論 文

[2221] Identification of Structural Characteristics of Frame Elements

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1. INTRODUCTION

The application of system identification and parameter estimation to real structures is difficult since a realistic model requires that the structure must be represented by many degrees-of-freedom (DOFs). The computation time required for convergence increases drastically with the number of DOFs and problems on divergence are usually encountered. For these reasons, studies applying the system identification problem to only a small portion of a structure by using substructuring so as to reduce the system under consideration have started to be considered by some researchers [1-2]. The identification approach [1] proposed recently by the authors was termed as "localized identification". This approach is reasonable and practical since the analysis can be concentrated at a local and critical part of a structure.

As an extension and improvement of the localized identification approach of the previous study, an application to a plane frame is now considered. Unlike in the previous formulation wherein the physical characteristics of the structure are indirectly determined from the identified elements of the stiffness and damping matrices, the present approach directly identifies the structural characteristics of the structural elements. In this study, the physical characteristics of a frame element such as the axial rigidity (EA), flexural rigidity (EI) and damping coefficients are directly identified. The present localized identification uses a state equation derived from the equation of motion of a frame element and the extended Kalman filter with weighted global iteration (EK-WGI) is applied to estimate the structural characteristics of a frame element. Numerical simulation studies were carried out on a two-story plane frame where the structural characteristics of a frame element were identified.

2. EXTENDED KALMAN FILTER

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In order to apply the extended Kalman filter to system identification problems, an appropriate set of state vector and measurement vector equations must be formulated, respectively as

$$\frac{d\mathbf{X}}{dt} = f(\mathbf{X}, t) \tag{1}$$

$$\mathbf{Y}(k) = h(\mathbf{X}(k), k) + \mathbf{v}(k) \tag{2}$$

where $\mathbf{X}(k)$ is the state vector at time $t = k\Delta t$, $\mathbf{Y}(k)$ is the measurement vector at time $t = k\Delta t$, $h(\mathbf{X}(k), k)$ is the measurement function relating the measurement vector to the state vector, $\mathbf{v}(k)$ is the observational noise vector with covariance matrix, $\mathbf{R}(k)$, and Δt is the sampling time interval.

In the EKF algorithm [3], the initial state vector, $\hat{\mathbf{X}}(0/0)$, and its error covariance matrix, $\mathbf{P}(0/0)$ are first assumed; and then as the measurement vector, $\mathbf{Y}(k)$ is processed, the state vector and the error covariance matrix are recursively updated. For convergence purposes in the parameter estimation, the extended Kalman filter with weighted global iteration (EK-WGI) developed by Hoshiya and Saito [4] is applied.

3. EQUATION OF MOTION OF A FRAME ELEMENT

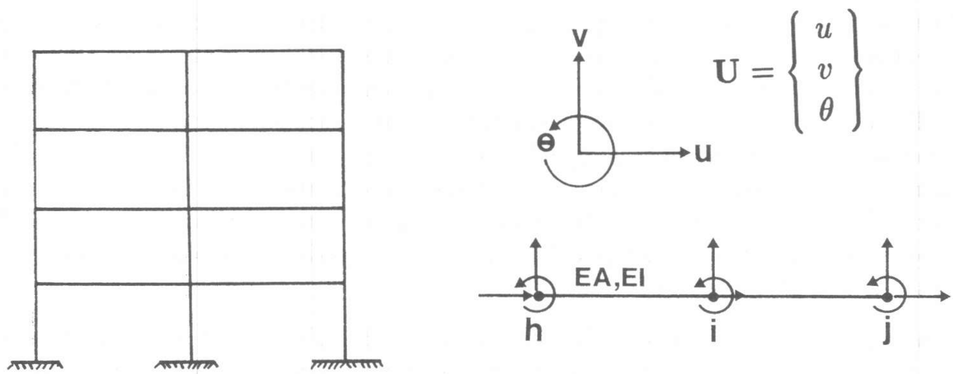

Figure 1: Plane Frame Structure and a Frame Element

Consider a frame element (Figure 1) which is extracted from the complete structure and introduce an internal node in the element. Each node consists of three DOFs which are the horizontal displacement (u), vertical displacement (v) and the rotation (θ). The equation of motion of the frame element may be written in partition form as follows:

