論文 Crack Spacing Analysis of Reinforced Concrete by Gradient Plasticity Model

K. M. AMANAT*1 , T. TANABE*2

ABSTRACT: A higher order continuum method, namely the gradient plasticity approach [1] is applied to analyze the fracture and crack propagation of reinforced concrete members in tension. Separate finite element modeling is done for each constituents of reinforced concrete - concrete, reinforcement and the bond-slip mechanism. A gradient enhanced continuum approach was used for concrete. Analyses was performed to investigate the capability of the gradient plasticity approach to produce discrete cracked zones at regular spacing. The effect of different reinforcement ratio on crack spacing and on the average stress-strain relation of steel is studied.

KEY WORDS: Fracture, Plasticity, Gradient Plasticity, Crack, Reinforced Concrete.

1. INTRODUCTION

In practical situations we usually encounter concrete with reinforcement whether the structure is a usual beam or pavement or it is a massive structure like the dams. In the previous works [2] we dealt with plain concrete. It helped us understanding the fundamental aspects of fracture and localization behavior of concrete. Tests on reinforced concrete [3] suggests that the behavior of concrete when reinforced with steel may significantly change from that of plain concrete and it may be argued that understanding of the characteristics of plain concrete may not be enough to predict the behavior of concrete in a RC (reinforced concrete) structure. Acknowledging this fact a number of theoretical model for the constitutive relation of reinforced concrete have been proposed in the recent past based on experimental studies. Pioneering work in this area are those of Belarbi and Hsu[4], Chang et al. [5], Hsu and Zhang[6] etc. However, it appears that all of the proposed theoretical models are based on simplified assumptions of the actual behavior found in experiments. Modeling of concrete is done upon the consideration of average stress-strain relation in the cracked and uncracked portion of concrete which is widely known as the tension stiffening effect of concrete. Although such models have found to be successful in simulating the overall load displacement behavior of concrete, they fail to do the same through the simulation of cracks discretely. To seek a more generalized approach, we will study the possibility of simulating the cracking behavior of reinforced concrete by gradient plasticity method in this paper.

2. NUMERICAL MODELING OF REINFORCED CONCRETE

When the overall macroscopic behavior of concrete is of primary interest we can model the reinforced concrete by finite element discretization where we use two or three dimensional elements for concrete and two or three dimensional truss elements or line elements for steel in such a way that the nodes where the steel elements are connected to the concrete element have same degrees of freedom. The disadvantage of such modeling is that they cannot simulate the actual cracking phenomenon. A more realistic model is to discretize the concrete and steel separately so that each components have completely independent degrees of freedom which are connected via special finite elements which simulate the bond-slip mechanism. Such modeling is more straightforward and we will use this approach in this paper.

2.1 Constitutive Model for Steel Reinforcement

The behavior of steel under uniaxial tension is much different than that of concrete. After yielding it shows some hardening and a rather long yield plateau. Also its behavior is much more predictable than

*1 Graduate Student, Dept. of Civil Engineering, Nagoya University, Japan, Member of JCI.
*2 Professor, Dept. of Civil Engineering, Nagoya University, Japan, Member of JCI.

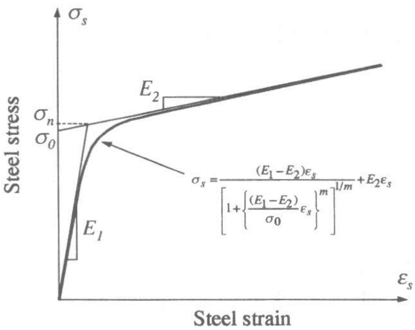

Fig.1 Stress-strain relation for steel

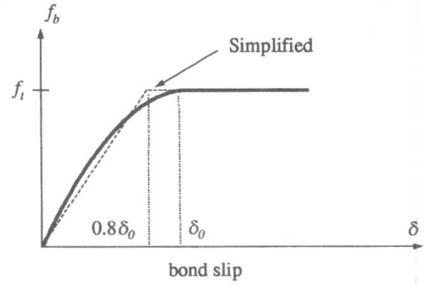

Fig.2 The bond - slip relationship of Dorr [8]

