論文 Application of Unified Concrete Plasticity Model Incorporating Fracture Energy and Tension Stiffening Effect

Supratic GUPTA [*1], Umehara HIDETAKA[*2] and Tada-aki TANABE [*3]

ABSTRACT: Gupta and Tanabe[2,3,4,5] had modified the Unified Concrete Plasticity Model [1] for the three dimensional analysis RC members(columns and beams) and showed that proper simulation of bending failure was possible after proper simulation of tension stiffening effect. In this paper, sparsely reinforced concrete members expected to fail in shear mode are analyzed to check the effect of implementation of fracture energy and tension stiffening effect. Plain concrete member and RC member with only bottom reinforcement of different sizes are analyzed.
KEYWORDS: Shear, fracture energy, tension stiffening effect, Concrete, plasticity, 3-D Analysis, Unified Concrete Plasticity Model

1. INTRODUCTION

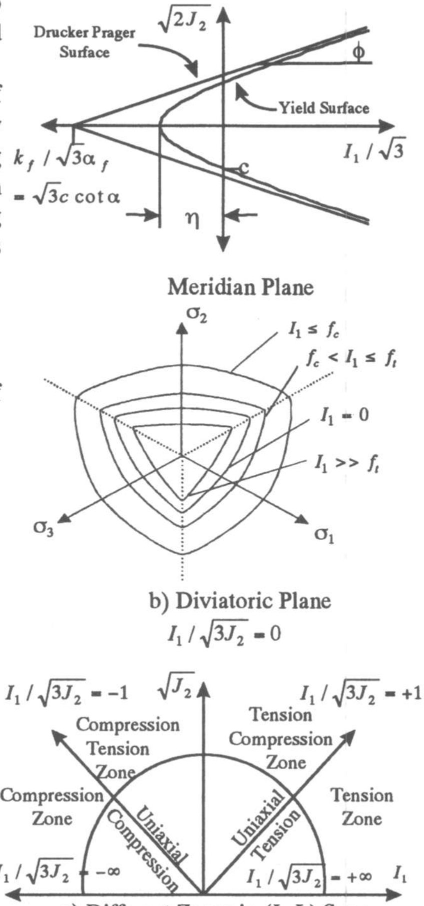
Development of method of three dimensional analysis of reinforced concrete structure has long been a goal of recent researches around the world. Recently, Tanabe et al.[1] had proposed Unified Concrete Plasticity Model for three dimensional analysis reinforced concrete members. They modified the Drucker-Prager model and introduced a parameter y in the denominator of Eq. 2, which is dependent on I_1 and θ (Eq.4) to get triangular shape in the tensile region and more circular shape in the compressive region so that the surface match Mohr-Coulomb core both at tensile and compressive meridian. As a result, this model can take care of stress-strain situation in both tensile and compressive zone in an ***unified*** manner. However some deficiencies existed because it tried to simulate both tensile and compressive properties by similar set of main controlling parameters namely cohesion and friction, by introducing η, the distance between the tip of surface from the origin, as an independent parameter. Gupta and Tanabe[2,3,4,5] showed that this assumption is not reasonable and recognized the cohesion and friction as the most important material parameters. Based on a stress term $X(=I_1/\sqrt{2J_2})$, independent variation of cohesion and friction in tensile($X \le a_1$) and compressive zone ($X > a_2$) was introduced and a gradual variation in between. Parameter η was made a dependent variable.

Even though the model works well in stress-strain and finite element level, different phenomenon like tension stiffening effect, fracture energy in plain, sparsely reinforced concrete(low reinforcement ratio) and highly reinforced concrete play an crucial role in the failure phenomenon. These phenomenon have been explained by different researchers using numerical and experimental work on simplified models. They have shown that these phenomenon requires the softening slope of stress-strain in tensile zone and shear modes to be different depending on the phenomenon occurring in the model. Setting the softening slope as required in these phenomenon is relatively a simple matter in this model and is also true in most other models. However, applicability of these phenomenon to complicated problems should be analyzed in more details. Gupta and Tanabe[2,3,4,5] had found that

*1 Department of Civil Eng., Nagoya Institute of Technology, Research Associate, Member of JCI
*2 Department of Civil Eng., Nagoya Institute of University, Professor, Member of JCI
*3 Department of Civil Eng., Nagoya University, Professor, Member of JCI

the applicability of the tension stiffening effect concept to reinforced concrete members failing in bending mode should depend on the actual crack pattern development.

