論文 Micromechanical Constitutive Relationship of Steel Fiber Reinforced Concrete with Strain Rate Effect

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ABSTRACT: A microstructural constitutive relationship that can describe the strain-rate effects on the mechanical properties of steel fiber reinforced concrete (SFRC) is presented. The constitutive properties are characterized separately on a microplane by normal deviatoric and volumetric strains and shear strain. The composite mechanical behavior which is affected by fiber and concrete matrix properties of each microplane is modeled. The material parameters are determined from test data. By comparison with experimental results from a literature, the appropriateness of modeling is confirmed.

KEYWORDS: steel fiber, concrete matrix, composite material, stress rate, strain rate, micromechanical constitutive relationship

1. INTRODUCTION

Concrete may be subjected to rapidly applied impact loads generating very high loading rates. Because concrete is a brittle and loading rate-sensitive material, a special consideration on these characteristics in the design of concrete structures subjected to impact loads is essential.

For concrete subjected to a tensile load, failure occurs first at the weak aggregate-mortar interfaces where some cracking exists even prior to load application. Propagation of these bond cracks may occur at a stress level far below the apparent proportional limit. At higher stresses, the bond cracks propagate rapidly under the imposed stress concentrations at their tips and penetrate into the matrix. With a further increase in the applied stress, cracks align themselves perpendicularly to the direction of load and eventually coalesce to precipitate a rapid tensile fracture. This weakness of concrete can be improved by addition of steel fibers. Fibers bridging with closing pressure control the opening of microcracks. The toughening effect of dispersed short steel fibers on the properties of steel fiber reinforced concrete (SFRC) can be used for specific applications for which conventional concrete materials are not adequate.

Although a constitutive model of SFRC considering the microstructural behavior has been accomplished [1], the strain rate effect which can be related to loading rate has not been taken into account. Therefore, in this study, a micromechanical constitutive model of SFRC considering strain rate effects is investigated. The constitutive properties are characterized separately on small planes of various orientations within the material referred to as microplanes. The state of each microplane is characterized by normal deviatoric and volumetric strains and shear strain. The composite mechanical behavior of fiber and concrete on each microplane is modeled and the material parameters are determined from experimental data.

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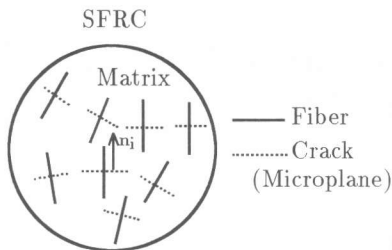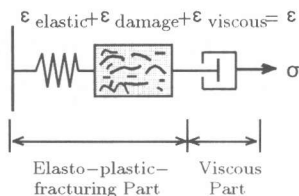
Fig.1 Fibers and Microplanes

Fig.2 Generalized Maxwell Model
for Each Microplane Component

2. MICROMECHANICAL CONSTITUTIVE MODEL

2.1 MODELING OF FIBER AND CONCRETE

The following assumptions are made to develop a micromechanical constitutive model for SFRC by incorporating the microplane model of concrete [2].

1. Fibers are uniformly distributed in concrete matrix (see Fig.1).
2. Fibers are bonded perfectly in concrete at the initial state.
3. Fibers are normal to microplanes.
4. Fibers are effective to both tension in the normal direction and shear in the tangential direction of a microplane.
5. The microscopic constitutive relationships for the normal and shear components on each microplane are mutually independent and are based on a generalized Maxwell viscoplastic model in which a viscous portion is coupled in series with an elasto-plastic-fracturing portion (see Fig.2).

