論 文

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[2215] Nonlinear Finite Element Analysis for Shear Behavior of Reinforced Concrete Beams Based on Fracture Mechanics

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1. INTRODUCTION

The phenomenon of shear behavior of reinforced concrete members has been studied by many researchers over the years, but because of its complexity, there is still no universal theoretical solution which can accurately predict this behavior. The shear capacity and the behavior of members are affected by numerous factors, such as size, crack pattern, ratio of shear span to effective depth, the compressive strength of concrete, etc. Shear failures generally are sudden and catastrophic, therefore, there is a strong incentive to predict shear behavior more accurately than is possible now.

There have been numerous experimental and analytical studies over the years that have indicated that the traditional approach utilizing ACI[1] whereby a constant value of shear value capacity, V_c, is computed for any size member, may not be keeping with the actual behavior exhibited. Shear equations considering the size effect are proposed for design by the JSCE[2], and CEB[3], respectively. Recently the application of fracture mechanics to the shear problem has shown great promise. In all of these alternative approaches, the idea is to analytically capture the behavior and failure of members under shearing forces and then to provide the designer with a simplified approach that can be readily used in the design office.

The objectives of this study are to numerically investigate the behavior of reinforced concrete beam without shear reinforcement. The shear capacity of geometrically similar reinforced concrete beam is calculated. The cracking of concrete is the major topic addressed. This study utilizes finite element analysis together with the fictitious crack modeling of identified diagonal tension crack. Firstly the application of fracture mechanics in the form of fictitious crack model to the shear problem is executed, and

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then the results predicted in this analysis are compared with those predicted through JSCE and CEB equations.

The Fictitious Crack Model with nonlinear rod elements [5] is used to predict the formation of cracks. In this model, the location and orientation of a diagonal crack are predefined, and the crack zones are presented by crack planes, whose material properties are associated with Mode I fracture energy. Accordingly, the shear model to be investigated consists of linear elastic finite element models with nonlinear rod elements joining one side to the other across the crack plane.

2. FINITE ELEMENT ANALYSIS

Because of computational time and accuracy consideration, four node linear isoparametric elements are used to present the concrete in the finite element analysis, and a diagonal crack is modeled using the Fictitious Crack Model. The concrete model is elastic in compression. The finite element analysis is employed to study the size effect of reinforced concrete beams without stirrups subjected to shear forces. Dowel action and interlock force are neglected. The reinforcement is considered only to carry axial stresses, therefore; a linear two node truss element, which can only resist axial loads, is used to represent the steel reinforcement. Each node of this element has two degrees of freedom. The reinforcing bar element is connected to the concrete, at the node points and follows the same displacement as the concrete, based on the assumption of perfect bond at the nodes. Perfect bond is not really exact but is assumed as an approximation. The steel reinforcement ratio used in this analysis is 2% to investigate the influence of the steel ratio on the behavior of reinforced concrete beams. Steel elements are located at the effective depth of a concrete beam.

Rod elements are used to simulate the crack; we can call these elements as link elements. Link elements are used to represent an assumed crack zone and these links exhibit nonlinear stress-strain behavior. Length of rod element is assumed as unity (L=1). The 1/4 Model softening curve is used for this stress-strain relationship. The main idea in using this curve is that the fracture energy remains constant and is equal to (100N/m) for this concrete. When the Fictitious Crack Model is applied to reinforced concrete members, the actual crack path needs to be predefined, and the link elements are modeled to be intentionally quite stiff before reaching the tensile strength. When the tensile stress is increasing, microcracks start to occur. The microcracks can still transfer partially tensile stresses, but decreases sharply as the strain increases.

3. ANALYTICAL RESULTS

The study for the behavior of reinforced concrete beam without shear reinforcement was conducted using the solution and modeling procedure discussed above. In this analysis beams are simply supported, without shear reinforcement. Two concentrated loads are used. The parameter included in this analysis is the crack orientation, that is the angle with which the crack intercepts the member, as well as its location. The ratio of shear span to depth of beam were set to 3.0. Because of symmetry, only half of the beam was modeled for analysis.

The finite element mesh with crack angle and orientation is shown in Fig.1, while Fig.2 is showing the assumed crack plane with rod elements.

