論文 Investigation and Modification of the Characteristic of Unified Concrete Plasticity Model

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ABSTRACT: The characteristics of the plasticity model named Unified Concrete Plasticity Model proposed by Tanabe et al. [1] are investigated by carrying out strain controlled calculations in constitutive level. Before carrying out FEM analysis, it is necessary to adjust the material parameter. The effectiveness of carrying out such strain controlled calculations for this purpose is presented. This calculation technique is used to modify the Unified Concrete Plasticity model significantly. Subsequently this model is applied to carry out FEM analysis of a unreinforced concrete cantilever structure.
KEYWORDS: plasticity, constitutive relations, Unified Concrete Plasticity Model

1. INTRODUCTION

Unlike metal, the behaviour of concrete under tension and compression is significantly different. Most of the constitutive models adopt separate constitutive modelling for tension and compression. In a real structure, except in special cases of uniaxial tension or compression or beam under two point loading which is a case of pure bending, the stress-strain behaviour is generally multi-axial in nature. As a result, one need a constitutive relation that is valid under multi-axial situation. Recently the Unified Concrete Plasticity Model was presented by Tanabe et al. [1] , which was a modified Drucker-Prager approach such that Mohr-Coulumb core match both at the tensile and compressive meridians. This model was numerically able to simulate various experimental results of proportional loading by Kufer[2], and uniaxial tension experiment by Petersson[3] within reasonable limits.

When the model was applied in finite element analysis, problems were encountered even in the numerical calculation of uniaxial compression. This lead the author to. further investigate the characteristics of this model. It was realised that carrying out finite element analysis is impossible when the chosen constitutive model exhibits material instability, like snapback in stress-strain relation etc. because of the choice of inappropriate set of material parameters. Later defects of this model was also detected and modifications are proposed.

2. INVESTIGATION OF THE UNIFIED CONCRETE PLASTICITY MODEL

2.1 THE UNIFIED CONCRETE PLASTICITY MODEL

The Unified Concrete Plasticity Model follows essentially the basic concept of classical theory of hardening plasticity. The subsequent failure surface is assumed to change its size continuously depending on the damage $\omega(W^p)$ accumulated in the concrete material. Associated flow rule is assumed.

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The yield surface is given by

$$f = f(\sigma_{ij}, \varpi(W_p)) = J_2 - (k_f - \alpha_f I_1)^2 + (k_f - \alpha_f \eta)^2 = 0 \quad (1)$$

$$k_f = \frac{6c\cos\phi}{\sqrt{3}(3 + y\sin\phi_1)}, \quad \alpha_f = \frac{2\sin\phi}{\sqrt{3}(3 + y\sin\phi_1)} \quad (2)$$

where ϕ_1 =14° is a material constant. Cohesion c, friction angle ϕ and η depends on the damage of that point ω and are defined by material constants ϕ_0, ϕ_f, c_0, η_0 as

$$c = c_o \exp\left[-(m\varpi)^2\right], \qquad \eta = \eta_0 \exp(-\varpi / b)$$

$$\phi = \begin{cases} \phi_0 + (\phi_f - \phi_0)\sqrt{2\varpi - \varpi^2} & \varpi \le 1 \\ \phi_f & \varpi > 1 \end{cases} \quad (3)$$

and y is a parameter that is used to match the Drucker-Prager based surface with Mohr-Coulumb core, both at the tensile and compressive meridian

$$y = \sqrt{a(\cos 3\theta + 1.00) + 0.01} - 1.10, \; a = 0.5r^2 + 2.1r + 2.2$$

$$r = \begin{cases} 3.14 & I \le f_c' \\ 2.93\cos(I_1\pi / f_c') + 6.07 & f_c' < I \le f_t \\ 9.0 & I > f_t \end{cases} \quad (4)$$

The classical plasticity approach is adopted here. The damage parameter $\omega(W^P)$ is defined as

$$\varpi = \frac{\beta}{\sigma_e \varepsilon_o} \int dW^p \quad \text{where} \quad dW^p = \sigma_{ij} d\varepsilon_{ij}^p = \sigma_e d\varepsilon^p \quad (5)$$

where $d\varepsilon_{ij}^p$ is the plastic strain, dW^P is the plastic work, σ_e and $d\varepsilon^p$ is effective plastic stress and strain, β and ε_O are material constants.

