

論文 Application of the Modified Lattice Model to Simulate Shear Failure of RC Beams

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ABSTRACT: Application of the modified Lattice Model is carried out for the different RC beams. Angle of inclination of the diagonal members and the rational discretization method for the truss member are very important parameters affecting the results of the Modified Lattice Model. These effect are studied in this paper for the different RC beams. Application of the modified Lattice Model for shear failure simulation is carried out. The change of the stress states in each member inside the beam is investigated. According to the simulation of the shear resisting mechanism, the Modified Lattice Model can simulate the shear failure mode with a very smart way.

Key words: Arch element, Modified Lattice Model, Shear resisting mechanism, subdiagonal element, total potential energy.

1. INTRODUCTION

Truss models have been successfully applied to determine the ultimate strength of reinforced concrete beams in torsion, bending and shear. Under the name of "strut-and-tie models", truss models have been implemented into the LRFD AASHTO code[1] and the Canadian Code [2]. Current truss models, however, did not yet realistically model the bar element and it's applications. Thus, current truss models lack the capability of explaining the simulation or the application of different failure modes. The newest truss model is the Lattice Model which is proposed by NIWA et al. [11] and extended later by the authors in three dimensions [4,5]. This Model still needs a huge study to clarify the behavior of the Model under a different failure modes. Lattice Model is modified latter by the authors using a new technique. This new technique is significantly depends on the calculation of minimum total potential energy of the structure at each step of the calculation during the different loading stages [6,7].

In this paper, the appropriate discretization of the Modified Lattice Model is studied widely to find out the suitable form of applying the Model to give a similar response close to the experimental results depending on the change of spacing of shear reinforcement and the subdiagonal angle. Angle of inclination of the diagonal members and the rational discretization method for the truss member are very important parameters affecting the results of the Modified Lattice Model. These effect are studied in this paper for the different reinforced concrete beams. That is in addition to an application of the Modified Lattice Model for shear failure simulation which is carried out and presented in this paper. The change of the stress states in each member inside the reinforced concrete beam is investigated to capture the real failure mode of the beam.

Finally, according to the simulation and considering the objectivity of the post processing of the calculated results and the simple representation of the shear resisting mechanism, the Modified Lattice Model can simulate the shear failure mode with a very smart way.

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2. OUTLINE OF THE MODIFIED LATTICE MODEL

The chosen element discretization and structural geometry of the Modified Lattice Model is illustrated in Fig. 1. The reason behind this truss discretization will be verified in the following chapters. The R.C. beam with a depth "d" has been simulated under bending and shear as simple truss components. The compressive stress in the upper part of the beam is resisted by concrete in the form of a horizontal strut with a cross-section area equal to the area of the upper rectangular in Fig. 2. The depth of the flexural compression member is determined to be equal to the depth of flexural compression zone at the flexural ultimate state with height $h_1 = (A_s f_r) / (0.68 f_c b)$. The tensile stress in the lower portion is taken by the bottom steel in the form of horizontal members in addition to the horizontal concrete fibers in the lower part with a cross section area equal to the area of the lower rectangular in Fig. 2. The depth of flexural tension member "h₃" is assumed to be twice of the distance between the centroid of reinforcing bars for flexural tension and the bottom of the beam. To resist the shear forces inside the beam, the truss model has diagonal concrete tension and compression members with the area as shown in Fig. 2, which can be fixed after the value of "t" is determined [6,7]. Also there are vertical steel members which represent the shear reinforcement in the web. Fig. 2 shows the cross section of a concrete beam modeled into the Modified Lattice Model.