The relationship between the microstrain ϵ or the strain at a microplane and the combined bond stress of several fibers at the microplane $\sigma^{\mathbf{f}}$ is modeled as shown in Fig.3 [1]. By the isostrain condition (Assumption 2) of a composite material, the microscopic stresses for fiber are modeled as follows:

$$\sigma_V^f = C_V^f(\epsilon_V)\epsilon_V \; ; \; \sigma_D^f = C_D^f(\epsilon_D)\epsilon_D \; ; \; \sigma_T^f = C_T^f(\epsilon_T)\epsilon_T \tag{1}$$

in which the fiber secant moduli for tension $\mathbf{C^f}$ are expressed as

$$C_V^f = \overline{C}_{V_0}^{\;f}(1 - \omega_V^f) \; ; \; C_D^f = \overline{C}_{D_0}^{\;f}(1 - \omega_D^f) \; ; \; C_T^f = \overline{C}_{T_0}^{\;f}(1 - \omega_T^f) \tag{2}$$

where $\overline{C}_{V_0}^{\;f}$, $\overline{C}_{D_0}^{\;f}$ and $\overline{C}_{T_0}^{\;f}$ are the fiber initial elastic moduli, ω_V^f, ω_D^f and ω_T^f are the microscopic fiber damage parameters for volumetric, deviatoric, and shear components, expressed as by the following equations.

$$\omega_V^f = exp\left[-\frac{\epsilon_V}{\zeta_1^f}\right]^{\kappa_1^f} \; ; \; \omega_D^f = exp\left[-\frac{\epsilon_D}{\zeta_1^f}\right]^{\kappa_1^f} \; ; \; \omega_T^f = exp\left[-\frac{\epsilon_T}{\zeta_2^f}\right]^{\kappa_2^f} \tag{3}$$

where $\zeta_1^f, \zeta_2^f, \kappa_1^f$, and κ_2^f are the fiber material parameters.

The microscopic stresses for concrete matrix $\sigma^{\mathbf{m}}$ are expressed as follows:

$$\sigma_V^m = C_V^m(\epsilon_V)\epsilon_V \; ; \; \sigma_D^m = C_D^m(\epsilon_D)\epsilon_D \; ; \; \sigma_T^m = C_T^m(\epsilon_T)\epsilon_T \tag{4}$$
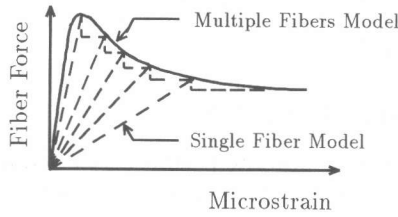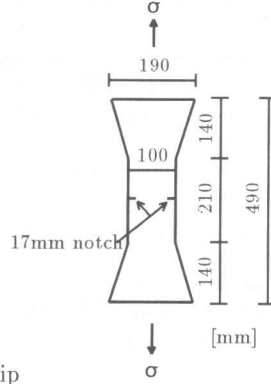
Fig.3 Fiber Force−Microstrain Relationship

Fig.4 Test Specimen[5]

in which the matrix secant moduli for tension $\mathbf{C^m}$ are expressed as

$$C_V^m = \overline{C}_{V_0}{}^m (1 - \omega_V^m) \; ; \; C_D^m = \overline{C}_{D_0}{}^m (1 - \omega_D^m) \; ; \; C_T^m = \overline{C}_{T_0}{}^m (1 - \omega_T^m) \qquad (5)$$

where ω_V^m, ω_D^m and ω_T^m are the microscopic matrix damage parameters for volumetric, deviatoric, and shear components, expressed by the following equations.

$$\omega_V^m = exp\left[-\frac{\epsilon_V}{\zeta_1^m}\right]^{\kappa_1^m} \; ; \; \omega_D^m = exp\left[-\frac{\epsilon_D}{\zeta_1^m}\right]^{\kappa_1^m} \; ; \; \omega_T^m = exp\left[-\frac{\epsilon_T}{\zeta_2^m}\right]^{\kappa_2^m} \qquad (6)$$

$\overline{C}_{V_0}{}^m, \overline{C}_{D_0}{}^m, \overline{C}_{T_0}{}^m$ and C_V^m, C_D^m, C_T^m are the initial elastic and secant moduli for concrete matrix. $\zeta_1^m, \zeta_2^m, \kappa_1^m$, and κ_2^m are concrete matrix material parameters.