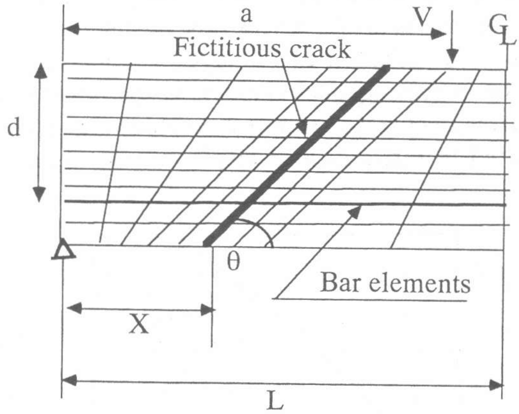

Fig.1. Finite Element Mesh For Reinforced Concrete Beam

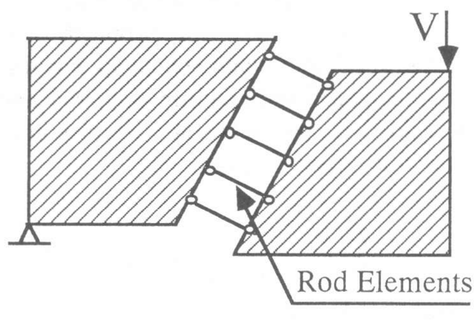

Fig.2. Assumed Crack Plane with Rod Elements

A diagonal crack was oriented in the angles 30°, 35°, 40°, or 45°. The location where the diagonal crack intersects the bottom of beam was assumed at 0.5d, d, or 1.5d, where d is the effective depth of the beam. The effect of these parameters on the nominal shear strength is shown in Fig.3 and Fig.4 respectively. It can be confirmed that the location and inclination of a diagonal crack definitely affects the shear strength of the beam.

Depending upon the results shown in Figs. 3 and 4, we cannot say which angle and location are suitable for the diagonal crack. Considering experimental results, we can say that crack should not be too flat nor too steep. We select the crack inclination at an angle of 40° and at the distance *d* from the support to proceed our analysis.

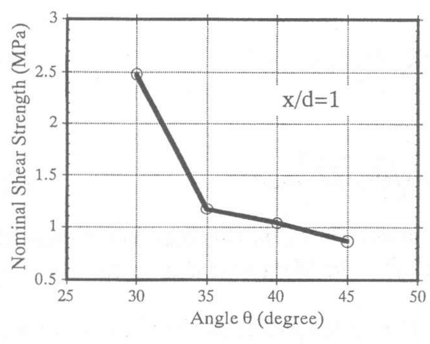

Fig.3. Variation of Nominal Shear Strength With Inclination of Diagonal Crack

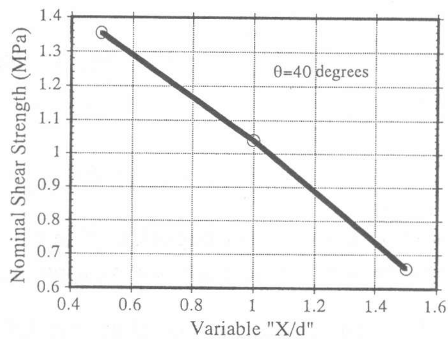

Fig.4. Variation of Nominal Shear Strength with "X"

Rod elements having the Mode I fracture energy $G_F = 100 N/m$, were provided perpendicularly along the pre-defined crack surface. In reality, shear cracks are supposed to have the Mode II fracture energy. However Mode II fracture energy can be neglected and replaced by Mode I fracture energy if the crack plane is properly modeled[4]. The initial stiffness of rod element is taken as 3×10^6 MPa. A bilinear tension softening curve called as 1/4 Model shown in Fig.5 is adopted in the analysis, where L is the length of rod element. The tensile strength of concrete is 3 MPa. In this analysis the loading of the reinforced concrete beam was imposed by increasing displacement. Displacement was increased until failure occurred and reinforced concrete beam was unable to resist any further displacement. Results are presented in terms of load-displacement curve, which is shown in Fig.6.

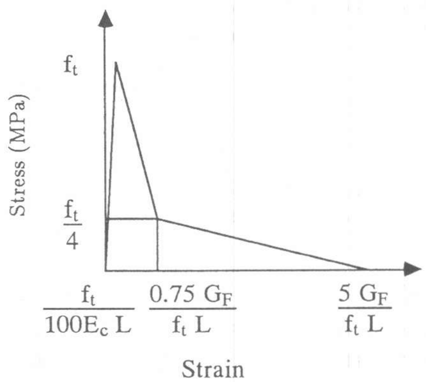

Fig.5. 1/4 Model Curve for Rod Elements

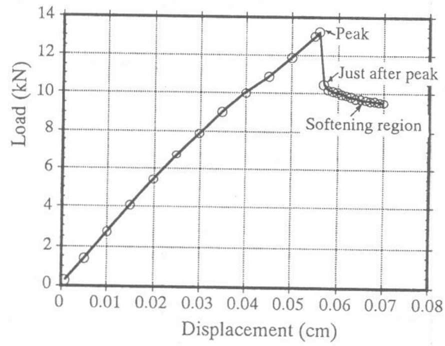

Fig.6. Load-Displacement Curve

Results were compared with the JSCE [2] and CEB[3] equations. These equations are as follows:

JSCE:

$$V_c = \left[0.19\ (100\rho_w f'_c)^{1/3} d^{-1/4}\right] b_w d \tag{1}$$

CEB:

$$V_c = \left[0.12\ (100\rho_w f'_c)^{1/3}\ (1 + \sqrt{(0.2/d)}\right] b_w d \tag{2}$$

where V_c is the shear capacity, f'_c is the concrete strength, b_w is the member width, d is the effective depth, ρ_w is the reinforcement ratio. f'_c is in MPa and d is in m.

It is important to note that the JSCE and CEB codes incorporate the size effect in their formulation. In the above equations the size effect is proportional to $d^{-1/4}$ or $[1 + \sqrt{0.2/d}]$, respectively.