In this paper, authors intend to check the applicability of these concepts of tension stiffening effect and fracture energy in plain, sparsely reinforced concrete beams by assuming different stress-strain curves for uniaxial tension. Recently, An et al.[6,7,8] successfully proposed a new approach of dividing the concrete in RC and plain concrete zones. This proposal is also applied and tested.



c) Different Zones in (I_1,J_2) Space

Fig.1 Unified Concrete Plasticity Model

2. THE UNIFIED CONCRETE PLASTICITY MODEL

Figure 1 shows the initial shape of the yield surface of the unified concrete plasticity model.

$$g(\sigma,\omega) = \sqrt{J_2 + k_f(1 - AA^*/\sqrt{3})^2} - (k - \alpha_f I_1) = 0 \quad (1)$$

$$k_f = \frac{6C\cos\phi}{\sqrt{3}(3 + y\sin\phi_1)}, \quad \alpha_f = \frac{2\sin\phi}{\sqrt{3}(3 + y\sin\phi_1)} \quad (2)$$

$$AA^* = \sqrt{3}c_o \cot\phi_o / \eta_o \quad (3)$$

$$y = \sqrt{a(\cos3\theta + 1.00) + 0.01} - 1.10, \; a = 0.5r^2 + 2.1r + 2.2,$$

$$r = \begin{cases} 3.14 & I \le f_c' \\ 2.93\cos\dfrac{f_t - I_1}{f_t - f_c'}\pi & f_c' < I \le f_t \\ 9.0 & I > f_t \end{cases} \quad (4)$$

$$c = \alpha c_0 \exp\left[(-m_1\omega)p_1(X) + (-m_2^2\omega^2)p_2(X)\right] + (1-\alpha)c_0,$$

$$\phi = \begin{cases} \phi_0 + (\phi_f - \phi_0)\sqrt{(\omega+k)(2-\omega-k)}\; p_2(X) & \omega \le 1 \\ \phi_0 + (\phi_f - \phi_0)p_2(X) & \omega > 1 \end{cases} \quad (5)$$

$$\{p_1(X) \;\; p_2(X)\} = \begin{cases} 0 \qquad\qquad 1 & X \le a_1 \\ \dfrac{1}{2}\cos\dfrac{X - a_1}{a_2 - a_1}\pi + \dfrac{1}{2} \quad \dfrac{-1}{2}\cos\dfrac{X - a_1}{a_2 - a_1}\pi + \dfrac{1}{2} & a_1 < X \le a_2 \\ 1 \qquad\qquad 0 & X > a_2 \end{cases} \quad (6)$$

where σ is stress tensor; I_1, J_2 and J_3 are the stress invariants; $\cos3\theta = (3\sqrt{3}J_3)/(2J_2^{1.5})$, $X = I_1/\sqrt{3J_2}$, ϕ_1, ϕ_0, ϕ_f, c_0, η_0 are material constants, k=10^{-3},a_1= -1 and a_2= -0.15.The *simple damage* model where $d\lambda$ is scalar plastic multiplier assumed equal to plastic strain is assumed

$$d\omega = \beta' d\varepsilon^P = \beta' d\lambda \quad (7)$$

3. STRESS STRAIN RELATIONSHIP OF CONCRETE IN TENSILE ZONE

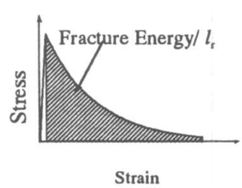


Fig3: Fracture Energy

For plain concrete structures or structures failing under shear failure, it has been pointed out by various researchers that concrete stress-strain behavior in tension and shear case should properly reflect the fracture energy. Basically, it boils down to controlling the softening slope and the following definition of fracture energy is assumed.