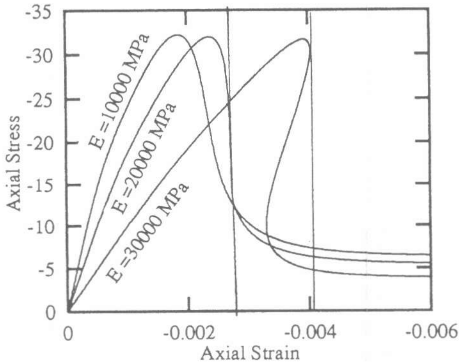
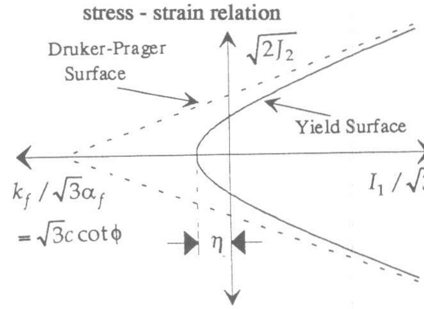
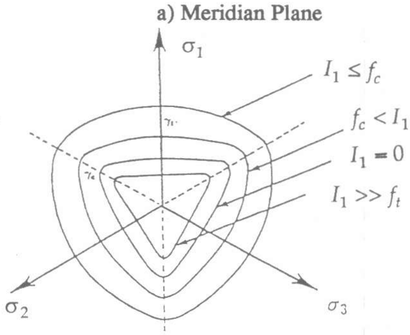


Fig 1: Snapback- example of ill conditioned stress - strain relation



a) Meridian Plane



b) Diviatoric Plane

Fig 2: The Yield Surface

2.2 CONSTITUTIVE LEVEL CALCULATION

Adjustment of the material parameter to simulate the correct concrete strength and ductility is an important step in finite element analysis. Finite element calculation is a very time consuming process. If, material instability exists, it is a very frustrating experience. The basic calculation carried out in the constitutive level is the calculation of *tangent modulus* D^{ep}. Stress is calculated from strain. This is more like displacement controlled process where instability occurs when softening curve is *ill conditioned* because the stress-strain curve shows snapback(fig. 1). To over come this problem, constitutive level calculation is found to be very effective. All results presented in this paper are results of constitutive level calculation. This method is later applied to understand and modify the Unified Concrete Plasticity Model. When finite element analysis gives undesired results, the strain history can be economically used in this process to understand what really is happening.

2.2 UNDERSTANDING THE CONSTITUTIVE MODEL: EFFECT OF c, ϕ AND η

The shape of the yield surface (fig. 2a and 2b) depends of the value of c , ϕ and η, which changes as the damage parameter ω increases incrementally. The stress strain relation largely depends on the rate of change of these parameters, whereas the ductility of the stress strain curve largely depends on the relation between plastic strain and ω , based on material parameters like β and ε_0.

(1) *Uniaxial compression*

Figure 3. presents a parametric study for uniaxial compression. Obviously, we get a perfectly elastic curve when c, ϕ and η are assumed constant. The stress-strain softening slope when only c changes while keeping ϕ and η constant, resemble the c-ω curve. The effect of ϕ looks like it changes the peak of the curve a little round. The lateral strain depends on the value of ϕ. The uniaxial behaviour under changing c and ϕ falls in between the previous two curves. η has almost no effect on uniaxial compression.

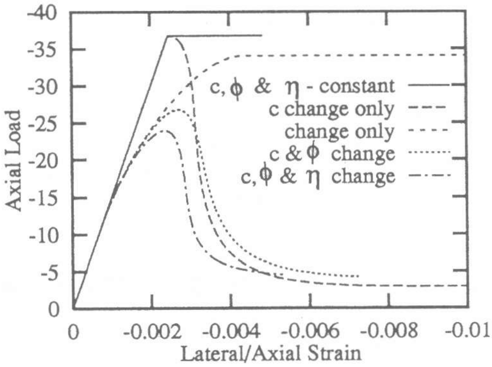


Fig. 3: Parametric study on uniaxial compression

(2) *Uniaxial tension*

Though it is not shown, we can understand that the peak of uniaxial tension curve depends on η_o,c_o and ϕ_o , the axial behaviour largely depends on how fast the tip of the yield surface moves towards the axis, i.e. rate of change of η. We get more ductile curve as the value of b increases as shown in the original paper[1]. It was stated that ϕ has less effect on the uniaxial tensile behaviour. This statement was found to be true only in case of the axial behaviour, it has significant effect on the lateral behaviour. Figure 4 shows the lateral and axial behaviour of concrete under tension with equal initial friction angle ϕ_o and different values of final friction angle ϕ_f.

