論文 Flexural Analysis of Two-Span Continuous Prestressed Concrete Beam with External Tendons

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ABSTRACT : This paper describes the analysis of flexural behavior of a two-span continuous concrete beam prestressed with external tendons. In the previous study[1], the analytical program based on the concept of deformation compatibility and secondary effect, in term of change of tendon's eccentricity, could provide a good agreement with experimental observations. Consequently, the attempt to adopt the analytical program mentioned above for a prediction of flexural behavior of a two-span continuous externally PC beam is made in this study. The comparison between analytical results and test observations is also made at the end of this paper.

KEYWORDS : two-span continuous, external tendon, change of tendon's eccentricity, compatibility of deformation

1. INTRODUCTION

The using of prestressed concrete beam associated with external tendons for continuous span is one of the extensive application in the field of bridge construction. Besides the advantages of externally PC beam such as; a)the construction and maintenance of bridge are easier, b) the old or damaged PC beam can be easily repaired or re-strengthened, and so on, the additional attractive advantage of PC beam on a continuous span-type is to reduce the number of expansion joints on bridge deck which can provide a better serviceability for passengers.

In the previous study[1], the analytical models based on the concept of compatibility of the deformation of concrete and elongation of prestressing tendons, and secondary effect, in term of change of tendon's eccentricity, were established to predict the flexural behavior of a simply supported externally PC beam. It was shown that an acceptable comparison with test results can be obtained. By the similar way, the attempt to adopt the analytical program based on the mentioned concept for a two-span continuous externally PC beam is made in this study.

2. ANALYTICAL MODELS

2.1 Basis of the method

The nonlinear analysis models were adopted from the following assumptions; (1) plane sections still remain plane after bending, (2) nonlinear of materials (concrete, reinforcements and prestressing steel) are considered through the use of constitutive relations of materials (see Fig.1), (3) compatibility of deformation, that is the total deformation of concrete located at prestressing tendons level equals to that of elongation of tendons, (4) only pure flexural deformation are considered, (5)

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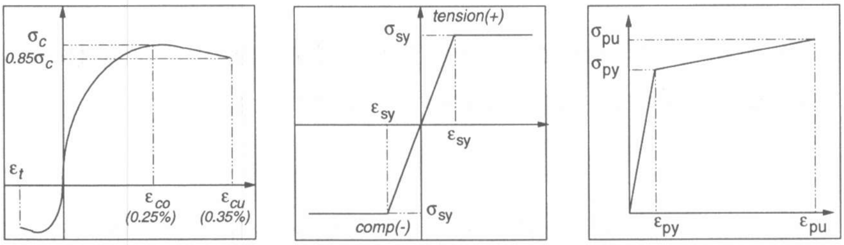


Concrete **Reinforcement** **Prestressing steel**

Fig.1 Models for constitutive relation of material

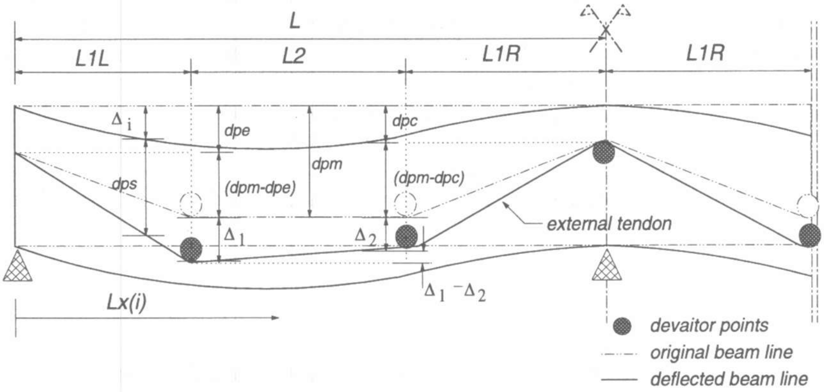


Fig.2 Evaluation of effective depth of external tendon

the ultimate limit stage is defined when either the concrete strain at extreme compression fiber (ε_{cu}) reaches 0.0035, or the tensile stress of reinforcement or prestressing tendons exceed the nominal tensile strength of each material.