In Fig. 1 the thick solid line represents the arch element which is assumed to be a flat and slender one connecting the nodes at both ends of the beam with an area as shown in Fig. 2. In this analysis, the arch element and the diagonal elements are separated, and each one of them has its stress and strain distribution. The reason of this element separation is that the structural action is normally a combination of series and parallel couplings of the cracking zones and the uncracked (elastic) zones. In the Modified Lattice Model, these zones are simulated to a continuous pairs of tension and compression members. Many codes assume two dimensional stress field; but if a member section is wide enough, the stress may not be uniform in the direction of member width. So, in this model we separate the arch element and the diagonal element, and each one of them has its stress and strain distribution. The arch element has the ability to resist a large portion of the applied load [9]. So it is very important to look for the change of the area of the arch element during the different loading stages. The thickness of the arch element is determined by minimizing the total potential energy among the different values of "t" inside each step of the calculation [6,7].

In the Modified Lattice Model, the diagonal tension member of concrete resists the principal tensile stress resulting from shear force. The stress-strain relation of tension member of concrete has been taken as expressed in Eq. (1) and Eq. (2) and as shown in Fig. 3.

$$\text{For ascending branch } (\epsilon_r < \epsilon_{cr}) \quad \sigma_r = E_c \epsilon_r \quad (1)$$

$$\text{For descending branch } (\epsilon_r \geq \epsilon_{cr}) \quad \sigma_r = (1 - \alpha) f_t \exp \left[-m^2 \left(\frac{\epsilon_r}{\epsilon_{cr}} - 1 \right)^2 \right] + \alpha f_t \quad (2)$$

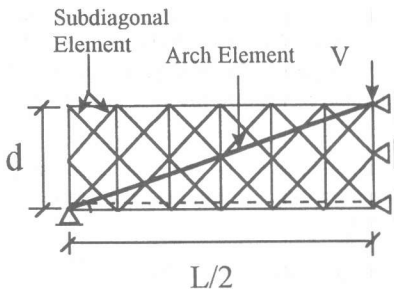


Fig. 1 Concrete Beam with the Modified Lattice Model

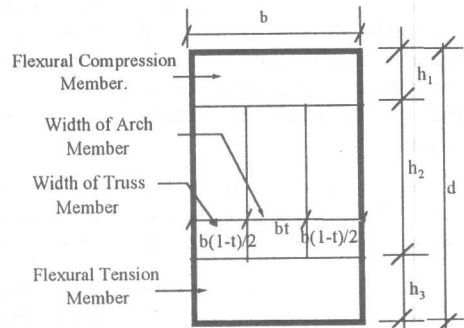


Fig. 2 Cross -Section of Concrete Beam in the Modified Lattice Model

Where ε_r and σ_r are the strain and the stress of the tension element respectively as shown in Fig. 3. Eq. (1) shows the elastic behavior before cracking. In Eq. (2) m can be varied to simulate appropriate fracture energy for plain concrete. Appropriate α can be chosen to simulate the residual stress in the final stage of damage for simulating tension stiffening effect in reinforced concrete [8]. In this research $m=0.5$ and $\alpha=0.0$ are adopted.

The diagonal compression member of concrete and the arch member shall resist the diagonal compression caused by shear. To consider the compression softening behavior of crushed concrete, the model proposed by Collins et al.[12] is adopted. In that model the softening coefficient was proposed as a function of the transverse tensile strain. So, the tension and compression members are considered as a pair together as shown in Eq. (3). The stress-strain relationship for reinforcing bars is assumed to be elasto-plastic for the case of tension and compression members.

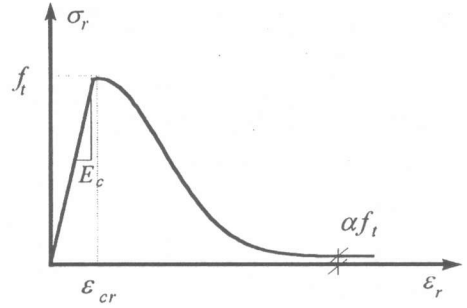


Fig. 3 Tensile Stress-Strain Curve of Concrete

$$\sigma_c = -\eta f_c' \left(2 \left(\frac{\varepsilon_c}{\varepsilon_o} \right) - \left(\frac{\varepsilon_c}{\varepsilon_o} \right)^2 \right) \quad (3a)$$

where,
$$\eta = \frac{1.0}{0.8 - 0.34 \left(\frac{\varepsilon_r}{\varepsilon_o} \right)} \leq 1.0 \quad (3b)$$

and the strain at the peak stress $\varepsilon_o = -0.002$.