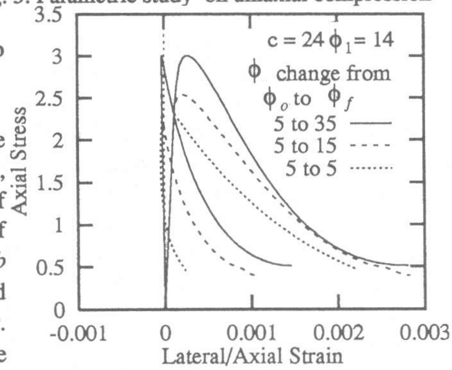


Fig 4: Parametric study ϕ of uniaxial tension showing the lateral strain

(3) *Pure Shear and mixed shear calculations*

Though there is no experimental basis to check these calculation results, this type of situation is very frequent in real structures, like cantilever. Though the cases of pure shear and compression-shear looks fine(not shown due to space limitation), a *hunch back* like curve is noticed in case of shear stress for shear-tension calculation(fig. 5). Figure 5 shows two cases with different ratios of applied tension:shear strain. The *hunch back* occurs when the value of η approximately becomes zero (not shown due to space limitation).

2.3 DEFECTS OF UNIFIED CONCRETE MODEL

The following two major defects were detected in the Unified Concrete Plasticity Model

a) The *hunch back* like curve creates numerical problems while solving the cantilever problem. This looks illogical too because concrete that has failed under tension, cannot take an increasing amount of shear stress.

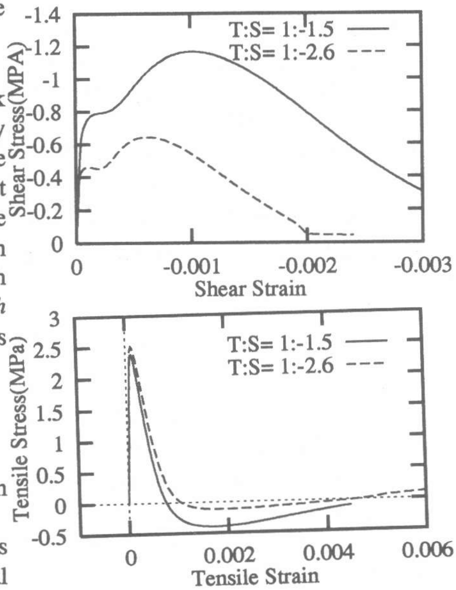


Fig 5: Stress-Strain relation when tensile strain:shear strain = T:S is applied

b) In the original paper[1], Tanabe et. al. had proposed that similar parameters for c, ϕ and η could be used in all situations(concrete used had η_o= 5.5 MPa, C_o=24MPa, ϕ_o =5° and ϕ_f =35°). In the original paper[1] these results(fig. 5) are not mentioned. It looks like concrete under uniaxial tension will expand laterally. This looks illogical because concrete cannot expand laterally while cracking.

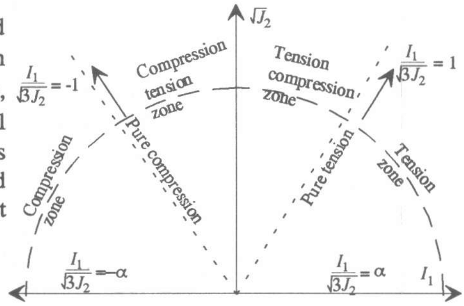


Fig. 6: Different zones in ($I_1, \sqrt{J_2}$) space

3. MODIFYING UNIFIED CONCRETE MODEL

The state of stress-strain, or even the shape of the yield surface highly depends on the *cohesion* and *friction* angle. Assuming that the initial material parameters are constant, though no experimental verification exists, it is not difficult to understand the rate of change of cohesion and friction angle should not be the same in case of uniaxial tension and uniaxial compression. But Tanabe et al. [1] in an attempt to create a unified theory which can be applied to all cases, introduced a material parameter η, which is the distance between the tip of the yield surface in tension zone from the origin. By making the rate of change of η independent of the rate of change of c and ϕ, the tip of the yield surface, and hence the tension zone is reduced to zero at a faster rate, even though the value of c is not yet zero. This gave excellent results in tension zone and compression zone. The author, did not check the results presented by Tanabe et. al.[1] in the case of tension-compression zone, however the uniaxial tension and compression calculation was checked and excellent results were obtained. The present author finds the assumption that the range of ϕ to be constant in all cases to be illogical because it presents illogical results for lateral behaviour in case of uniaxial tension. The present author also finds the fact that rate of change of η to be independent of rate of change of c and ϕ as not logical.