2.2 Secondary effect due to change of eccentricity of external tendons

In the external prestressing system, tendons are arranged outside the cross-section of a prestressed concrete beam. Unlike internal unbonded tendons, external tendons are formed the profile by deflecting at the deviators. When the externally PC beam subjected to bending, the tendon would not follow the beam deflection, except at deviator points. The large deflection of PC beam, particularly when the beam is close to failure, could induce the secondary effect on flexural strength of PC beam, namely *"change of tendon's eccentricity"*. In this study, the effect of change of tendons's eccentricity is taken in account in the analysis models to provide an accurate prediction of flexural strength of externally PC beam. Fig.3 shows the evaluation of effective depth of external tendon. For a case of deflected beam, the value of d_{ps} can be expressed as followings;

a) for Lx(i) < L1L ;

$$d_{ps} = d_{pe} + \frac{(d_{pm}-d_{pe})+\Delta_1)*Lx}{L1L} - \Delta_i \qquad .....(1)$$

b) for L1L < Lx(i) < L1L+L2 ;

$$d_{ps} = d_{pm} + \Delta_1 - \frac{(\Delta_1 - \Delta_2) * (Lx - L1L)}{L2} - \Delta_i \quad .....(2)$$

c) for Lx(i) > L1L+L2 ;

$$d_{ps} = d_{pc} + \frac{(d_{pm} - d_{pe}) + \Delta_2) * (L - Lx)}{L1R} - \Delta_i \quad .....(3)$$

where d_{pe}, d_{pm}, d_{pc} are the effective depth of tendon at end support, middle span, and center support section, respectively; Δ_1, Δ_2, Δ_i are the deflections of beam at deviator points and considering point, respectively. In calculation of load-deflection response, the value of d_{ps} is needed to be renewed for every step of calculation.

3. FLOWCHART OF ANALYSIS

Fig.4 shows flowchart diagram which describe the comprehensive procedures of calculation for the computerized program. The model of 2-span continuous PC beam using in this study is graphically drawn in Fig.5. The PC beam is divided into n small elements (in this study, n=27), and i integration point along the beam axis. The analytical program consists of three main parts this is, data input, computational loop, and output of results at ultimate stage. In the computation step, the followings iterative loops have to be satisfied at each step of calculation.

3.1 Force equilibrium (iterative loop No.1)

Once the beam was divided into i number of integration points, the internal forces of the section consisting of forces in concrete, nonprestressed reinforcement and external tendons were calculated by using the constitutive relation of materials (see Fig.1). The analysis initiates at the center support section ($i^{th} = 0$, where the maximum bending moment was taken place) by assuming the concrete strain at extreme compressive fiber, $\Delta\varepsilon_{c,ith=0}$ and $\Delta\varepsilon_{ps}$. Then, the strain distribution of section is assumed until the equilibrium of internal forces (iterative No.1) is achieved. By discreting element of section[2] (see Fig.6), the forces of each component can be calculated as follows:

- *compression force in concrete*

$$C_{conc} = \sum_{x_n=0}^{x_n=c} \sigma_c(\overline{x_n}) b \Delta x - \sum_{x_n=h_f}^{x_n=c} \sigma_c(\overline{x_n})(b - b_w)\Delta x \quad .....(4)$$

- *tensile force in reinforcements and prestressing tendons*

$$T_s = A_s\sigma_s - A_s'\sigma_s' + A_{ps}\sigma_{ps} \quad .....(5)$$

where $\sigma_c(x_n)$, σ_s and σ_{ps} are stress of concrete, reinforcement and prestressing tendon which are calculated from material's constitutive relations.

After force equilibrium is achieved, the internal moment Mn(i=0) can be calculated by summing moments from all force components. At this step, the moment distribution along the span length of beam can be formed by using the value of Mn(i=0) which represents the bending moment due to externally applied load.

3.2 Moment equilibrium (iterative loop No.2)

In this iterative loop, it involves performing a moment equilibrium of all integration points (i^{th}=1,2,...n) other than the center support section. The calculation of internal force is repeated in iterative loop No.1 by assuming the compressive concrete strain at extreme fiber $\varepsilon_{c(i)}$ and strain distribution until force equilibrium is achieved, then calculating the internal moment, Mi(i^{th}=1,2,...). Here, the iterative loop of equilibrium of moment are made by adjusting the value of $\varepsilon_{c(i)}$ until the difference of Mn(i^{th}=1,2,...) and Mi(i^{th}=1,2,...) is less than a reasonable tolerance, that is $|$Mn(i^{th}=1,2,...) - Mi(i^{th}=1,2,...)$| < \Delta_{allowable}$, which is about 1%.