3. APPROPRIATE SUBDIAGONAL ANGLE FOR THE MODIFIED LATTICE MODEL

Next thing to do is the clarification of the appropriate discretization of lattice members with angle may be firstly predetermined as 45 degrees. To investigate the extent of the discretization, three different truss models depending on the number of the pairs of diagonals along the depth of the beam are investigated. Mod 1, Mod 2 and Mod 3, represent one, two and three pairs of subdiagonal members respectively along the depth of the beam. The results for these cases are studied and compared with the experimental results. It has been found that, it is preferable to use two pairs only of the subdiagonal members along the depth of the beam to implement the Modified Lattice Model, since the numerical results in this case is more close to the experimental results than any other discretization with the same angle of inclination 45 degrees [6,7].

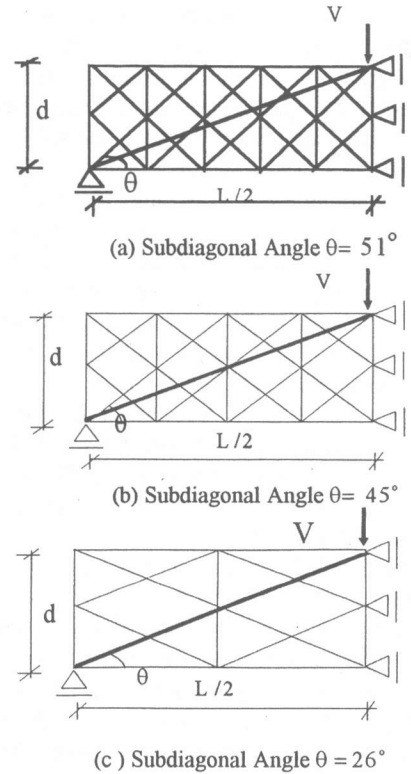


Fig. 4 Three Different Subdiagonal Angles

Table 1 The outline of Experimental Data

| Exp. | Cross Sec. | b cm | h cm | d cm | a/d | f_c MPa | A_s cm ² | f_y MPa | A_w cm ² | f_{wy} MPa | s |
|--------|------------|---------|---------|---------|------|--------------|--------------------------|--------------|--------------------------|-----------------|------|
| Clark | R | 20.3 | 50.8 | 42.5 | 2.15 | 31.0 | 23.1 | 530 | 1.42 | 530 | 1.33 |
| Ohuchi | R | 45.0 | 60.0 | 52.5 | 2.86 | 66.2 | 95.7 | 383 | 1.43 | 355 | 15.0 |