where G_f =113 N/m is assumed as taken by An et. al.[6] and l_r is the characteristic length. In this model, the stress-strain behavior of concrete in tension and shear cases can be very easily controlled by changing only m_1 and α without effecting the stress-strain behavior in compression. Gupta and Tanabe[2,3,4] found that the uniaxial tensile stress-strain curve is linear up to the peak and resemble the shape of c-ω relationship in uniaxial tension as friction ϕ remains constant. For a typical set of material parameter (Sec. 6) and for different value of m_1 and α, the stress-strain curve in uniaxial tension is shown in Fig. 3 which matched exactly with the following stress-strain relationship

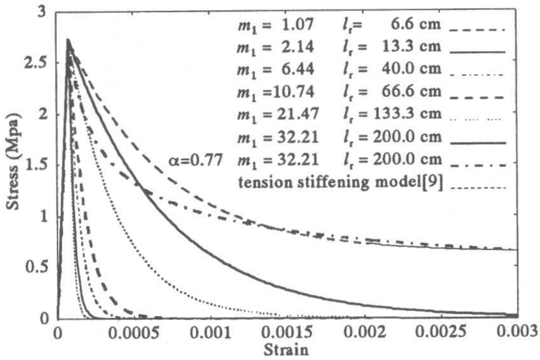


Fig. 3. Various Stress-Strain Models

$$\sigma = \alpha\sigma_0 \exp\left(-m_1\xi\left(\varepsilon-\varepsilon_0\right)\right)+\left(1-\alpha\right)\sigma_0 \tag{8}$$

The numerical basis for the above and its generality is being investigated and is probably because simple damage rule(Eq. 6) is assumed. For α =0, and substituting Eq. 8 in Eq. 7, it can be found that m_1 is proportional to l_r. Softening curve for Tension Stiffening effect as proposed by Tamai et al.[9] (also adopted by Hsu and his colleagues[10,11]) can also be easily simulated by selecting appropriate parameters in this model.

4 UNREINFORCED CONCRETE BEAM

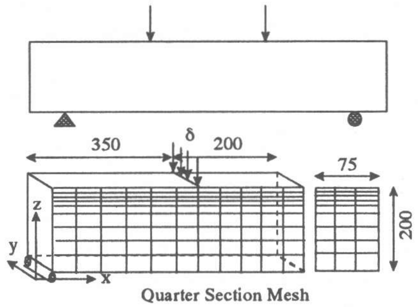


Fig. 4. Dimensions and Mesh of the Beam

Fig. 4 shows the unreinforced concrete(or plain concrete) beam and the mesh used in the analysis under two point loading to cause fracture at the center. Three cases with different softening slope as shown in Fig. 5 or Table 1 are considered. The other material parameter are E_c=34600 MPa, μ=0.22, f_t=3.15 MPa, f_c' = 33.75 MPa, c_o = 22.58 MPa , $\phi_1 = 14.3°$, $\phi_o = 5°, \phi_f = 36°$, m_2=0.83, η_o =6.0 MPa, k = 1.0 x 10^{-3}, $\beta' = 35$, a_1= -1.0 and a_2= -0.15.

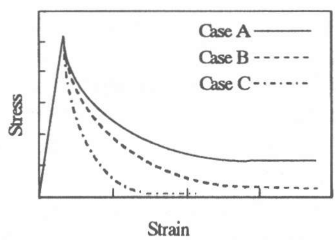


Fig. 5. Uniaxial Stress-Strain Curve Adopted

Table 1. Material parameters

	α	m_1
Case A	0.92	4.0
Case B	0.99	4.0
Case C	0.99	10.0

Results were reasonable(Fig. 6). The point of first crack is same in all the cases(Fig. 6). As expected, we find that the peak strength of the beam is higher then the strength at first crack point. On decreasing the residual stress(Case A to Case B), the strength at the final stage decreases. When the value of m_1 is increased(Case C), it implies the fracture energy of the tension has been decreased to get a steeper slope for the softening slope. This is quite well reflected in the load displacement diagram. Moreover, the peak load decreased for Case C as compared to Case A and B. We find that the behavior becomes more unstable as we take sharper stress-strain diagram or smaller fracture energy and load displacement diagram becomes almost vertical with tendency of snapback. Hence we can say that implementation of fracture energy is properly reflected in the three dimensional analysis by the Unified Concrete Plasticity Model.

