論文 Analysis of Two Continuous Span Prestressed Concrete Beam with External Cables Considering Shear Deformation

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ABSTRACT: External prestressing concrete bridges have become very popular in recent years because of their advantages such as reduction of both construction time and construction cost and possibility of maintenance and repair. However structural behaviors between external cables and concrete, especially friction coefficients at deviators are not yet fully understood. In this papers were developed FEM for analysis of two continuous span of PC beam considering shear deformation with arbitrary loading scheme. The analytical results were also compared with test results which is available in some literature.
KEYWORDS: External cable, PC beam, deviators, moment, displacement, strain, friction, reaction.

1. INTRODUCTION

In external prestressing system, cables are arranged outside the cross section of prestressed concrete beam. The externally prestressed concrete beam can be treated as unbonded prestressing members because no bonding between concrete and cables exists. However unlike internal unbounded cables, external cables are formed the profile by deviating at deviators. The main difference between internal and external unbonded prestressing cables lies in deflected shape of beam and cable. The deflected shape of internal cables usually follow the deflected shape of beam itself throughout the entire span. However when the externally prestressed concrete beam subjects to bending , the cables would not follow the concrete beam deflection, except at deviator points.

Unlike the analysis of beam prestressed with bonded cable, analysis of beam prestressed with external unbonded cable is treated with assumption no bonding between prestressing steel and surrounding concrete. In this case strain compatibility at critical section can not be maintained rather the stress in prestressing steel at any load level during the response history depend on total change in length of concrete at level of prestressing steel between end anchorages. This makes the deformation thus the stress in member dependent rather than section dependent and proper modeling of overall beam deformation becomes necessary.

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2. REVIEW OF PREVIOUS LITERATURE

Terao .D [1] developed a finite element model for nonlinear analysis of simple support beam considering shear deformation. This model was taken into account friction coefficient at deviator and cable strain at both side of deviator was different by portion which is caused by friction force. This difference of strain can be expressed in term of friction coefficient k_{Di}

$$\Delta\varepsilon_{s(i+1)} - \Delta\varepsilon_{s(i)} = \frac{k_{Di}}{l_{i+1} + l_i} \int_0^{l_{i+1}+l_i} \Delta\varepsilon_{cs} dx \qquad (2.1)$$

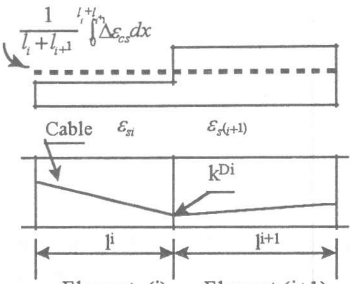

Fig.1. Strain difference at deviator

and total cable strain must be equal to total concrete strain between end anchorage's

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \int_0^{L} \Delta\varepsilon_{cs} dx \qquad (2.2)$$

Where $\Delta\varepsilon_{si}$, $\Delta\varepsilon_{si+1}$ are increase strain of *(i)* and *(i+1)* prestressed cable elements respectively; $\Delta\varepsilon_{cs}$ is increase strain of *(i)* concrete element at location of prestressed cable; l_i , l_{i+1} are length of *(i)* and *(i+1)* cable elements respectively; *L* is total length of prestressed cable.

Formulation of stiffness matrix with considering shear deformation could be found in [1],herein only presented a result. Stiffness matrix of concrete element *[Kc]* is expressed

$$[K_C] = \iiint_{dV} [A]^T E[A]dV + \iiint_{dV} [B]^T G[B]dV \qquad (2.3)$$

By using layered model of concrete, the element is divided *M* layers on *X*-direction and *N* layers on *Y*-direction (see Fig.2) and Eq.(2.3) becomes

$$[K_C] = \sum_{j=1}^{M} \sum_{k=1}^{N} b_{jk} \left(E_{jk} [A]^T [A] + G_{jk} [B]^T [B] \right) \qquad (2.4)$$

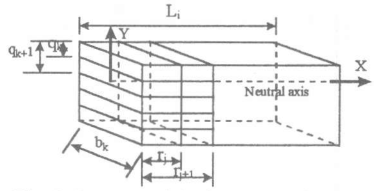

Fig. 2. Layer model of concrete element

Where matrices *[A]*, *[B]* are considered bending and shear deformation respectively, E_{jk}, G_{jk} young modulus and shearing modulus of layer *jk*, b_{jk} is the width of layer *jk*.

In this model the author solved problem by applying of two displacement controlled points for symmetrical loading scheme of simple supported beam, but did not consider asymmetrical loading scheme.