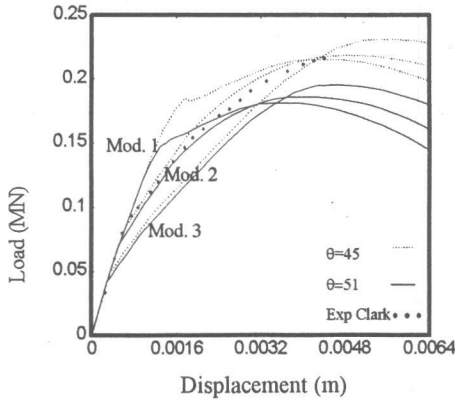


Fig. 5 Load-Displacement Diagram in Case of $\theta=51^\circ$ and 45°

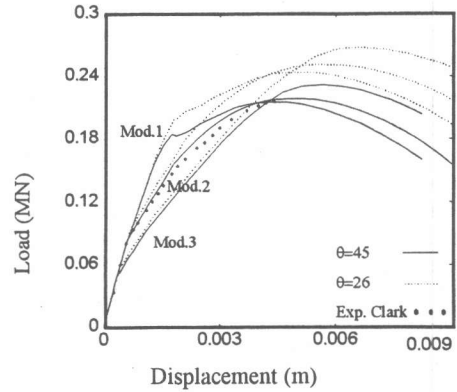


Fig. 6 Load-Displacement Diagram in Case of $\theta=26^\circ$ and 45°

To study the direction of initial cracking in each of the solved beam under the three different number of the subdiagonal members mentioned before, three different values of angle of inclination of the subdiagonals in the Modified Lattice Model are suggested. The suggested values are 51, 45 and 26 degrees as shown in Fig. 4 (a), (b) and (c) respectively. In each case the summation of all the stirrups are equal to the total area of all the stirrups in the original beam. Table 1 shows a summarized description for the experiment work which sued for the comparison. Fig. 5 and Fig. 6 show the load-displacement diagrams for the reinforced concrete beam of Clark's experiment [3] for a different subdiagonal angle and also for the different models which are mentioned before according to the number of the subdiagonal pairs. The numerical results using the Modified Lattice Model are compared with the experimental results. It has been found that the results using two pairs of subdiagonal member and by using subdiagonal angle 45 degrees are very close to the experimental results. Under any other inclination of diagonal angle, the relation of the load-displacement goes diverging from the experimental results up or down. In case of angle 51 degree the length of the diagonal members are decreased comparing with the case of 45 degree, so the relation of load-displacement is kept lower than the relation

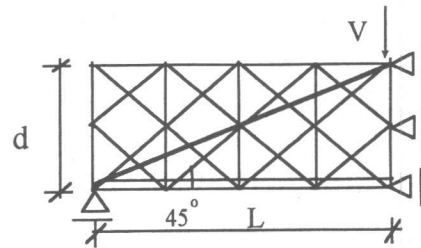
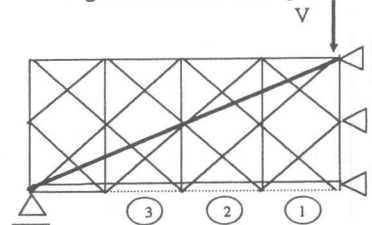
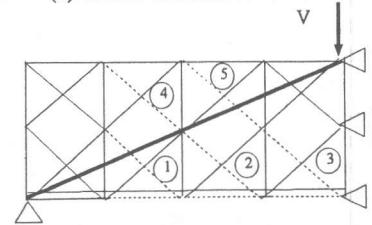


Fig. 7 Illustrative Example



(a) Cracks in Bottom Member



(b) Cracks in the Diagonal Members

Fig. 8 Propagation of Cracks

using 45 degree diagonal angle as shown in Fig. 5. But in the case of the same models using diagonal angle 26 degree the results become upper than the relation using diagonal angle 45 degree as in Fig. 6. Actually this happens because the increasing of the diagonal member length increases the elastic energy and the stiffness of the structure. This behavior was the same using the three different models, and also for each angle of inclination for the diagonal members.

4. APPLICATION OF THE MODIFIED LATTICE MODEL FOR SHEAR FAILURE SIMULATION

To investigate the change of the stress states in each member inside the reinforced concrete beam using the Modified Lattice Model, Clark's experiment [3] was chosen as a subject for the simulation. Fig. 7 shows the Modified Lattice Model for Clark's reinforced concrete beam. The stresses in diagonal members of concrete and stirrups and the stress of arch member are examined. From the output results of the simulation of this beam using the Modified Lattice Model, it is found that at the primary cracking stage, the concrete elements in the bottom cord start to crack firstly and in a simultaneous fashion, according to the numbers which inside the circles in Fig. 8 (a). Then the initiation of the diagonal cracking happens also in a simultaneous manner according to the numbers inside the circles in Fig. 8 (b). After that, the initiation of the yielding of stirrups starts to take place. Although the stirrups start yielding and the diagonal tension elements have cracked, but the beam still continues to be loaded up to the complete failure. That is because of the existence of some stirrups without yielding and also the arch element which continues to carry load up to the complete failure of the structure. At the final stage, all the stirrups yielded. At that time the arch element crushed immediately. From this simulation for the failure of that beam we can consider it as a shear failure.