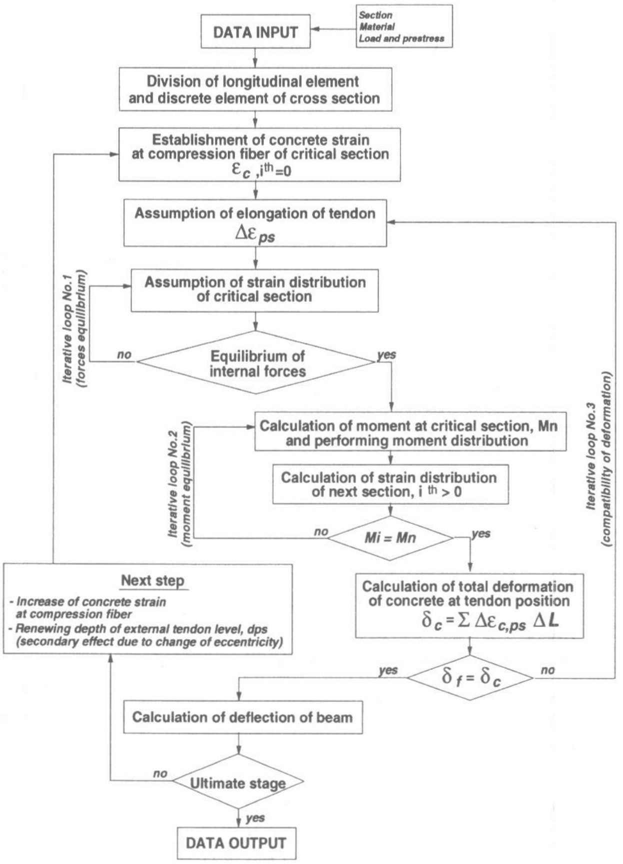


Fig.3 Flowchart of analysis

3.3 Compatibility of deformation (iterative loop No.3)

Once the equilibrium of force and moment are achieved for all i integration point, the total deformation of concrete at tendons level, δ_c, can be calculated by summing the average strain increase in concrete at tendons level, $\Delta\varepsilon_{c,ps}$, between the end anchorages, ($\delta_c=\Sigma\Delta\varepsilon_{c,ps}\cdot\Delta L$). Meanwhile, the total elongation of external tendons (δ_f) can be obtained from the value of strain increase in tendon, $\Delta\varepsilon_{ps}$ ($\delta_f=\Sigma\Delta\varepsilon_{ps}\cdot L_t$), as shown in Fig.7. At this step, the additional iterative loop No.3 is required to manage the compatibility of deformation until $|\delta_c-\delta_f|<\Delta_{allowable}$. If the compatibility condition is not satisfied, the value of $\Delta\varepsilon_c$ and $\Delta\varepsilon_{ps}$ have to be re-assumed again until all of iterative loop mentioned above are satisfied.

Finally, the deflection of beam can be computed by numerical integration of whole span length. In order to consider the effect of change of tendon's eccentricity (secondary effect) mentioned in the item 2.3, the tendon's eccentricity (or depth of tendon level, d_{ps}) has to be renewed for the next step of calculation. The completion of calculation is defined when one of the ultimate condition of

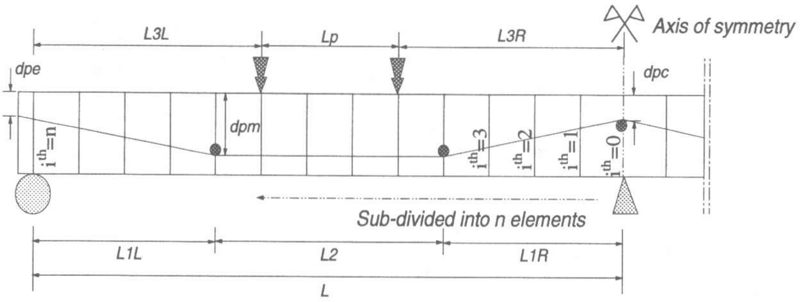


Fig.4 Modelling of PC beam used in analysis

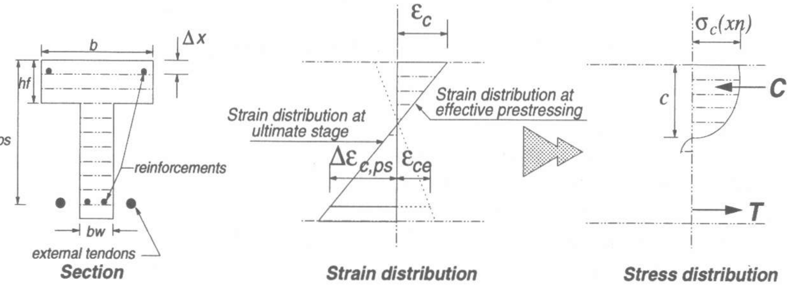


Fig.5 Discreting section and stress-strain distribution of section

materials are reached that is compressive strain of concrete reaches 0.0035, or the tensile stress of reinforcements or tendons exceed the nominal tensile strength. (4,000 and 17,500 kg/cm^2 for reinforcement and prestressing tendons, respectively)