Subscripts V, D and T express the volumetric, deviatoric and shear components and superscripts f and m stand for fiber and matrix, respectively.

For SFRC which is considered as a composite material, the secant moduli $\mathbf{C}$ become as follows:

$$C_V = C_V^m(1 - V_f) + C_V^f V_f \; ; \; C_D = C_D^m(1 - V_f) + C_D^f V_f \; ; \; C_T = C_T^m(1 - V_f) + C_T^f V_f \quad (7)$$

where V_f is the volume fraction of fiber.

For compression, the matrix secant moduli are expressed as in Ref. [2] and the fiber secant moduli are neglected.

2.2   COMPOSITE ELASTIC MODULI

The initial elastic moduli of fiber and concrete are expressed as follows by composite material consideration [1].

$$\overline{C}_{V_0}{}^f = (C_{V_0}{}^f - C_{V_0}{}^m)\Lambda + C_{V_0}{}^m \quad ; \quad \overline{C}_{V_0}{}^m = (C_{V_0}{}^f - C_{V_0}{}^m)V_f\Lambda + C_{V_0}{}^m \qquad (8)$$

$$\overline{C}_{D_0}{}^f = (C_{D_0}{}^f - C_{D_0}{}^m)\Lambda + C_{D_0}{}^m \quad ; \quad \overline{C}_{D_0}{}^m = (C_{D_0}{}^f - C_{D_0}{}^m)V_f\Lambda + C_{D_0}{}^m \qquad (9)$$

$$\overline{C}_{T_0}{}^f = (C_{T_0}{}^f - C_{T_0}{}^m)\Lambda + C_{T_0}{}^m \quad ; \quad \overline{C}_{T_0}{}^m = (C_{T_0}{}^f - C_{T_0}{}^m)V_f\Lambda + C_{T_0}{}^m \qquad (10)$$

in which

$$C_{V_0}{}^f = \frac{E_f}{E_m}C_{V_0}{}^m \; ; \; C_{D_0}{}^f = \frac{E_f}{E_m}C_{D_0}{}^m \; ; \; C_{T_0}{}^f = \frac{G_f}{G_m}C_{T_0}{}^m \qquad (11)$$

$$C_{V_0}^m = \frac{E_m}{1-2\nu_m} \ ; \ C_{D_0}^m = \eta_0 C_{V_0}^m \ ; \ C_{T_0}^m = \frac{1}{3}\left[\frac{5(1-2\nu_m)}{1+\nu_m} - 2\eta_0\right] C_{V_0}^m \qquad (12)$$

where Λ is the reduction ratio of composite effective modulus, E_f, E_m are the elastic moduli of fiber and concrete matrix, respectively, G_f, G_m are the respective elastic shear moduli, ν_m is the Poisson's ratio of the concrete matrix and $\eta_0 = (1-2\nu_m)/(1+\nu_m)$. Eqs.(7)–(11) are assumed from the expression of the elastic modulus of a composite material reinforced by aligned straight fibers [3].

2.3 MACROSCOPIC STRESS-STRAIN RELATIONSHIP

Using the principle of virtual work to approximately enforce the equivalence of forces on the microscale and the macroscale, the following macroscopic stress-strain relationship can be obtained.

$$\Delta\sigma_{ij} = C_{ijkm}\Delta\epsilon_{km} - \Delta\sigma_{ij}'' \qquad (13)$$

where C_{ijkm} denotes the incremental stiffness tensor (elastic modulus tensor)

$$C_{ijkm} = \frac{3}{2\pi}\int_S \left[(C_D^t - C_T^t)n_i n_j n_k n_m + \frac{1}{3}(C_V^t - C_D^t)n_i n_j \delta_{km} \right.$$
$$\left. + \frac{1}{4}C_T^t(n_i n_k \delta_{jm} + n_i n_m \delta_{jk} + n_j n_k \delta_{im} + n_j n_m \delta_{ik})\right] f(\mathbf{n}) \ dS \qquad (14)$$