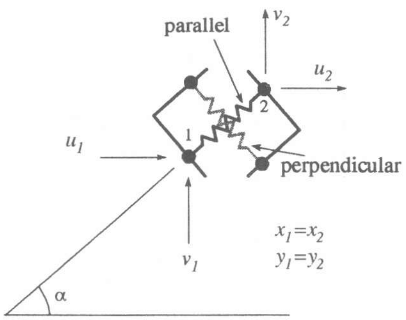

Fig.3 The bond link element

concrete. Uniaxial tests on steel reinforcing bars shows that the stress strain curve is initially a straight line having a slope equal to the Young's modulus. After yielding, hardening begins and the slope of the curve gradually decreases which ultimately results in a long and almost horizontal yield plateau. The shape of the curve just described can best be represented by the following equation proposed by Richard and Abbott[7],

$$\sigma_s = \frac{(E_1 - E_2)\varepsilon_s}{\left[1 + \left\{\frac{(E_1 - E_2)}{\sigma_0}\varepsilon_s\right\}^m\right]^{1/m}} + E_2\varepsilon_s \qquad (1)$$

where σ_s is the tensile stress in the reinforcing bar, ε_s is the strain, E_1 is the elastic modulus or the slope of the initial straight portion, E_2 is the modulus in plastic regime i.e. the slope of the second straight portion, σ_0 is the vertical intercept of the second straight portion and m is a shape parameter for the curved knee. The curve is shown in Fig.1. The plastic modulus E_2 is taken as the slope of the strain-hardened region. Although the test shows that this slope is almost zero resulting in a horizontal line, for the purpose of numerical finite element procedure it is taken as a small positive value because the experience shows that with iterative incremental finite element approach absolute zero value poses some disturbance in the convergence of iterations. The parameter m controls the curvature of the bent between the two straight lines. Lower values of m results in a more gradual bent while high values of m gives sharp bent. It is found that the use of m=25 is enough to obtain a very sharp kink.

2.2 Bond Stress - Slip Relationship

For analytical applications several linear and non-linear approximations of the bond stress - slip relationship are available. All the models describe the bond stress as a function of relative slip. But there is still debate on the amount of maximum slip and the corresponding bond stress level at which perfect slip occurs. It appears that with so many influencing factors like the amount of confinement, spacing of ribs in reinforcement, tensile strength of concrete etc. the relative slip δ alone is not enough to define the bond stress - slip relationship. In the present chapter, our aim is only to see the applicability of non-local gradient plasticity approach in simulating the crack propagation in concrete and hence we will adopt a simpler bond stress - slip relationship similar to that of Dorr [8] but with different maximum bond stress and with different values of slip δ_1 at which perfect slip occurs. Dorr proposed one nonlinear function relating the bond stress with the tensile strength and relative slip as,

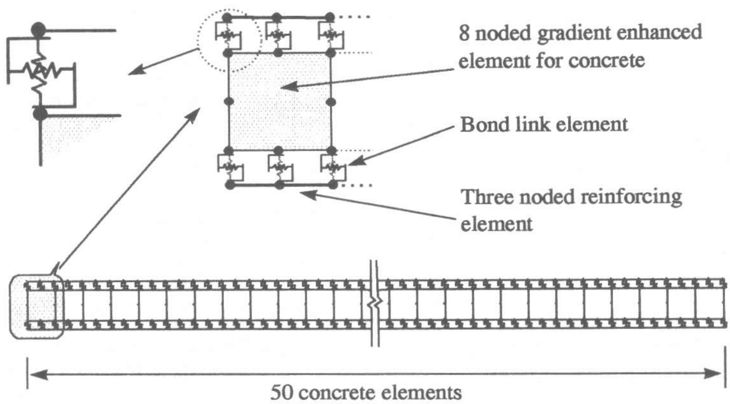

Fig.4 The finite element mesh.

$$f_b = f_t\left[5(\delta / \delta_0) - 4.5(\delta / \delta_0)^2 + 1.4(\delta / \delta_0)^3\right] \ , \ 0 < \delta < \delta_0 \qquad (2)$$

In the above expression, f_t is the tensile strength in N/mm^2 and δ_0 is the amount of slip at which perfect slip occurs which is usually taken as 0.06 mm. For $\delta > \delta_0$ the value of f_b is constant at $1.9f_t$. We will also investigate a simplified form of the same as shown in Fig.2.