3. APPLICATION OF DISPLACEMENT CONTROL METHOD

Let's assume that the single incremental external force, ΔF applying at displacement controlled point at which incremental displacement, ΔU_k was given and external forces, $\Delta F_{1k}, \Delta F_{2k}, .., \Delta F_{nk}$ applying at the points, at which displacements, ΔU_u are unknown (see Fig.3).

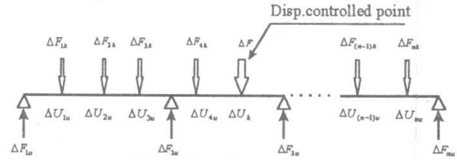

Fig.3. Loading scheme

These forces can be expressed in term of proportional forces $\lambda_{1k}\Delta F, \lambda_{2k}\Delta F, .., \lambda_{nk}\Delta F$ where $\lambda_{1k}, \lambda_{2k}, .., \lambda_{nk}$ are defined as $\lambda_{ik} = \Delta F_{ik} / \Delta F$. The unknown reactions $\Delta F_{1u}, \Delta F_{2u}, .., \Delta F_{nu}$ are applied to the fixed points at which displacements are equal zero. The proportional force vector is written in general form $(\lambda_k \Delta F, \Delta F, \Delta F_u)^T$, correspondingly the incremental displacement vector is $(\Delta U_u, \Delta U_k, 0)^T$. From force-displacement relation $[K]\{\Delta U\} = \{\Delta F\}$ can be written as

$$\begin{bmatrix} K_{11} & K_{12} & K_{31} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} \Delta U_u \\ \Delta U_k \\ 0 \end{Bmatrix} = \begin{Bmatrix} \lambda_k \Delta F \\ \Delta F \\ \Delta F_u \end{Bmatrix} \tag{3.1}$$

rewrite again

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta U_u \\ \Delta U_k \end{Bmatrix} = \begin{Bmatrix} \lambda_k \Delta F \\ \Delta F \end{Bmatrix} = \begin{Bmatrix} \lambda_k \\ 1 \end{Bmatrix} \Delta F \tag{3.2}$$

To solve equation (3.2) by using invert matrix operation

$$\begin{Bmatrix} \Delta U_u \\ \Delta U_k \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^{-1} \begin{Bmatrix} \lambda_k \\ 1 \end{Bmatrix} \Delta F = \begin{bmatrix} M \\ N \end{bmatrix} \Delta F \tag{3.3}$$

Finally, unknown displacement vector ΔU_u is obtained as $\Delta U_u = M \dfrac{\Delta U_k}{N}$ (3.4)

4. ANALYTICAL MODEL

4.1. INTRODUCTION OF ANALYTICAL MODEL

The nonlinear analysis was taken following experimental work[2] for two span continuous PC beam with external cable (as shown in Fig.5) which was conducted by Saitama University and was adopted following assumptions

1. Plane sections still remain plane after bending
2. Shear deformations are considered
3. Compatibility of deformations that is the total deformation of concrete at prestressing tendon level equals to that of elongation of tendon
4. Strain-stress relation model for concrete and prestressed cable as shown in Fig.4

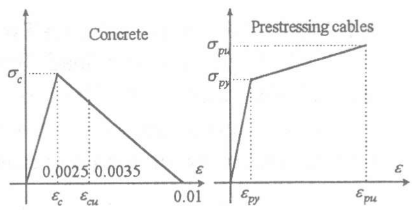

Fig. 4 . Stress - Strain relation

The analytical beam is having T shaped section, using two cables of SWPR7A type with diameter 12.4mm. Asymmetrical loading scheme was accepted as shown in Fig.5. It means that loading on the right span is equal to 30% of applied loading on the left span. Material properties are shown in table 1.

Table 1. Material properties Unit : N/mm^2

Concrete					Steel		Prestressed cables				
σ_c	Es	ε_c (%)	ε_{cu} (%)	ε_t (%)	σ_{sy}	E_s	σ_{py}	σ_{pu}	ε_{py} (%)	ε_{pu} (%)	E_p
40	3x10^4	0.25	0.35	0.015	350	2.1x10^4	1500	1750	3.5	0.79	1.9x10^5

In Eq.(2.1), coefficients k_{Di} are not Coloumb friction. These coefficients are considered to be a function of inclination of cables. It means that they depend on the angles between the cables and horizontal line. Furthermore they have a values 1 and 0 correspondingly with cases of cables having perfectly fixed or free slip, respectively. Otherwise, their values lie between 0 and 1 depending on the angles of cables.