However the good results predicted by the Unified Concrete Plasticity Model cannot be neglected. The tension behaviour is largely controlled by the rate of change of the distance between the tip and the origin(value of η). So we have to mentain similar η-ω relation. We know that the tip of the Draker-Prager surface model is given by $\sqrt{3}c\cot\phi = k/(\sqrt{3}\alpha)$ (fig. 2a). Assuming the same η_o as before[1], η is assumed to be

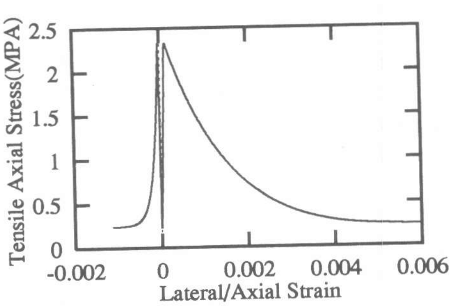
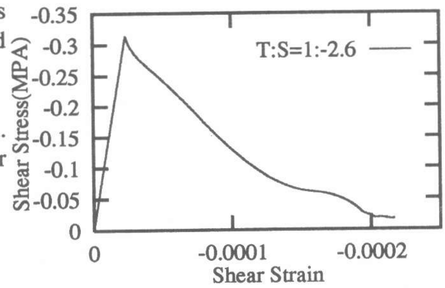


Fig. 7: Uniaxial tension behaviour showing lateral strain

$$\eta = AA*\sqrt{3}c\cot\phi = AA*k/(\sqrt{3}\alpha) \qquad \text{where } AA* = \sqrt{3}c_o\cot\phi_o/\eta_o \qquad (6)$$

Assuming ϕ to be constant for tension zone, and changing from ϕ_o to ϕ_f in compression zone, it is obvious that the rate of change of c in tension zone should be exactly similar to that of η. That means that c-ω will also look like the softening curve in case of uniaxial tension. This conclusion is similar to an observation in sec. 2.2.1 where we found that the c-ω curve also resembles the softening curve in case of uniaxial compression. In case of uniaxial compression or tension, the relation between the softening curve, hence the rate of change of strength, and the c-ω curve which is nothing but the rate of change of cohesive force with damage looks logical.

The author found it difficult to decide how the intermediate values of c and ϕ should be. The author has assumed a gradual variation between the tension to compression zone based on the parameter $I_1/(\sqrt{3J_2})$ whose variation is shown fig. 6. The author has no justification for this assumption except for the fact that the above two defects does not arise in this present model as shown in fig. 7 where we find the lateral behaviour to be significantly different. The author is not sure if this lateral behaviour is correct or not because of the unavailability of experimental data, however the illogical lateral expansion of concrete under tension does not appear after this modification. Fig. 8 shows the behaviour of concrete when proportional shear strain and tensile strain is applied, according to this model. We do not notice the *hunch back* as noticed before.



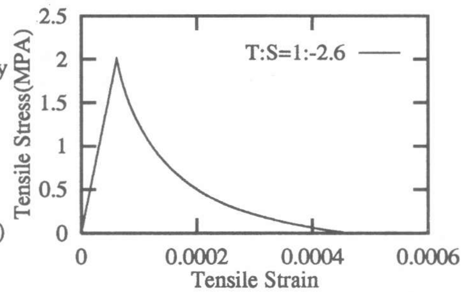


Fig 8. Stress-strain when tensile strain:shear stress=T:S is applied using modified Unified Concrete Plasticity Model

Here the modified unified concrete model is presented. Equations that have not changed are not presented for brevity.

$$\begin{aligned} f &= f(\sigma_{ij}, \varpi(W_p)) \\ &= J_2 - (k_f - \alpha_f I_1)^2 + (k_f - \alpha_f \eta)^2 \qquad (7) \\ &= J_2 - (k_f - \alpha_f I_1)^2 + (k_f - AA/(\alpha_f \sqrt{3}))^2 \\ &= J_2 - (k_f - \alpha_f I_1)^2 + k_f^{\,2}(1 - AA/\sqrt{3})^2 = 0 \end{aligned}$$

where the material property is defined as previously explained to match tension and compression zone

$$c = c_o \exp\left[p_1\left(\frac{I_1}{\sqrt{3J_2}}\right)(-m_1\varpi) + p_2\left(\frac{I_1}{\sqrt{3J_2}}\right)(-m^2\varpi^2) \right]$$

$$\phi = \begin{cases} \phi_0 + (\phi_f - \phi_0) p_2\left(\dfrac{I_1}{\sqrt{3J_2}}\right)\sqrt{(\varpi + k)(2 - \omega - k)} & \varpi \le 1 \\ \phi_0 + (\phi_f - \phi_0) p_2\left(\dfrac{I_1}{\sqrt{3J_2}}\right) & \varpi > 1 \end{cases} \qquad (8)$$