2.3 Constitutive Model for Concrete

For concrete we use the gradient plasticity based constitutive relations originally proposed by Borst and Mühlhaus [1] which was later applied to early age concrete cracking problems by the authors [2]. Due to space limitation we are not explaining the same here. Interested readers are suggested to read the above mentioned papers for further details.

3. FINITE ELEMENT MODELING OF RC MEMBERS

For the discrete modeling of single bars or group of bars with the same location in two or three dimensional analyses we usually use general three dimensional truss elements. These truss elements are typically two noded having constant strain between the two end nodes. The deformation of this element is not compatible with the eight noded two-dimensional elements which has quadratic shape functions for the displacement field. To maintain deformation compatibility, we must use three noded truss elements having similar quadratic shape functions for the axial deformation. In this paper we used the three noded truss element.

The bond stress - slip relationship is simulated by the special contact elements. These contact elements are dimensionless bond link elements, connecting a single concrete node to a corresponding reinforcement node (Fig.3). This bond element is basically a pair of simple springs placed orthogonally. One spring is in the direction of the reinforcement (parallel) which simulate the bond stress - slip relationship and the other one is perpendicular to the direction of reinforcement and it may model dowel action. If dowel action is not of any interest then a high stiffness can be assigned to this perpendicular spring or a compatibility condition of zero relative displacement in perpendicular direction between the reinforcement node and the corresponding concrete node can be assigned. In this paper we are dealing with pure tensile loading and as such, a higher stiffness value was assigned to the perpendicular spring. The bond element shown in Fig.3 is dimensionless, i.e. nodes 1 and 2 have same physical coordinate. For the purpose of calculation the length of the link elements is taken as unity. The cross sectional area of the link elements is taken in such a way that it is equivalent to the surface of friction where bond stress is developed.

For concrete we use the eight noded gradient enhanced elements [2]. The softening function proposed by Amanat and Tanabe [2] along with a seperate exponential function for the gradient influence factor was used to describe concrete softening. Simple Rankine's principal stress criterion was used for modeling mode-I fracture of concrete.

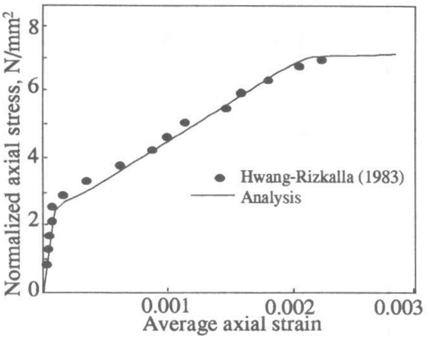

Fig.5 Load-displacement response for Rizkalla & Hwang's [3] test.

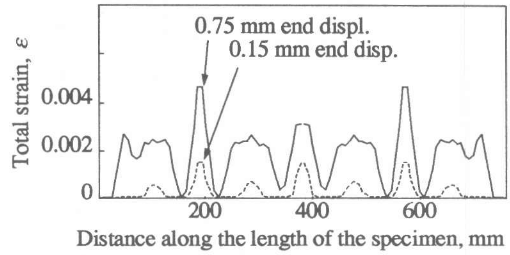

Fig.6 Strain distribution obtained for Rizkalla & Hwang's [3] test.

4. NUMERICAL ANALYSIS

Before we begin our parametric study with the present configuration of the analytical model it is necessary to check the model against experimental results. Rizkalla and Hwang[3] did an extensive test on RC members in tension. But only a few were without lateral reinforcement. Rizkalla et al[9] pointed out that in the presence of lateral reinforcement, the crack spacing is highly influenced by the location of such lateral reinforcements. In the present study we are not considering any effect of lateral reinforcement in the analysis and as such, test specimens without lateral reinforcement only is considered. Here we choose the specimen no. 7 of the second series of test made by Rizkalla and Hwang [3]. For this specimen the material properties are as follows: Young's modulus for concrete E_c=27800 N/mm^2 , concrete tensile strength f_t=2.7 N/mm^2 , Young's modulus for steel E_s=200000 N/mm^2 , yield strength of steel f_y=468 N/mm^2 , internal length scale l= 6mm. Fifty gradient enhanced eight noded elements in a row were used to simulate the concrete with three noded reinforced elements attached to the concrete elements via bond link elements similar to that shown in Fig.4. The load displacement response obtained is shown in Fig.5 where we see good match between the test and numerical analysis. The distribution of strain is shown in Fig.6 when the steel just yielded. Initially only one weak element was introduced at center. During loading, due to the interaction of steel and concrete elements through the bond-link elements, stress redistribution occurred and other distinct cracked zones were developed automatically. We observe in total seven distinct damaged zones of which three can be considered major cracks and the rest ones are minor cracks. In the actual tests five cracks were observed across the length of the specimen. We may consider this acceptable as a beginning since the effects of different model parameters on crack spacing are still not studied. In the next paragraphs we will make a parametric study to investigate the influence of different parameters.

