasing rate of straining is seen to improve the strength of the confined as well as un confined concrete. Based on previous studies [1,2,8,11], the form in eq. 15 is proposed where, $\dot{\varepsilon}$ is the rate at which the strain is applied, for the prediction of confined strength under higher strain rates.

$$f_{cc} = f_{co}+6[\sigma_m]^{\frac{3}{4}} \qquad \textit{...stresses in MPa} \tag{14}$$

$$[f_{cc}]_{\dot{\varepsilon}} = f_{cc}*(1+0.06\ \log(\dot{\varepsilon}-10)) \qquad \textit{...for } \dot{\varepsilon} > 10\ \mu \textit{strain}/\text{sec} \tag{15}$$

Comparisons of analytical predictions with experimental results of different sections and reinforcement arrangements by several previous researchers are given in fig. 6.

7. CONCLUSIONS

Through this study, form of the macro model was developed based on micro models, which was calibrated by the results of carefully conducted idealized experiments. Final form of the macro model is a result of micro and macro behavior of confined RC. Following deductions can be concluded.

1 The confinement efficiency of a particular lateral reinforcement arrangement can be quantified as the ratio of confining stress attained at the peak load to the maximum that can be provided. This ratio also can be expressed as the average steel stress attained at peak load to yield strength of the steel for normal strength steels used in practice.

2. The afore mentioned ratio is a function of mainly, amount of lateral steel, shape of the section, relative spacing of discrete ties, their flexural rigidity (in case of rectilinear), and arrangement of lateral and longitudinal reinforcement, which can be expressed through the proposed model.

3. It is necessary to account for the flexural stiffness of rectilinear lateral ties for a rational formulation of confinement effectiveness, though in previous studies this factor is not generally assumed explicitly.

4. The spatially averaged confinement obtainable from confinement effectiveness coefficient can be related to the strength of the confined concrete by considering the lateral and longitudinal distribution uniformity and strength of uniformly confined concrete. Proposed model for strength takes these factors into account and predicts the strength gain with good accuracy.

REFERENCES

1. Ahmad, S. H., and Shah, S. P., "Behavior of Hoop Confined Concrete Under High Strain Rates," American Concrete Institute Journal, Vol. 82, No. 55, Sep.-Oct., 1985, pp 634-647.
2. Dilger, W. H., Koch, R., and Kowalczyk, R., "Ductility of Plain and Confined Concrete Under Different Strain Rates," ACI Journal Proceedings, Vol. 81, No. 11, Jan.-Feb., 1984, pp. 73-82.
3. Maekawa, K. et al., "Plasticity in Concrete Nonlinearity Under Triaxial Confinement," Proceedings of JSCE, vol. 2, No. 2, Jan., 1993.
4. Maekawa, K. et al., "Triaxial Elasto-Plastic Fracture Model for Concrete," Proceedings of JSCE, vol. 9, No. 3, Feb., 1993.
5. Iyengar, K. T. S. R., Desayi, P., and Reddy, K. N., "Stress Strain Characteristics of Concrete Confined in Steel Binders," Magazine of Concrete Research," Vol. 22, No. 72, Sep., 1970, pp. 173-184.
6. Karsan, I. D., and Jirsa, J. O., "Behavior of Concrete Under Compressive Loadings," Journal of the Structural Division, ASCE, Vol. 95, No. ST12, Dec, 1969, pp.2543-2563.
7. King, J. W. H., "Further Notes on Reinforced Concrete Columns," The Structural Engineer, Nov., 1946, pp. 609-616.
8. Mander, J. B., Priestly, M. J. N., Park, R., "Observed Stress-Strain Behavior of Confined Concrete," Journal of Structural Engineering, ASCE, Vol. 114, No. 8, Aug., 1988, pp. 1827-1849.
9. Saatcioglu, M., and Razvi, S. R., "Strength and Ductility of Confined Concrete," Journal of Structural Engineering, ASCE, Vol. 118, No. 6, June, 1992, pp. 1590-1607.
10. Shiekh, S. A., and Uzumeri, S. M., "Strength and Ductility of Tied Concrete Columns," Proceedings, ASCE, Vol. 106, ST5, May, 1980, pp. 1079-1102.
11. Scott, B. D., Park, R., and Priestley, M. J. N., "Stress-Strain Behavior of Concrete Confined by Overlapping Hoops at Low and High Strain Rates," ACI Journal Proceedings, Vol. 79, no. 1, Jan.-Feb., 1982, pp. 13-27.