論文 Path-Dependent Nonlinear Model of Reinforced Concrete Shells

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ABSTRACT: Geometrical nonlinear and material path-dependent constitutive models are presented for the analysis of reinforced concrete shells through finite element method. These models cover loading, unloading and reloading paths. Cracked reinforced concrete is modeled as an orthotropic material using smeared crack approach. Compression softening and tension stiffening effects are included in the derivation of constitutive equations. Layered formulation is used to discretize reinforced concrete shells in thickness. Analytical results are verified using data from experimental works under various loading conditions.

KEYWORDS: reinforced concrete shells, constitutive equations, finite element analysis

1. INTRODUCTION

To correctly understand the behavior of reinforced concrete shells and to predict its response under external loads, realistic constitutive laws for reinforced concrete are crucial. In the past various constitutive models have been proposed [1,2]. However, most of these constitutive models only deal with monotonic loading case where path-dependency is less important than the case of cylic loading.

At present, path-dependent constitutive models for cracked concrete in compression, in tension, in shear, and a path-dependent model of reinforcement in reinforced concrete are available [3]. These constitutive models cover loading, unloading and reloading conditions and have been successfully used to predict the response of concrete panel under in-plane loads [3].

In this study, these models are incorporated through layered formulation to predict the behavior of reinforced concrete shells with combined in-plane and out-of-plane loads and also under cyclic loads through an efficient nonlinear finite element algorithm. Formulation of element and verification with test data are presented in this paper.

2. ELEMENT FORMULATION

Eight-node Serendipity isoparametric element with six degree-of-freedom in each node, three translations and three rotations, was used in the analysis. Reissner-Mindlin formulation was adopted to take into account shear deformation of concrete shells. The formulation is made based on the following assumptions:

1. Normals to the mid-surface remain straight but not necessarily normal to the mid-surface after deformation.
2. Stresses normal to the mid-surface are negligible.

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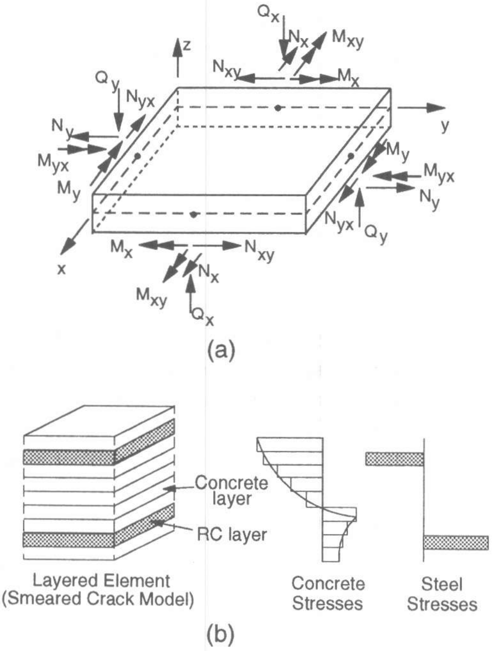

Fig. 1 Layered Element: (a) Forces Acting on RC Shells; (b) Plain Concrete and RC Layers

The use of exact numerical integration to obtain stiffness matrix of element tends to cause shear strain to impose constraints $\gamma_{xz}=\gamma_{yz}=0$ in the total potential energy when the limiting thin shells are approached. This constraint is widely known as shear locking [4]. To avoid the locking in this analysis, numerical integration with the reduced second order of Gaussian quadrature was performed. The element has been tested to be free of shear-locking for the thickness of the specimens used in the verification. Geometrical nonlinearity was accounted by total Lagrangian formulation.

Shell element is divided into several layers of panel where in-plane constitutive models [3] were applied to take into account material nonlinearities as discussed in chapter 3. In the depth of its mid-surface, one integration point was used for each layer of panel. Each layer is classified as plain concrete or reinforced concrete layer with reinforcing bars being smeared in the layer. This classification is important to define the location of reinforcement in the calculation of internal bending moment of the shells. Since the area of concrete which actively contributes to tension stiffening is not known exactly, it is assumed that whole concrete contributes uniformly to the tension stiffening in this analysis and does not depend on the location from reinforcement. In other words, in this analysis tension stiffening used in every layer – with and without reinforcement – is similar and based on tension stiffening with reinforcement smeared in the whole concrete. Average stress-average strain relationship of steel reinforcement in concrete is also derived based on this assumption. Internal forces are calculated by integrating the corresponding stresses from each layer over the thickness of the element. Figure 1 gives the illustration of layered element and forces acting on the shells.