5. INVESTIGATION OF STRESS STATES INSIDE THE MODEL

To investigate the change of the stress states in each member inside the reinforced concrete beam using the suggested Modified Lattice Model Ohuchi's experiment [11] was chosen as a subject for a solved example. The change of the average stress in diagonal members of concrete and stirrups and the stress of arch member are examined. Fig. 9 shows the modified lattice model for Ohuchi's beam. The average stress of members located in the center of shear span, which are drawn by solid lines in Fig. 9, is calculated and shown in Fig. 10 with the increase in the displacement of the loading point. From Fig. 10, it is clear that the average tensile stress of diagonal tension members of concrete is decreasing rapidly after the initiation of diagonal cracking. On the other hand, the average compressive stress of diagonal compression members of concrete and

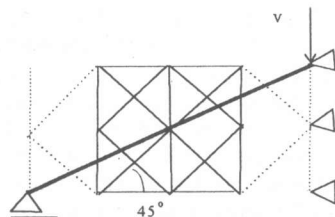
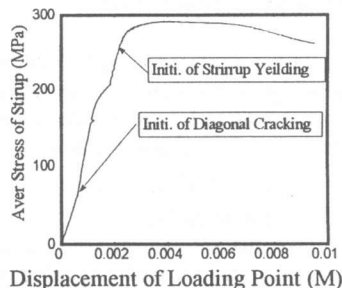
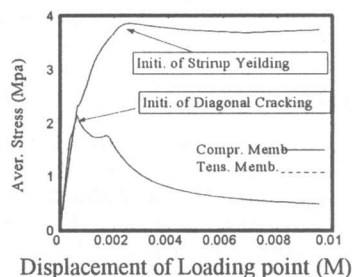


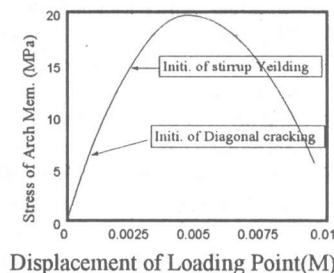
Fig. 9 The Solved Example



(a) Average Stress of Stirrups



(b) Average Stress of Diagonal tension and Compression Members



(c) Stress of Arch Member

Fig. 10 The Change of Average Stress of each Member

average tensile stress of stirrups are increasing significantly as shown in Fig. 10(a), and (b). The average stress of diagonal compression members has the tendency to maintain almost constant value after exhibiting a certain amount of the increase in the average stress. The average stress of stirrups is slightly increasing with the increase in the displacement after the initiation of yielding. But the compressive stress of arch member exhibits the significant increase after the initiation of stirrup yielding, however, due to the softening in compression, the arch member reaches to the ultimate state as shown in Fig. 10(c). The maximum stress of the arch element before crushing was 13.0 MPa after the peak as shown in Fig. 10 (c). From this example it is the predicted shear failure mode for this beam is compression failure of arch member after the initiation of stirrup yielding. This is quite similar to the experimental results.

6. CONCLUSIONS

In the newly developed Modified Lattice Model, a concrete beam subjected to shear force is converted into simple truss and arch members by the consideration of the minimum total potential energy for the structure at each step of calculation. A nonlinear incremental analysis is performed. Conclusions obtained from this research are as follows:

1. Modified Lattice Model has the tendency to estimate the stiffness of the beam closer to the experimental results. However, predicted displacement at the peak is almost similar to the experimental results, specially by using two pairs of diagonal members along the beam depth.
2. 45 degrees is the best angle of subdiagonal members to implement the Modified Lattice Model.
3. Using the different forms of the Modified Lattice Model, the ultimate load is almost kept constant but the cracking load is decreasing depending on the strain energy of the cracked element.
4. Although the Modified Model is more simplified method than the normal F.E.M., it can capture the shear behavior of concrete beams throughout the change of the shear resisting mechanism.

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