4. EXPERIMENTAL PROGRAM

4.1 Test setup

A test specimen having T-shaped cross section was post-tensioned with external tendons over two-span continuous with 4.05 m span length each. Two-point load pattern with loading length of 0.75 m were symmetrically applied to the specimen. The detail of test specimen and test setup were shown in Fig.7. In order to monitor the stress increase in the external tendons during testing, four load cells were installed at each end of the external tendons. The reaction forces of all supports were also observed by load cells fixed under each support so that the moment distribution along the span of an indeterminate continuous span-type can be investigated. The electrical strain gauges were attached at surface of concrete, reinforcements and external tendons to observed the variation of strain in each components. The deflections of specimen were monitored by large deformation transducers at the particular points such as deviators points, mid-span section and loading points.

4.2 Discussion on test results and comparison with analytical prediction

From the load-deflection relation (see Fig.7a), it is noticed that the specimen showed an elastic behavior until first crack was observed at 6.5 tonf of each span loading approximately, and then the yielding of reinforcements started at 11 tonf. The observed failure mode of test specimen was the crushing of concrete at midspan section near exterior loading point which caused by the sufficient ductility of center support section. The comparison between test results and analytical predictions were

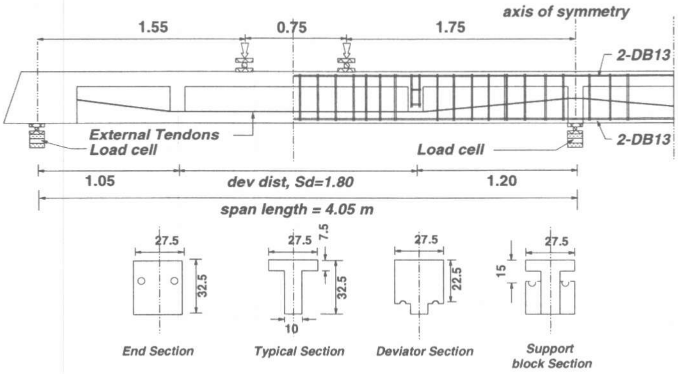
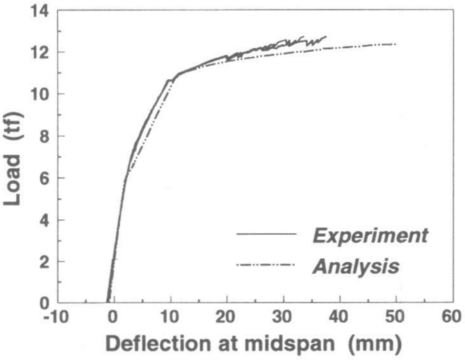


Fig.6 Detail of test specimen and test setup

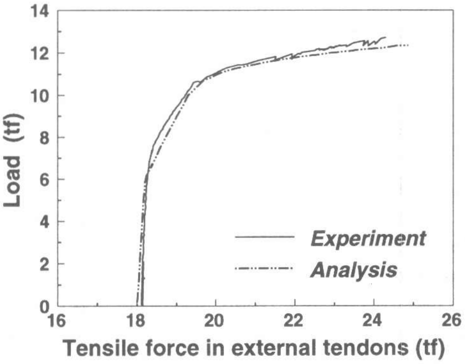
made on the load-deflection and load-tensile force in tendons responses. (see Fig.7a and Fig.7b). According to the analysis models described above, it is found that the analytical program can provide a satisfied agreement with test results. The ultimate flexural strength was estimated with difference of 5% far from test observations.(12.72 tonf and 12.30 tonf for test and predicted values, respectively)

5. CONCLUDING REMARKS

In this study, the attempt to establish the flexural analysis of such a indeterminate two-span continuous concrete PC beam with external tendons was carried out. In order to provide an accurate prediction of flexural strength, the effect of change of tendon's eccentricity was included. The experimental investigation of the test specimen was conducted to confirm the analytical prediction. In the comparison, it is found that the analytical prediction can show a good agreement with the test results.



a) Load-deflection response



b) Load-tensile forces in external tendons response

Fig.7 Comparison of test results and analytical predictions

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