3. MATERIAL MODELING

A smeared fixed crack approach of multi-direction was incorporated in the material modeling of cracked concrete. It consists of tension stiffening model, compression model and shear transfer model. A path-dependent nonlinear model of reinforced concrete has been constructed by combining those models with a model of reinforcing bars in concrete. Prior to cracking, concrete is modeled as an elasto-plastic and fracture material with the introduction of fracture parameter as an indicator of the reduction of elastic modulus in the unloading process [3].

Tension model is independent of spacing of cracks, direction of reinforcing bars and reinforcement ratio. It is modeled in the form of average stress versus average strain of concrete. After average strain reaches cracking strain, concrete stress decreases gradually to take into account tension stiffening effect. In the reversed cyclic loadings, concrete stress is the sum of stress transmitted from the reinforcing bars and that transmitted from the closing of the cracks [3].

Compression model is based on elasto-plastic fracture model similar to the pre-cracking model. The effect of compression softening due to the present of transverse cracks, which causes the reduction of strength and stiffness, is accounted by modifying the value of fracture parameter for the cracked concrete from the uncracked one as a function of strain perpendicular to the crack plane [3].

Shear model is based on contact density function [3]. The model defines the form of a crack based on two parameters, shear displacement and crack width, and applicable to any loading histories. The model gives the shear stress in term of the ratio of those two parameters, regardless of the width of the crack. Compressive stress associated with shear displacement is formulated in the same manner. In the reversed cyclic loading, shear stiffness of uncracked portion, which causes the sudden increase on the closing of the crack, is included.

The modeling of reinforcing bars in concrete is given in the form of average stress versus average strain in use of bilinear line. It has a clear offset point for initiation of strain hardening with the strain hardening rates held constant and is derived for the post-yielding model of bar under monotonic loading. The strain hardening rate is influenced by steel ratio, angle between the bars and the normal to the crack plane, yield strength of the bar, compressive strength of concrete and the bond [3]. In this model, the bond distribution between cracks is modeled as tensile stress distribution in the form of cosine function.

4. ELEMENT SUBJECTED TO BENDING AND IN-PLANE LOADS

A series of tests from the University of Toronto [5] was used in verifying the response of shell element subjected to the combination of bending moment and in-plane loads. The size of the specimens was 1524 x 1524 x 316 mm with two layers of deformed bars in each of the two orthogonal directions. The specimens were subjected to various loading combinations of bending moment and in-plane loads. The details of specimens and loading conditions are given in Table 1.

Specimens were discretized by 16 equal-size finite elements. Through the thickness ten layers per element were used. Comparisons of analyses and experiments are given in Figs. 2 to 5.

In all slabs, tension stiffening model plays an important role to obtain good predictions. Generally the analysis predicts the behavior of specimens well in terms of yield and ultimate moments, except for specimen SM4 where the prediction of ultimate moment is higher than the experiment. This difference might be due to the rotation of crack direction of specimen SM4 when the load was increasing and the yielding of y-reinforcement (weaker reinforcement) was approached as reported in reference [5]. Fixed crack approach used in the finite element analysis provides additional constraints, which causes higher prediction of ultimate moment.

Table 1. University of Toronto Slab Specimens [5]

Specimen	Concrete	Reinforcement					Applied [b] Loading
	f_c' (MPa)	θ (degrees)	ρ_x (%) [a]	f_{yx} (MPa)	ρ_y (%) [a]	f_{yy} (MPa)	M_1:M_2:P
SM1	47	0	1.25	425	0.42	430	1:0:0
SM2	62	0	1.25	425	0.42	430	0.25 m:0:1
SM3	56	0	1.25	425	0.42	430	3.2:1:0
SM4	64	45	1.32	425	0.44	430	0.25 m:0:1

f_c' : compressive strength of concrete
θ : angle of the orientation of reinforcing bars to the specimen
ρ_x , ρ_y : reinforcement ratio in x and y directions
f_{yx} , f_{yy} : yield strength of steel in x and y directions