Table 2. Friction coefficient

Friction coefficient	Before decom-pression	After decom-pression
Deviator D1	0.85	0.85→ 0.5
Deviator D2	0.85	0.85→ 0.5
Center support	0.95	0.95→ 0.75

In this analysis, friction coefficients were assumed that at the both ends of beam cables were perfectly fixed, at deviators and center support cables could be allowed slip. Before decompression, friction coefficients have a constant value and after decompression they have been changed. The change of friction coefficients at deviators is shown in table 2. The sign of friction coefficients at deviators is defined and based on its combination which is giving a maximum displacement at midspan.

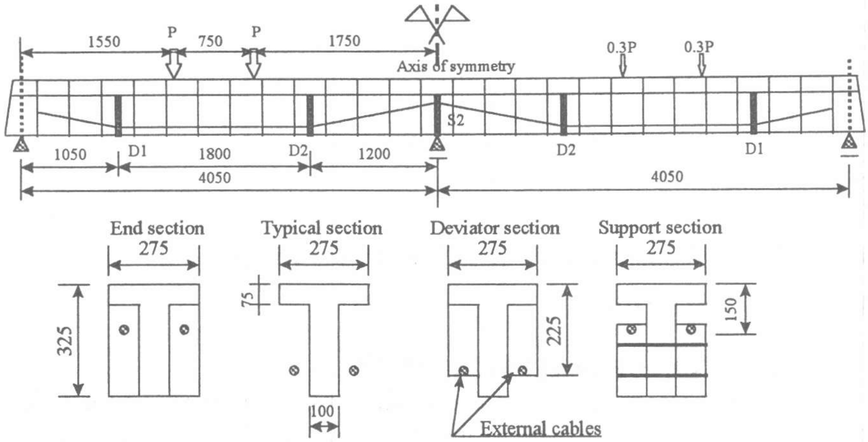

Fig 5. Layout scheme of two span continuous beam (Dimension in mm)

4.2. DISCUSSION OF ANALYTICAL RESULTS

In Fig.6 is shown load-displacement relationship of midspan section on the left span and on the right span. The first crack appears in tensile zone when loading reaches to 33.53kN and crushing concrete occurs in compression zone for midspan section and above support section when loading reaches to 72.4kN and 72.85kN, respectively. After crushing

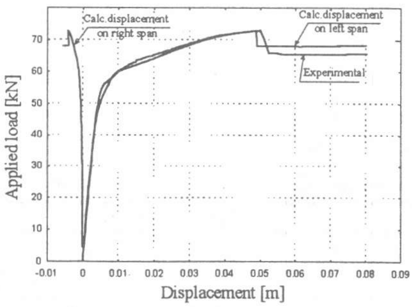

Fig.6. Load -Displacement relation

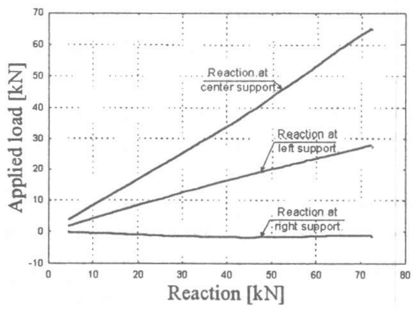

Fig.7. Reaction at supports

concrete, the load is suddenly dropped but displacement is gradually increased up to completely failure of structure. From Fig.6 it could be seen that the displacement path follows almost the same as experimental test. Although loading on the right span is equal to 30% of applied loading of left span, but displacement path has upward convex, correspondingly reaction at the right support has opposite sign to compare with two the others. Maximum reactions are 28.21kN and 65.12kN for left and center supports respectively, while at the right support is -1.41kN. The reactions at supports which caused by applied loading are shown in Fig.7.

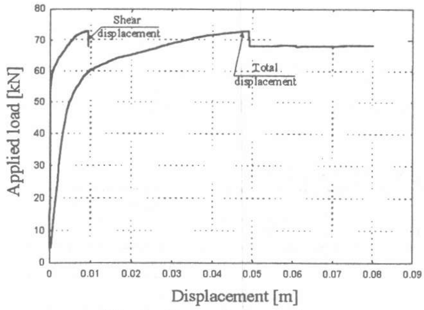

Fig 8. Shear deformation

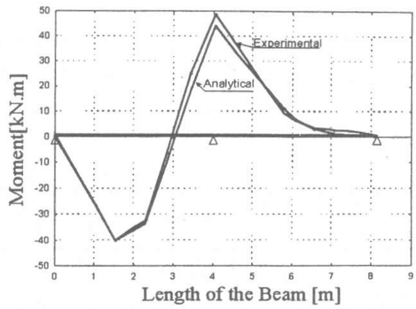

Fig.9. Distribution of moment along the beam

The deformation can be divided into two parts (the shear and flexural deformation). In Fig.8 the shear deformation is shown by comparison with total deformation. It could be seen that at elastic zone shear deformation is very small, almost nearly to zero. But after decompression, shear deformation increases very fast and is about 17% of total displacement at the ultimate stage