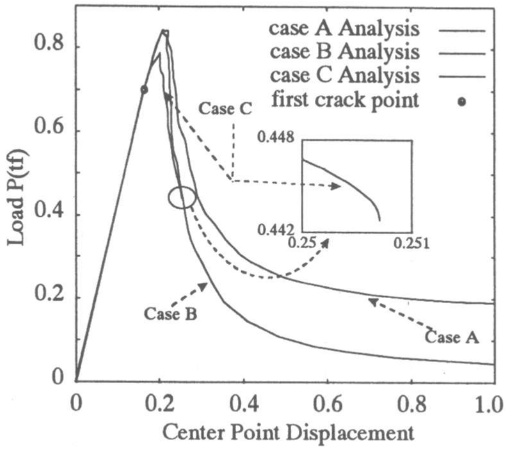


Fig. 6. Load-Deflection Behavior of Plain Concrete Beam under Two Point Bending

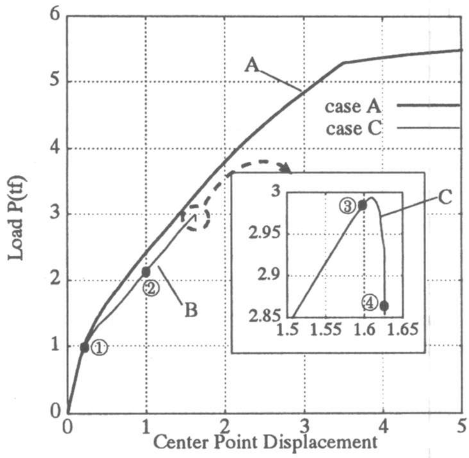


Fig. 7. Behavior of Beam with only bottom reinforcement failing in Shear

5 RC BEAM FAILING IN SHEAR

Dimensions of the beam and material parameter similar to previous example is adopted with reinforcement at the bottom (2-D13). Two nodded truss element is used as rod element. with elastic perfectly plastic stress-strain curve is adopted. Elastic perfectly plastic stress-strain relationship is adopted as the beam is expected to fail in shear even before the reinforcement yields. For concrete two Case A and Case C of Fig. 5 are considered.

When tension stiffening effect (case A) is implemented, it does not fail in shear(Fig. 7). However when Case C with sharper slope is implemented, it fail suddenly showing clear shear failure. The calculation could not proceed beyond this point due to displacement controlled algorithm as steep drop in load was noticed as seen in the magnified zone(Fig. 7) indicating failure of the beam in shear. From the point near the peak, the step size was made very small and calculation proceeded till the load-deflection diagram became almost vertical.

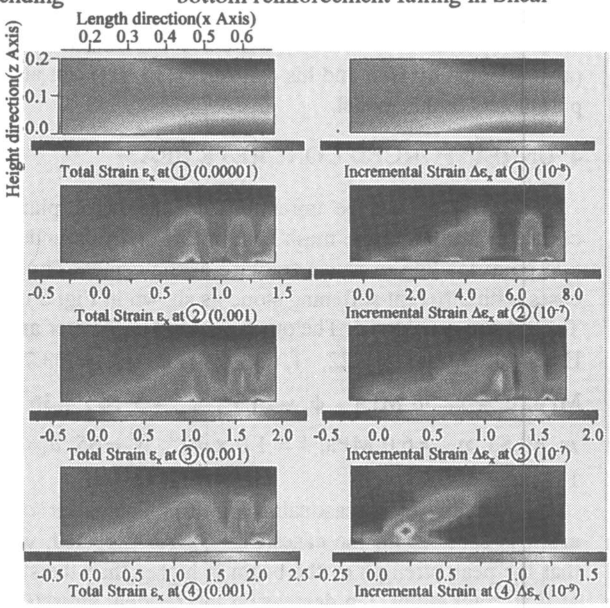


Fig. 8 Strain and Incremental Strain at Different Stages

Fig.8 shows the strain and incremental strain distribution at different stages for Case C. This results are quite interesting. Multiple cracks are formed in the initial stage and in the final stage, snapback like phenomenon is noticed with formation of localized diagonal crack (incremental strain).