[a] Per layer [b] See Figs. 2 to 5

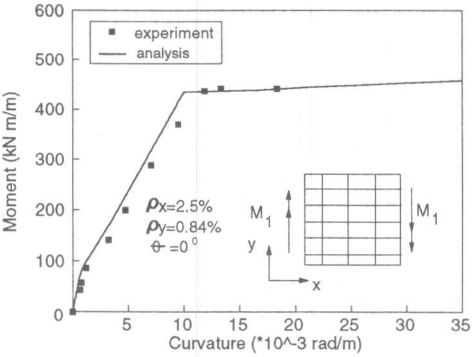

Fig. 2 SM1 under Bending Moment

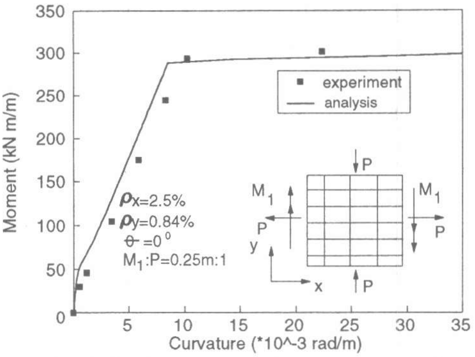

Fig. 3 SM2 under Bending Moment and In-Plane Loads

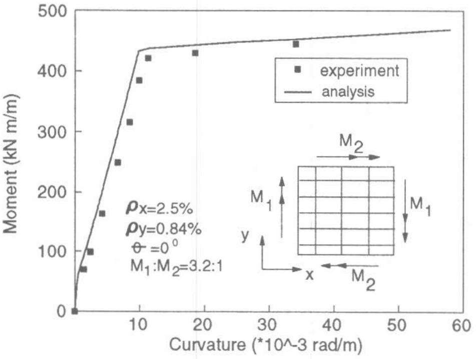

Fig. 4 SM3 under Biaxial Bending Moments (stronger reinforcement)

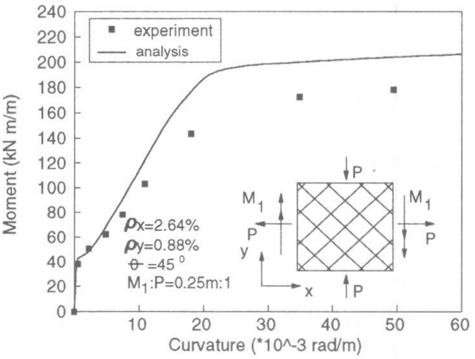

Fig. 5 SM4 under Bending Moment and In-Plane Loads (skew reinforcement)

5. ELEMENTS SUBJECTED TO IN-PLANE AND TRANSVERSE LOADS

A series of tests from the University of Alberta [6] was used for checking the applicability of path-dependent constitutive models in predicting the response of shell element subjected to combined loads of in-plane and transverse loads. Compared to the previous tests, loading condition is not uniform around the element. Depending on the slab's aspect ratio, the test slabs were divided into various series. Type-A slabs were square with outside dimension of 1,830 x 1,830 mm, while type-B slabs were rectangular with the dimension of 2,744 x 1,830 mm. The thickness of the slabs is approximately 65 mm. The specimens were reinforced with two layers of deformed bars placed in orthogonal directions, with the reinforcement ratios for the top and bottom layers being equal. The specimens are simply supported at certain points around the perimeter. In-plane loads were applied along the outside layers of reinforcement while transverse loads were applied at nine points for the A-type slabs, and at 12 points for the B-type slabs [See Figs. 6 and 7]. The detail of specimens and the amount of in-plane loads are given in Table 2.

A-type slabs were divided into a mesh of 6 x 6 elements, while the B-type slabs were divided into a mesh of 6 x 8 elements. Seven layers per element were used for the integration through the depth. Comparisons of analyses and experiments are given in Figs. 6 and 7.

Table 2. University of Alberta Slab Specimens[6]

Specimen	Concrete Properties		Reinforcement			In-Plane
	f_c' (MPa)	E_c (MPa)	ρ_x (%) [a]	ρ_y (%) [a]	f_y (MPa)	(kN/m)
A1	32.3	22,970	0.336	0.390	504	962
A2	32.3	23,010	0.350	0.400	504	765
B1	40.3	25,580	0.500	0.590	504	874
B2	40.2	25,550	0.500	0.590	504	634