where $k=10^{-3}$ (or any small value) is introduced to get rid of the singularity caused by $\dfrac{\partial\phi}{\partial\omega} = \infty \quad at \quad \omega = 0$,

$$\left\{ p_1\left(\frac{I_1}{\sqrt{3J_2}}\right) \quad p_2\left(\frac{I_1}{\sqrt{3J_2}}\right) \right\} = \begin{cases} 0 \qquad 0 & \dfrac{I_1}{\sqrt{3J_2}} \le -1 \\ \dfrac{1}{2}\sin\left(\dfrac{I_1}{\sqrt{3J_2}}\dfrac{\pi}{2}\right) + \dfrac{1}{2} \qquad -\dfrac{1}{2}\sin\left(\dfrac{I_1}{\sqrt{3J_2}}\dfrac{\pi}{2}\right) + \dfrac{1}{2} & -1 < \dfrac{I_1}{\sqrt{3J_2}} \le 1 \\ 1 \qquad 1 & \dfrac{I_1}{\sqrt{3J_2}} > 1 \end{cases} \qquad (9)$$

and y is a parameter that is used to match the Draker-Prager based surface with Mohr-Coulumb core, both at the tensile and compressive meridian. This equation is also modified to make it continuous in all ranges.

$$y = \sqrt{a(\cos 3\theta + 1.00) + 0.01} - 1.10, \quad a = 0.5r^2 + 2.1r + 2.2$$

$$r = \begin{cases} 3.14 & I \le f_c' \\ 2.93\cos\left(\dfrac{f_t - I_1}{f_t - f_c'}\pi\right) + 6.07 & f_c' < I \le f_t \\ 9.0 & I > f_t \end{cases} \qquad (11)$$

4. NUMERICAL PROBLEM: CANTILEVER

A cantilever (fig. 9), with a length of 2.5 m and a cross-section area of 250mm x 500 mm with end condition fixed is chosen. The set of material parameters are chosen after due consideration of the constitutive level instabilities. Material parameters chosen are

E_c=15000MPa, f_t=1.46MPa, f_c=14.6MPa, c=15MPA, ϕ_o=4°, ϕ_f=25°, ϕ_1 =14°, η_o=3.5 MPa, ε_o=0.002, m_1 =10, m_2=1.0

Present author was unable to obtain convergence with the original formulation[1] because of the *hunch back* problem. However in the present model, this problem did not arise and a very nice load deflection diagram was achieved. The load at which the extreme fibre reaches tensile strength is calculated as:

Approximate load= $(0.5*f_t*(bd/2)*2d/3)/L$ = 6.08KN

Numerical Peak value = 9.09 KN

We achieved slightly higher value, which probably can be attributed to the stress redistribution after softening starts in the extreme fibre after it reaches the peak stress.

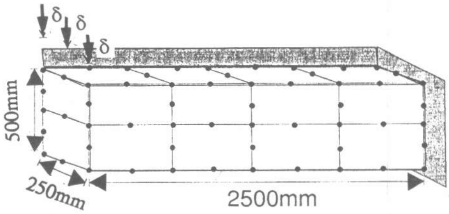


Fig. 9: The Finite Element mesh

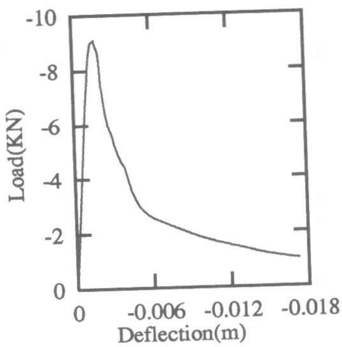


Fig. 10: Load Deflection Diagram

5. CONCLUSION

The Unified Concrete Plasticity Model has been analysed and modified. This proposed model is not yet perfect and has to be checked for various other condition. But we can conclude the following

a) *Constitutive level calculations* are very necessary before conducting any finite element analysis. By this method we can avoid frustrating experience due to material instability.

b) The uniaxial softening curve is very similar in shape as the c-ω curve. So it is important to recognise variation of c and ϕ at various range appropriately. In tension it is recognised that first order (negative) exponential curve as more appropriate where as higher order is more appropriate in compressive zone

c) The initial value of friction angle is assumed to be constant. It was found that almost constant ϕ for uniaxial tension and gradually increasing ϕ for uniaxial compression provides better results for lateral effect.

d) By making η dependent on c and ϕ , the *hunch back* like curve could be avoided.

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