and $d\sigma_{ij}''$ denotes the inelastic stress increments

$$\Delta\sigma_{ij}'' = \frac{3}{2\pi}\int_S \left[n_i n_j(\Delta\sigma_V'' + \Delta\sigma_D'') + \frac{1}{2}(n_i \delta_{rj} + n_j \delta_{ri} \right.$$
$$\left. - 2n_i n_j n_r)\Delta\sigma_{T_r}'' \right] f(\mathbf{n}) \ dS \qquad (15)$$

where C_V^t, C_D^t and C_T^t are the incremental elastic moduli for the current loading step for a microplane and n_i is the microplane direction cosine, $d\sigma_N''$ and $d\sigma_{T_r}''$ are the inelastic normal and shear stress increments, S is the surface of a unit hemisphere and $f(\mathbf{n})$ is the weighting function of the normal unit vector $\mathbf{n}$. The unloading and reloading criteria for the composite behavior are assumed to follow those of the microplane model [2].

3. RATE EFFECT IN MICROMECHANICAL MODEL

3.1 VISCOPLASTIC MODEL

A series coupling of a linear viscous element and an elastoplastic fracturing element is adopted to specify the strain rate dependent composite microscopic constitutive relationship for each microplane [2]. Calculations of microstrains ϵ_V, ϵ_D and ϵ_T are the same for those in Ref. [2]. For the sake of brevity of notation, let ϵ and σ represent any of the microstrains ϵ_V, ϵ_D and ϵ_T and microstresses σ_V, σ_D and σ_T.

The microscopic stress-strain relationship for each microplane is described by the following differential equation.

$$\dot{\sigma} = \check{C}\dot{\epsilon} - \frac{\sigma}{\gamma} \qquad (16)$$

where the tangent modulus, C_t and the apparent relaxation time, γ are expressed as follows:

$$C_t = \check{C} + \epsilon\frac{\Delta\check{C}}{\Delta\epsilon} \ ; \ \frac{1}{\gamma} = \frac{1}{\beta} - (C_t - \check{C})\frac{\dot{\epsilon}}{\sigma} \qquad (17)$$

Table 1 Optimum Values of Material Parameters

Specimen	$\dot{\sigma}$ [MPa/s]	$\dot{\epsilon}$ [s^{-1}]	E_{imp} [MPa]	β [10^{4}s]	$\zeta_1^f = \zeta_1^m$ 10^{-4}	$\zeta_2^f = \zeta_2^m$ 10^{-3}	$\kappa_1^f = \kappa_1^m$	$\kappa_2^f = \kappa_2^m$
SFRC"A"	0.001	4.49 ×10^{-8}	22300					
with	1.10	4.14 ×10^{-5}	26500					
Hook–ended	54.0	1.85 ×10^{-3}	29300	0.03	0.33	0.50	0.80	1.50
Steel Fiber	720	2.31 ×10^{-2}	31200					
SFRC"B"	0.001	4.12 ×10^{-8}	24300					
with	1.20	4.14 ×10^{-5}	29000					
Straight	56.0	1.75 ×10^{-3}	31900	0.30	0.22	0.40	0.50	1.80
Steel Fiber	700	2.06 ×10^{-2}	34000					

where $\check{C}$ is the secant modulus for the microplane strain component and β is a material parameter representing the relaxation time of the viscous part. The rate effect is introduced into the microplane model by extending the exponential algorithm [4]. The exact solution of the differential equation [Eq.(16)] for the loading step under the assumption that the loads, the material properties, and the prescribed rates are constant in time gives the following form of a pseudoelastic microscopic stress-strain relationship.

$$\Delta\sigma = D\Delta\epsilon - \Delta\sigma'' \tag{18}$$

$$D = \frac{1}{\Delta z}(1 - \exp[-\Delta z])\check{C} \ ; \ \Delta\sigma'' = (1 - \exp[-\Delta z])\sigma_\tau \tag{19}$$

where $\Delta z = \Delta t/\gamma$ and Δt is a time increment.