$$\begin{bmatrix} \mathbf{M}_{hh} & \mathbf{M}_{hi} & 0 \\ \mathbf{M}_{ih} & \mathbf{M}_{ii} & \mathbf{M}_{ij} \\ 0 & \mathbf{M}_{ji}0 & \mathbf{M}_{jj} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}}_h(t) \\ \ddot{\mathbf{U}}_i(t) \\ \ddot{\mathbf{U}}_j(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{hh} & \mathbf{C}_{hi} & 0 \\ \mathbf{C}_{ih} & \mathbf{C}_{ii} & \mathbf{C}_{ij} \\ 0 & \mathbf{C}_{ji} & \mathbf{C}_{jj} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}}_h(t) \\ \dot{\mathbf{U}}_i(t) \\ \dot{\mathbf{U}}_j(t) \end{Bmatrix}$$

$$+ \begin{bmatrix} \mathbf{K}_{hh} & \mathbf{K}_{hi} & 0 \\ \mathbf{K}_{ih} & \mathbf{K}_{ii} & \mathbf{K}_{ij} \\ 0 & \mathbf{K}_{ji} & \mathbf{K}_{jj} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_h(t) \\ \mathbf{U}_i(t) \\ \mathbf{U}_j(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_h(t) \\ \mathbf{F}_i(t) \\ \mathbf{F}_j(t) \end{Bmatrix} \tag{3}$$

where dot denotes time derivative; **M**, **C**, **K** are the mass, damping and stiffness matrices, respectively; $\mathbf{U}(t)$ represents the displacements and $\mathbf{F}(t)$ represents external forces applied at the nodes. The subscripts h and j denote the DOFs at the ends of the element which will be referred to as boundary DOFs and i refer to the internal DOFs.

4. STATE EQUATION

Since we are interested in identifying the structural characteristics of a frame element, only the equation of motion of the internal node is used in the formulation of the state equation, ie.,

$$\mathbf{M}_{ii}\ddot{\mathbf{U}}_i(t) + \mathbf{C}_{ii}\dot{\mathbf{U}}_i(t) + \mathbf{K}_{ii}\mathbf{U}_i(t) = \mathbf{F}_i(t) - \mathbf{F}_{ih}(t) - \mathbf{F}_{ij}(t) \tag{4}$$

where

$$\mathbf{F}_{ih}(t) = \mathbf{M}_{ih}\ddot{\mathbf{U}}_h(t) + \mathbf{C}_{ih}\dot{\mathbf{U}}_h(t) + \mathbf{K}_{ih}\mathbf{U}_h(t) \tag{5}$$

$$\mathbf{F}_{ij}(t) = \mathbf{M}_{ij}\ddot{\mathbf{U}}_j(t) + \mathbf{C}_{ij}\dot{\mathbf{U}}_j(t) + \mathbf{K}_{ij}\mathbf{U}_j(t) \tag{6}$$

The equation of motion of the internal node of the frame element will now be used to derive the state equation. First, the elements of the stiffness matrices $\mathbf{K}_{ih}$, $\mathbf{K}_{ii}$ and $\mathbf{K}_{ij}$ are expressed in terms of the axial rigidity (EA) and the flexural rigidity (EI). The damping matrices are assumed to be proportional, i.e., $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$. Now we define the state vector as follows:

$$\mathbf{X} = [\mathbf{X}_1^T \quad \mathbf{X}_2^T \quad \mathbf{X}_3^T \quad \mathbf{X}_4^T]^T \tag{7}$$

in which

$$\mathbf{X}_1 = \{u_i,\ v_i,\ \theta_i\}^T,$$

$$\mathbf{X}_2 = \left\{\dot{u}_i,\ \dot{v}_i,\ \dot{\theta}_i\right\}^T,$$

$$\mathbf{X}_3 = \{EA,\ EI\}^T,$$

$$\mathbf{X}_4 = \{\alpha,\ \beta\}^T$$

The state equation of the system can be written as

$$\dot{\mathbf{X}} = \left\{\begin{array}{c}\dot{\mathbf{X}}_1\\ \dot{\mathbf{X}}_2\\ \dot{\mathbf{X}}_3\\ \dot{\mathbf{X}}_4\end{array}\right\} = \left\{\begin{array}{c}\mathbf{X}_2\\ \mathbf{M}_{ii}^{-1}[-\mathbf{C}_{ii}(\mathbf{X}_3,\mathbf{X}_4)\mathbf{X}_2 - \mathbf{K}_{ii}(\mathbf{X}_3)\mathbf{X}_1 + \mathbf{F}_i - \mathbf{F}_{ih}(\mathbf{X}_3,\mathbf{X}_4) - \mathbf{F}_{ij}(\mathbf{X}_3,\mathbf{X}_4)]\\ 0\\ 0\end{array}\right\} \tag{8}$$