E_c : modulus of elasticity of concrete

[a] Per layer

Note: x-direction is normal to the applied in-plane load

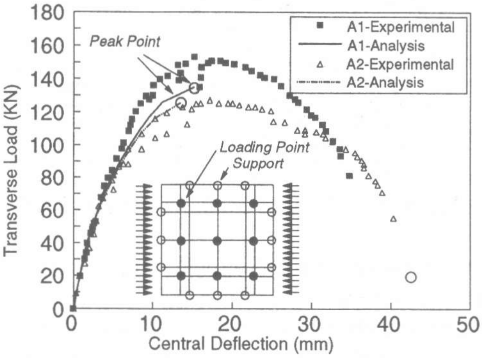

Fig. 6 Type-A Slab Series

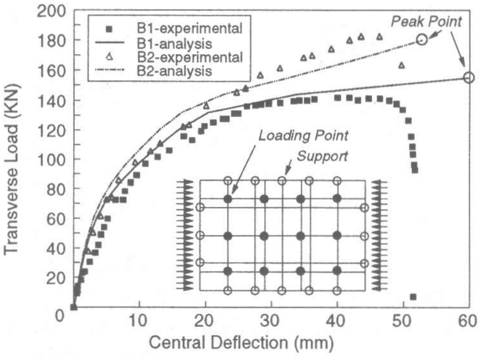

Fig.7 Type-B Slab Series

Since the thickness of slabs is very small compared to the width of slab and furthermore due to the presence of relatively high in-plane loads, geometrical nonlinearity is significant for these cases. These series of tests can provide additional check for the capabilities of the analysis procedure. Generally the model can predict the load-deflection curve of slabs well. It should be noted that since the analyses were carried out by using an increasing load control, the falling branch of the load-deflection curve could not be obtained from the analyses, but the peak capacity point is identified.

6. ELEMENTS SUBJECTED TO CYCLIC TRANSVERSE LOADS

A series of tests consisting of two slabs was conducted under cyclic transverse load at the center of the slabs. The purpose of test is to obtain the data for verifying the capability of shell element in predicting the behavior of slabs under cyclic loadings. The size of the specimens was 1800 x 1800 mm. The slabs were simply supported around the perimeter of the middle part of the slabs of 1400 x 1400 mm. Hence, only this part was considered in the analysis. The specimens were reinforced with two layers of deformed bars placed in orthogonal directions, with the reinforcement ratios for the top and bottom layers being equal. One of the slabs has isotropic arrangement of reinforcing bars, and the other one has anisotropic arrangement. The detail of specimens is given in Table 3.

Specimens were modeled using a mesh of 6 x 6 finite elements. Ten layers per element were used for the integration through the depth. Comparisons of analyses and experiments' cases for both slabs are given in Figs. 8 and 9.

Table 3. Slab-Series Test Specimens

Specimen	Concrete	Reinforcement		
	f_c' (MPa)	ρ_x (%) [a]	ρ_y (%) [a]	f_y (MPa)
SLAB-1	37.0	0.78	0.78	380
SLAB-2	37.0	0.78	0.39	380

[a] Per layer

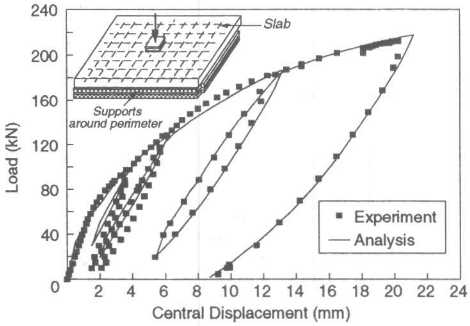

Fig. 8 Slab-1 under Cyclic Transverse Load

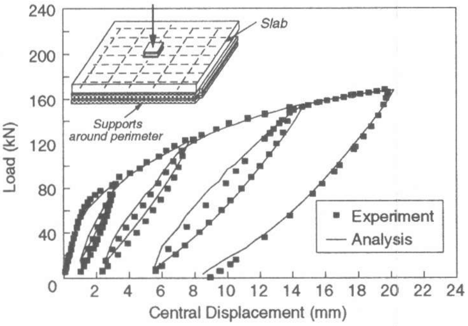

Fig. 9 Slab-2 under Cyclic Transverse Load

The envelope curve of load-deflection for both isotropic and anisotropic arrangements of reinforcing bars were well predicted. The cyclic loop in the case where reinforcing bars in the middle of slabs have been yielding, which also means the crack width is relatively large, was also well predicted. However, in the case where cracking has just occurred, due to complex stress conditions at the closing and opening of cracks, there are some discrepancies in the prediction of cyclic loop in the lower level of transverse loads. This is the target of future development.

7. CONCLUSIONS

A path-dependent nonlinear finite element model has been developed for the analysis of reinforced concrete shells. Through layered formulation, the model has successfully utilized the constitutive models of cracked concrete and reinforcing bar under one dimensional stress condition to predict the behavior of reinforced concrete slabs under the combination of bending moments and in-plane loads, the combination of in-plane and transverse loads , and under cyclic transverse loads.

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