3.2 IMPACT MODULUS OF ELASTICITY

The influence of tensile stress rate on the macroscopic modulus of elasticity of concrete is estimated by using the following equations in CEB Model Code 1990.

$$\frac{E_{c,imp}}{E_{ci}} = \left[\frac{\dot{\sigma}_c}{\dot{\sigma}_{cto}}\right]^{0.025} \tag{20}$$

where $E_{c,imp}$ is the impact modulus of elasticity of concrete, E_{ci} is the static modulus of elasticity of concrete, $\dot{\sigma}_c$ is the stress rate (MPa/s) and $\dot{\sigma}_{cto} = 0.1$MPa/s.

The strain rate is estimated from the stress rate given in test data by the following equation using Eq.(20).

$$\dot{\epsilon}_c = \frac{\dot{\sigma}_c}{E_{c,imp}} \tag{21}$$

The strain rate effect for steel fiber is neglected in this study.

4. COMPARISON WITH TEST RESULTS

In order to verify the present model, the experimental data by Glinicki, M.A. [5] is used. Mortars reinforced with steel fibers with mix proportions of 1:4.1:0.6 by weight
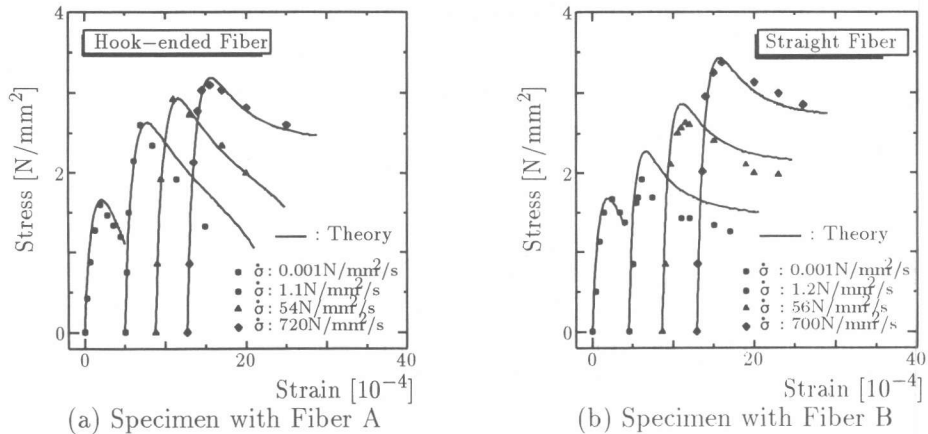
(a) Specimen with Fiber A   (b) Specimen with Fiber B

Fig.5 Stress−Strain Curves with Rate Effect

(cement:sand:water) were used. Two types of steel fibers were used: Fiber A [hook-ended, 30.0mm long, 0.40mm in diameter] and Fiber B [straight, 32.3mm long, 0.80mm in diameter]. The fiber volume fraction was of 1.0 and 1.4 percent for Fiber A and B, respectively. Different volume fractions were chosen to obtain the same static tensile strength of composites. The paddle-shaped specimens shown in Fig.4 were used. The specimens were subjected to direct tension in a testing machine at a controlled rate of cross-head displacement.

Applying spherical integration formula with 21 microplanes, the optimization by which the fits have been achieved was carried out simply by a trial-and-error approach. By fitting SFRC experimental data with various loading rates as shown in Fig.5, the optimum material parameters were obtained and are listed in Table 1. With the present model, the triaxial strain rate-dependent response of SFRC can be simulated although experimental data is not enough.

5. CONCLUSION

A micromechanical constitutive model based on the microplane model is proposed for SFRC. It is confirmed that the present model is suitable to express the influence of strain rate on the macroscopic mechanical behavior of SFRC. The present model can be used in a finite element analysis of SFRC.

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