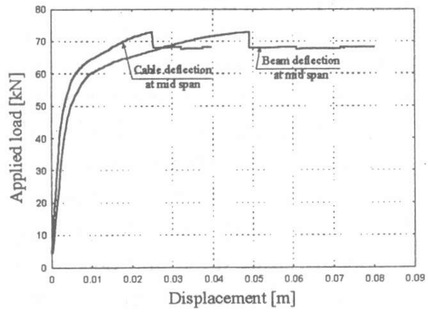

Fig.10. Difference of deflection between cable and beam

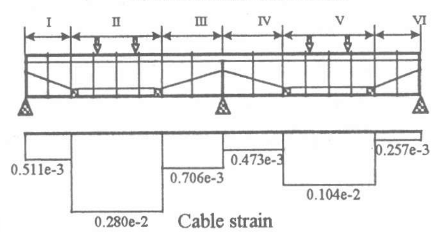

Fig 11. Schematic distribution of increase cable strain a long the beam

In Fig.9 is shown the distribution of moment along the beam and also compared with test data. The value of moment at midspan section on the left span is approximately 40kN.m, while at center support is about-43.8kN.m. This is very good agreement with experimental test accepted moment at middle support is slightly lower than test data.

In analytical process, the change of deflection shape of external cable due to applied load was also calculated. The difference of deflection shape of cable and concrete beam at midspan section on the left span is presented in Fig.10. It could be seen that deflection of the external cable is always less than deflection of beam, maximum deflection of cable is about 2.5cm. From calculated results, at the end of loading stage cable stress is still less than the ultimate value. This obviously explained that structural failure was caused by crushing concrete in compression zone and this phenomenon was found as the same as observation in experiment which was mentioned in [2]

In Fig 11 is shown distribution of cable strain along the beam. It could be seen that at ultimate stage on section II gives maximum value of cable strain. It satisfies loading condition as asymmetrical loading scheme. Cable slip at center support is calculated by assumption that total cable elongation minus total concrete elongation at position of cable on the same span. This is expressed as $\Delta l = \int_0^L \varepsilon_{cs} dx - \int_0^L \varepsilon_{si} dx$, where $\varepsilon_{si}, \varepsilon_{cs}$ are cable strain and concrete strain at position of cable, respectively. The result of calculation is presented in table 3. At ultimate stage the cable slip at center support is 2.4mm from calculation of left span and 2.3mm from calculation of right span, respectively. This difference could be explained as the accumulation of very small amount of error in each step up to ultimate stage.

Table 3. Cable slip at center support

Items	Left span				Right span			
Section	I	II	III	Total	IV	V	VI	Total
Length., m	1.05267	1.80000	1.20234		1.20234	1.80000	1.05267	
Cable strain, ε_{si}	0.485e-3	0.155e-2	0.587e-3		0.393e-3	0.578e-3	0.244e-3	
Cable elongation , m	0.511e-3	0.280e-2	0.706e-3	0.402e-2	0.473e-3	0.104e-2	0.257e-3	0.177e-2
Concrete strain , ε_{cs}	0.425e-4	0.367e-2	-0.183e-3		-0.300e-3	-0.772e-4	-0.255e-4	
Concrete elongation , m	0.446e-4	0.660e-2	-0.220e-3	0.642e-2	-0.361e-3	-0.139e-3	-0.269e-4	-0.527e-3
Cable slip at support, m				0.240e-2				-0.230e-2

3. CONCLUSION

The nonlinear analysis for estimating of structural behavior of span continuous PC concrete beam with external cables considering shear deformation was established by using of finite element method and fiber model. The following can be concluded from this study

1. By applying of one displacement controlled point and strain compatibility between concrete and external cables as above mentioned, structural behavior of externally PC beam can be obtained and estimated more accurately and closely to the test data.
2. Structural behavior of externally PC beam sensitively depends on initial condition at prestressing stage and current loading state. It means that depend on prestressing force and friction coefficient at deviators, more accurately calculated initial condition, the getting results are more closely to the experimental test
3. The values of friction coefficients are not constant after decompression so it is necessary to change these values in compliance with loading condition. The values of friction coefficients should be properly changed during loading step to get more accurate results
4. Shear deformation is about 17% of total displacement, the load-displacement response is very good agreement with test data.

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