To investigate the crack propagation in concrete a specimen with two longitudinal reinforcement was analyzed. The length of the specimen was 300 mm long having a cross sectional area of 100mm^2. The specimen was discretized longitudinally with 50 gradient enhanced elements as shown in Fig.4. Tensile load was applied at the ends of the reinforcement. It is a general postulate that reinforcement ratio is an influencing factor in determining the average spacing of cracks. As such, a parametric study was made for different reinforcement ratio. The reinforcement ratio that were studied are 1%, 1.5%, 2%, 2.5% and 3.0%. Values of other material parameters were as follows: Young's modulus 20,000 N/mm^2 , internal length scale l = 3 mm, ultimate value of equivalent fracture strain, κ_u = 0.01, concrete tensile strength f_t = 3.0 N/mm^2 . The load displacement response obtained for different reinforcement ratio is shown in Fig.7 which clearly shows the typical effect of reinforcement ratio on the ultimate load. This is quite in agreement with the experimental observations. It is seen that different reinforcement ratio produces different ultimate strength and the load at which concrete begins cracking (the first bend in the load-displacement response) is also somewhat proportional to the reinforcement ratio (Fig.7).

The distribution of axial strain along the length of the member for different cases of reinforcement is shown in Fig.8 through Fig.12. For every case two different distributions are presented - one at end displacement 0.22 mm which is just a few load steps after the first cracking of concrete and another one at end displacement 1.00 mm which is of the value when the steel is completely yielded. For 1 percent steel ratio we observe that there are three cracks more dominant than other cracks at both of the load level. For this