where a matrix with $(\mathbf{X}_i)$ means that the elements of that matrix are functions of the elements of vector $\mathbf{X}_i$. The structural characteristics of the frame element which are to be identified are EA, EI, α and β. These parameters are augmented as $\mathbf{X}_3$ and $\mathbf{X}_4$ in the state vector.

5. MEASUREMENT EQUATION

The measurement equation can be derived based on the measured responses. If the two displacements, u_i and v_i, are observed, the measurement equation can be written as

$$\mathbf{Y}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{X}(k) + \mathbf{v}(k) \tag{9}$$

On the other hand, if the two velocities, $\dot{u}_i$ and $\dot{v}_i$, are observed, the measurement equation takes the following form

$$\mathbf{Y}(k) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{X}(k) + \mathbf{v}(k) \tag{10}$$

The measurement equations above are both linearly dependent upon the state variables. In these equations, $\mathbf{v}(k)$ represents a noise vector. Incorporating the state and measurement equations into the Kalman filter algorithm, the parameters can then be estimated.

6. NUMERICAL STUDY

To test the localized identification of structural characteristics of a frame element, a numerical simulation was carried out on a two-story plane frame shown in Figure 2.

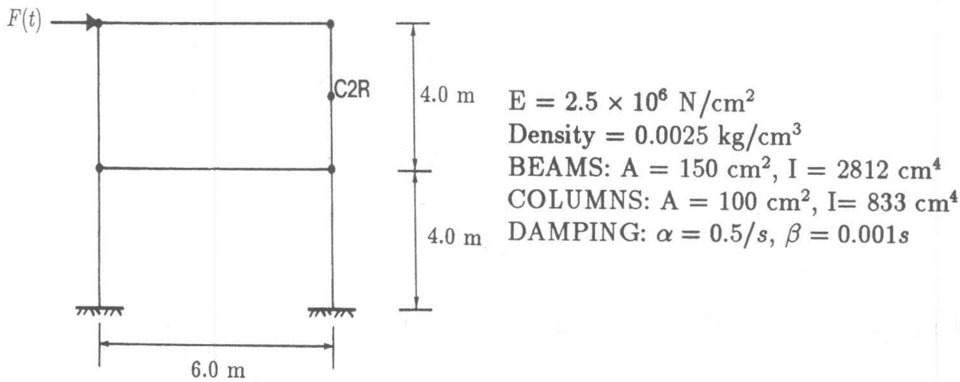

Figure 2: Plane Frame and Structural Characteristics

The plane frame was first modeled using finite elements with three DOFs at each node and the dynamic response due to an external force $F(t) = 5\ SIN(5t)$ kN was obtained

Table 1. ESTIMATED STRUCTURAL PARAMETERS OF COLUMN C2R

Parameter	True Value	Initial Value	Estimated Value	
			Displacement Data	Velocity Data
EA	2.50×10^8	0.0	2.50×10^8	2.50×10^8
EI	2.0825×10^9	1.0×10^9	2.082×10^9	2.083×10^9
α	0.50	0.0	0.4995	0.5007
β	0.001	0.0	0.001	0.001