6 ANALYSIS OF LARGE BEAM FAILING IN SHEAR

A large beam as adopted by Iguro et al.[12](Specimen 6) is adopted. In the experiment, uniformly distributed load was applied to the beam. Authors aims to conduct detailed analysis of these cases in future. However as first stage of the application to shear problems, two point displacement controlled loading is adopted. The aim of this part is to check the general behavior as reflected in the analysis.

In this section, the idea presented by An et al.[6,7,8] is tested. They proposed to divide the concrete area into plain concrete and reinforced concrete volume. This task of dividing the concrete volume in RC and plain concrete zones is not easy in normal structure. However this set of experiments allows such implementation and results obtained by An. et al. matched very well with the experimental results.

However a critical differences exist between the method of analysis adopted by An et al.[6,7,8] and the present approach. An et al.[6,7,8] adopted distributed reinforcement element, where in the present case two nodded truss element is adopted. For the reinforcement element only elastic-perfectly plastic stress-strain relationship is adopted for reasons explained in Sec. 5. An et al.[6,7,8] used two dimensional direct stress-strain approach developed by Okamura, Maekawa group, whereas present analysis adopts three-dimensional classical plasticity approach. The difference and advantages and disadvantages of each approach is worth studying and will be carried out in future.

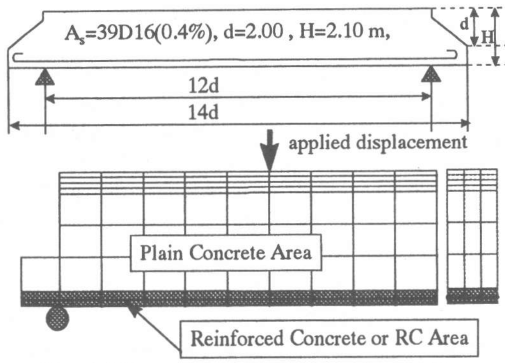


Fig. 9: Dimensions and Mesh of Large Beam (Specimen No. 6[12])

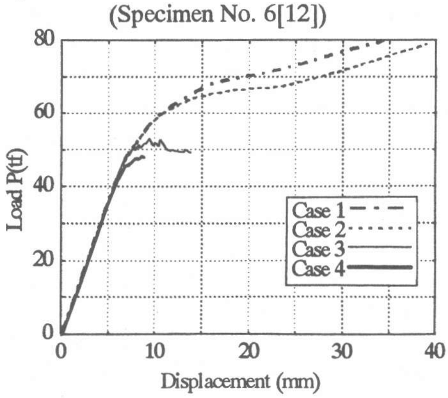


Fig. 10: Load Deflection behavior for Large Beam under Different stress-strain conditions

Fig. 9 shows a large beam(Specimen 6) and the mesh used in the analysis. It is proposed to check the effect of the characteristics of the softening curve in this analysis. The residual stress(represented by α) and the sharpness in the fall of stress-strain diagram(controlled by factor m_1) as shown in Sec. 3 is used as control parameters.

The common material parameter are E_c=33050 MPa, μ=0.22, f_t=2.73 MPa, f_c' = 28.5 MPa, c_o = 23.2 MPa , $\phi_1 = 12.5^\circ$, $\phi_o = 5^\circ$, $\phi_f = 24.5^\circ$, m_2 =0.83, η_o =5.25 MPa, k = 1.0 x 10^{-3}, $\beta' = 100$, a_1= -1.0 and a_2= -0.15. Four cases are considered. Case 1 represent full area has concrete properties as RC. In Case 2, the residual stress is decreased. In Case 3, sharper stress-strain is used in the plain concrete portion. In Case 4, plain concrete characteristics is implemented in the full area. Fig. 10 shows the load deflection behavior. We can see that like in previous section, Case 1 and 2 shows ductile failure. Case 3 shows some what weak behavior. Case 4 showed even more weakness. This looks logical. Hence, it looks more detailed study of stress, strain and incremental strain distribution is important to understand the actual phenomenon occurring in the beam in these different types of analysis. After the authors completes the understanding of the phenomenon occurring in the analysis, the comparison with actual experiment of beams with different sizes and will be carried out.