Thus, the intent of this investigation was to ascertain the validity of fictitious crack modeling to the "correct" behavior of reinforced concrete members subjected to shear and whether or not this approach could be accurate in the overall prediction of behavior.

4. SIZE EFFECT

The behavior of reinforced concrete beam is significantly affected by the beam size, and changes ductile to brittle as the beam size increases, moreover, the shear strength capacity decreases as the beam size increases. This behavior is caused by the size effect.

The results of this study with the design equation of JSCE and CEB are shown in Fig.7. According to this figure, the tendency of shear strength to decrease with an increase in beam size was obtained from finite element analyses and shows the good agreement with JSCE and CEB design equations in which the size effect has been incorporated. For the smallest beam (d=1cm), we could not get the same result as for the JSCE and CEB design equations. In design practice it is very rare to design such a small beam. So results of such a small beam are not very important because we are not sure about the exact behavior. Other beams showed the significant decrease in shear strength with increasing in size. The comparison of size effect with JSCE and CEB shear equations is shown in Fig.7.

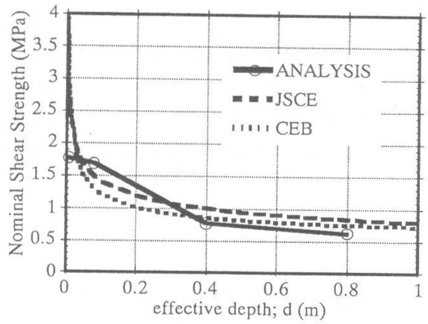

Fig.7. Comparision of Analysis with JSCE and CEB Equations

5. STRESS DISTRIBUTION

Stress distribution shown along the crack path is actually the inclined path but on the graph this line was drawn vertically. Stress distribution at peak showed that once the rod element at the mid height of beam reaches to the tensile strength of concrete, the crack propagates rapidly towards the compression zone and the beam showed the maximum resistance. Peak resistance does not mean the fracture of all rod elements, because at the peak resistance all rod elements did not reach the tensile strength of concrete.

Similarly, the results of stress distribution just after peak showed that just after the peak the neutral axis along the crack moved up and the stress of rod element just below the neutral axis reached to the tensile strength of concrete, after

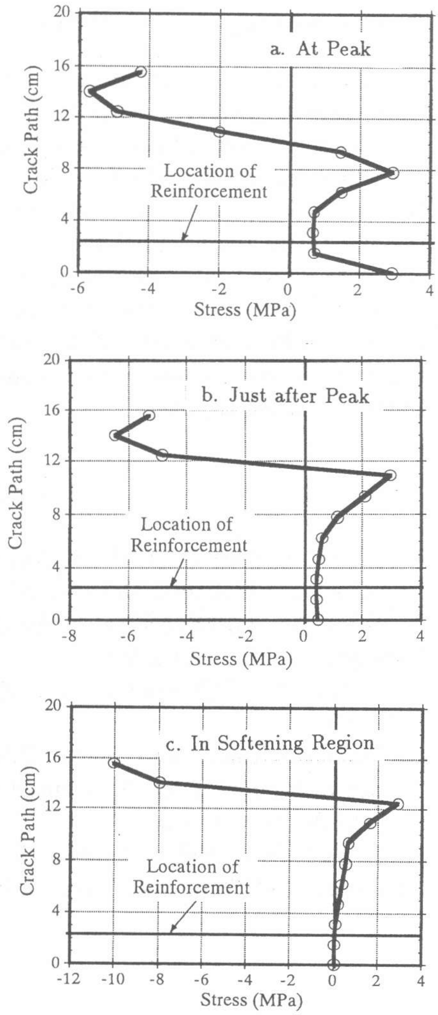

Fig.8. Stress Distribution of Rod Elements

that stresses of rod elements are decreased. The stress distribution of the point in the softening region showed that the neutral axis moved towards the compression zone and the stress of rod element near the neutral axis reached to the tensile strength of concrete and the rest of the elements are in the softening zone. The results of the stress distribution of rod elements along the crack path is shown in Fig.8.

5. CONCLUSION

From this study we conclude that the behavior of reinforced concrete beam without shear reinforcement is much affected by the inclination and the location of the diagonal crack where the crack intersects the bottom of the beam. It can be observed that the nominal shear strength is decreased as we increased in the inclination. Similarly the nominal shear strength is also affected by the location of crack. We observed that as the location is going away from the support, the value of nominal shear strength is decreased.

It can be observed that there is indeed a size effect that can be analytically shown to exist when investigated using a finite element technique. Shear capacity predicted by the finite element method shows good agreement with the JSCE and CEB shear equations. All three showed that the behavior of reinforced concrete beam is generally affected by the beam size. The behavior ranges from ductile to brittle if the beam size increases, in the same time the nominal shear strength decreases. The stress distribution for the rod elements showed that the peak resistance does not mean the fracture of all rod elements, because at the peak resistance, all rod elements did not reach the tensile strength of concrete. The neutral axis of the crack plane at peak was nearly at the mid height of the beam but it moved towards the compression zone as the response of the beam entered in the softening region.

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