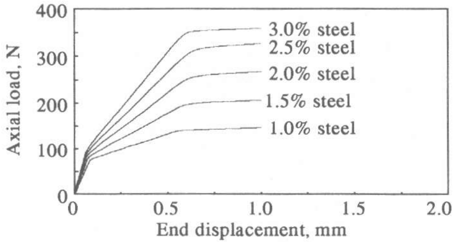

Fig.7 Load-displacement response for different steel ratio

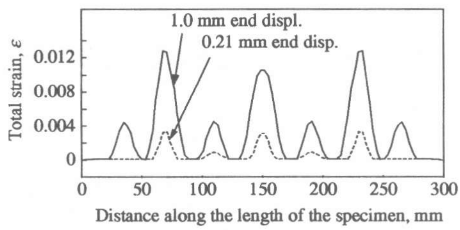

Fig.8 Strain distribution in concrete for steel ratio of 1%

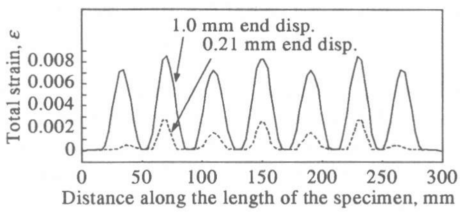

Fig.9 Strain distribution in concrete for steel ratio of 1.5%

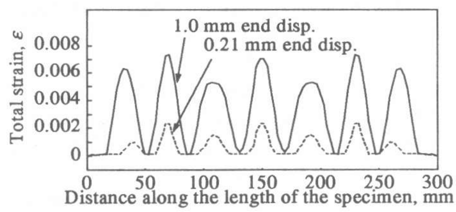

Fig.10 Strain distribution in concrete for steel ratio of 2%

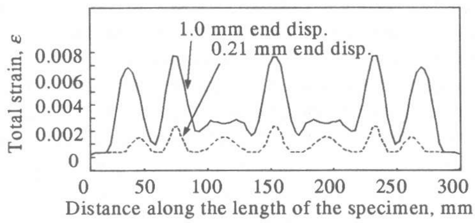

Fig.11 Strain distribution in concrete for steel ratio of 2.5%

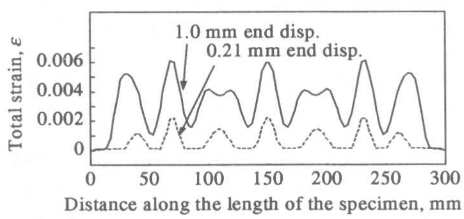

Fig.12 Strain distribution in concrete for steel ratio of 3%

case the three cracks might be considered as primary cracks while the other localized zones might be considered secondary cracks. For 1.5 percent steel ratio we also observe three major localized zones and other secondary localized zones for end displacement 0.21 mm. However, at later stages of loading we observe that the initially smaller localized zones grow almost equal to the initial primary localized crack zones and we get a total seven almost equally damaged localized zone. The development of damaged zone for 2 percent steel is similar to that of 1.5 percent steel except that the width of the localized zone appears to be wider. For 2.5 percent steel we initially get seven separate damaged zones but later we observe that the damaged zones are merged between each other. However, at 1.0 mm displacement we have two distinct peaks in the damage distribution in between the initially dominant cracks. This may be the indication of a tendency to grow two cracks in-between the initial cracks thus giving a total of 9 cracks. For 3 percent steel similar phenomenon is observed as that of the 2.5 percent steel ratio but the effects are more pronounced.

5. AVERAGE STRESS-STRAIN RELATION FOR STEEL

In the analysis and design of reinforced concrete structures the average stress-strain relation of steel bars are an important quantity. Belarbi and Hsu [4] calculated a series of average stress-strain curves for steel which is shown in Fig.13. It is seen that in reinforced concrete the average stress strain relation of steel is affected by the steel ratio. In this study we also calculated average stress-strain relation for steel based on the parametric study and found similar response as shown in Fig.14. However the post yield slopes for the present analytical result seem to be too high compared to the results obtained by Belarbi and Hsu [4]. This is due to the fact that in the present analysis the slope of the steel stress - strain relation in the post yield region

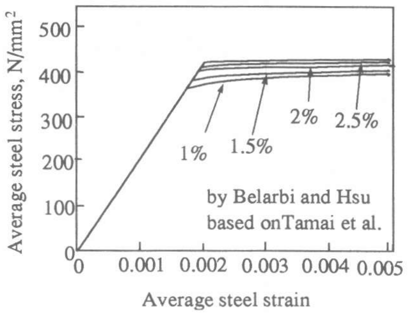

Fig.13 Average stress-strain relation obtained by analysis.

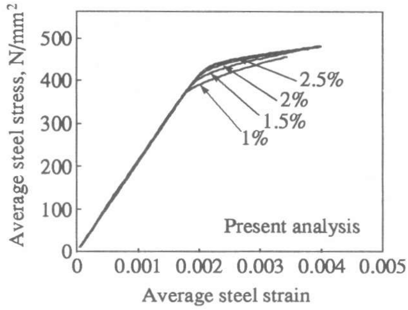

Fig.14 Average stress-strain relation obtained Belarbi and Hsu [4].

was taken considerably high value (e.g. 5% of initial Young's modulus). It was necessary to achieve convergence in calculation after the steel elements starts yielding. It was observed that if the slope of steel stress - strain relation in the post peak region is too small then the calculation ceases to converge. This is a limitation of the present finite element algorithm. However, from the results of the analysis we can at least infer that the present formulation is capable of producing the physically observed phenomenon.

6. CONCLUSIONS

A preliminary investigation on the applicability of the gradient plasticity theory in the analysis of reinforced concrete members is made. In the finite element modeling of the RC member separate elements were used for concrete, steel and the bond interface behavior. It was shown that with such a formulation it is possible to simulate experimental results acceptably. The parametric study with different reinforcement ratio suggests that it has some influence on the number and spacing of the crack bands. With the increase of steel ratio the width of the individual crack bands gets wider and there is a tendency of increasing the number of cracks. We calculated the average stress-strain relation for the reinforcement and found that the average yield limit of steel in concrete depends on the reinforcement ratio. The obtained results supports previous experimental observations. In short, we came across some interesting results during the parametric study whose detailed interpretation is beyond the scope of the present paper. A more thorough analysis is necessary before any quantitative description of the observed phenomena is made.

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