Table 2: Material Parameter for Large Beam

	RC Portion		Plain Concrete Portion	
	α	m_1	α	m_1
Case 1	0.77	1.073	0.77	1.073
Case 2	0.77	1.073	0.98	1.073
Case 3	0.77	1.073	0.98	6.44
Case 4	0.98	6.44	0.98	6.44

7. CONCLUSION

In this paper, three dimensional finite element analysis is carried out for plain concrete members failing by fracture mode and sparsely reinforced concrete members failing in shear modes. As a first step simplified analysis is carried out to check the effect of various parameters. Following can be concluded about the applicability of the Unified Concrete Plasticity Model:

a) For the selected set of parameters, it was found that fracture energy or the softening slopes for uniaxial tension can be easily controlled by controlling a and m1. The numerical reason and generality is however under consideration.

b) For the plan concrete beam, the implementation of fracture energy is properly reflected in the three dimensional analysis by the Unified Concrete Plasticity Model.

c) When higher slope was implemented for the beam expected to fail in shear, the beam failed in bending mode. When sharp stress-stain diagram was implemented , it failed in shear mode. The final stage showed localized diagonal crack formation.

d) It was seen that implementation of different RC and Plain concrete zones had effect in the load-displacement behavior. However, the effect of implementation of different sets of α and m_1 in different zones needs more attention. Detailed study of the stress-strain-incremental strain distribution might be useful in understanding the actual phenomenon occurring in each of these cases.

REFERENCES

1. Tanabe,T., Wu,Z. and Yu,G., Aug., "A Unified Plasticity Model for Concrete" JSCE, Vol.24, No.296, pp. 21-29,1994.
2. Gupta, S. and Tanabe, T., "Investigation and Modification of the Characteristics of the Unified Concrete Plasticity Model", JCI,pp.1305-1310 ,1995.
3. Gupta, S. and Tanabe, T., "Modification of the Unified Concrete Plasticity Model and its characteristics", JSCE, Vol. V, No. 571,pp. 225-234,Aug,1997.
4. Gupta, S. and Tanabe, T., "Unified Concrete Plasticity Model in Three Dimensional Analysis of Reinforced Concrete Structures",JCI,Vol.19, No.2.,pp.561-566,1997.
5. Gupta, S., "The Development of the Unified Concrete Plasticity Model for the Three Dimensional Analysis of Reinforced Concrete members", Ph.D. dissertation, Department of Civil Engineering, Nagoya University, 139 pp, July, 1997.
6. An, X., "Failure Analysis and Evaluation of Seismic Performance for Reinforced Concrete in Shear", Ph.D. dissertation, Department of Civil Engineering, University of Tokyo, 198 pp, July, 1996.
7. An, X., Maekawa, K. and Okamura, H., "Numerical simulation of size effect in shear strength of RC beams," Journal of Materials, Conc. Struct., Pavements, JSCE, No.564,V-35,pp. 297-316, May, 1997.
8. Okamura, H. and Maekawa, K, "Nonlinear Analysis and Constitutive Models of Reinforced Concrete," ISBN4-7655-1506-0 C3051, Gihodo, Tokyo., 1990.
9. Tamai, S., Shima, H., Izumo,J. and Okamura, H., 1988, June, 'Average Stress-Strain Relationship in Post Yield Range of Steel Bar in Concrete', Concrete Lib. JSCE, No. 11, pp.117-129.
10. Hsu, T. T.C., 1993, "Unified Theory of Reinforced Concrete", CRC Press.
11. Belarbi, A. and Hsu, T.T.C. , 1994, "Constitutive Laws of Concrete in Tension and Reinforcing Bars Stiffened in Concrete," ACI Struct. , V.91, No.4, pp. 465-474.
12. Iguro, M, Shioya, T., Nojiri, Y. and Akiyama, H., "Experimental Studies on Shear Strength of Large Reinforced Concrete Beams under Uniformly Distributed Load", pp. 137-154, Concrete Library of JSCE No. 5, August 1985.