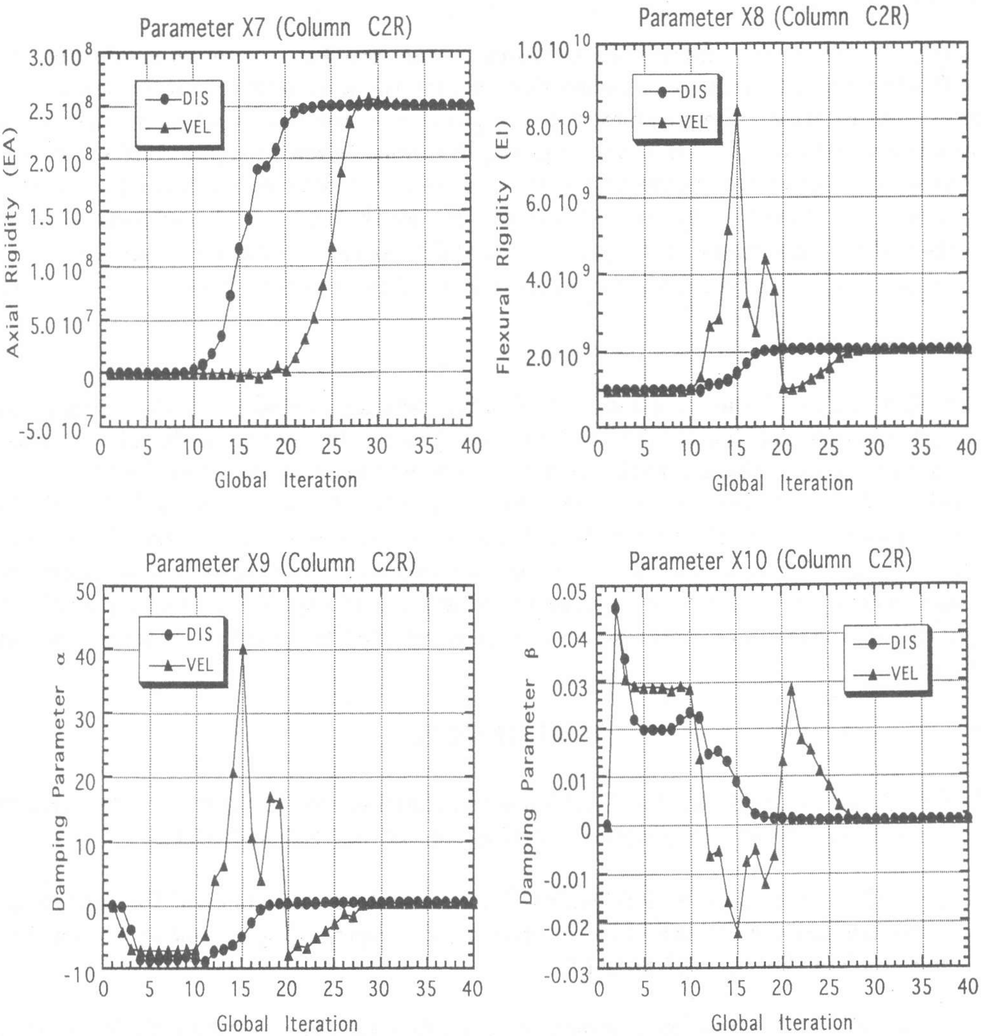

Figure 3: Convergence Behavior of Parameters of Column C2R

by using numerical integration at increments of 0.01 s. The calculated responses at the nodes were then used in the identification.

In the numerical study, the structural characteristics of beam and column elements of the frame were estimated. Due to space limitations, only the results of the identification of the structural characteristics of column C2R will be presented. Assuming initial values for the parameters, EA, EI, α and β and using the displacements, u_i and v_i, and the velocities, $\dot{u}_i$ and $\dot{v}_i$, as observations, the parameters were identified by EK-WGI method. Table 1 shows the true, initial and estimated values of the parameters using both the displacements and velocities as observations. The estimated values are in close agreement with the true values.

Figure 3 shows the convergence behavior of the structural parameters during the identification by EK-WGI. It is shown that at first the parameters were unstable, but after several iterations, the parameters converged to the true values. Convergence of the parameters was obtained after about 20 global iterations. Even with poor initial guesses, the parameters ultimately converged to the true values. Convergence of the parameters can be affected, however, by several factors such as the number of parameters to be identified, the initial values of the parameters and their error covariance, sampling time and sampling length. Hence, a study of the effects of these factors is necessary.

7. CONCLUSION

An application of the localized identification was conducted on a plane frame to identify the structural characteristics of a frame element. In the present localized identification, the physical characteristics of a frame element such as the axial rigidity (EA), flexural rigidity (EI) and damping coefficients are directly estimated using EK-WGI. Numerical simulation studies were carried out on a two-story plane frame wherein the structural characteristics of a frame element were reasonably identified. This noteworthy result necessitates that more comprehensive studies and applications of the localized identification approach on other structures be conducted to verify its capabilities and usefulness.

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