論文 Post-Peak Analysis of Reinforced Concrete Panels Using Localization Signal

Mohammad Reza SALAMY[*1] and Tada-aki TANABE[*2]

ABSTRACT: In this paper, a unified plastic model for concrete ,in which only one criterion is considered to describe the behavior of concrete under different stress states, is used. This model can treat a major nonlinear phenomenon of concrete such as cracking, shear transfer degradation, tension stiffening and compressive strength reduction. Moreover ,localization signal is used to define shear band direction as well as mode of failure when the element passes peak point of loading. This phenomenon occurs when the material acoustic tensor A(n) looses its positive definiteness (*det.*$\mathbf{A}(\mathbf{n})$=0). The method has been evaluated by experimental results of Vecchio and Collins.

KEYWORDS: reinforced concrete ,localization phenomenon, tension stiffening, shear failure, shear band, non associated flow rule,

1. INTRODUCTION

The Drucker-Prager yield function is widely used in plasticity based analysis of pressure sensitive material such as concrete and rock. This criterion gives acceptable results for pure concrete while needs to be modified for concrete cooperated with reinforcement. It has been recognized that the major nonlinear factors for concrete in reinforced concrete structures can be briefly categorized in three items as follows:

1. tension stiffening effect
2. compression strength reduction due to transverse tensile strain
3. shear transfer at the crack surface

which should be considered in comprehensive models.

The already existing models can perhaps correctly reflect one of these various kinds of nonlinearity effects but fail to deal with others. Therefore we may encounter the judgment problem of appropriateness of choice of various nonlinear models for aforementioned factors. For these reasons, it is worthwhile proposing a unified plastic concrete model[1] in which only one model is used to describe the various nonlinear behavior of concrete under different stress states including tension stiffening and compressive strength reduction. The results of well known experiments done by Vecchio and Collins[2] have been used to show the capability of the method and evaluate predicted responses of the model for two modes of failure I and II. In mode I of failure, the final crack direction can be easily calculated by principal stress direction which is almost the same as initially formed cracks while in mode II, the final crack direction is considerably different than initially formed cracks so can not be found by the conventional method. In the other words, for the most cases of failure in

*1 Department of Civil Engineering, Nagoya University, Member of JCI
*2 Department of Civil Engineering, Nagoya University, DR, Fellow of JCI

mode-II, in fact at the final load stages near to the peak point, a shear band will be formed which is independent to initially formed cracks. This shear band is formed due to accumulation of strain inside of this band while in the neighboring points, strain reduces due to unloading so strain field show a jump and discontinuity in the border of shear band. This phenomenon is usually called localization. The localization phenomenon is signaled when the material acoustic tensor A(n) ceases to be positive definite or *det.***A**(**n**)=0. Solving the mentioned equation for vector **n** gives direction of shear band.

2. MODIFIED DRUCKER-PRAGER FAILURE SURFACE

Modified Drucker-Prager failure surface which is used in this paper can be expressed as follow:

$$f = J_2 - (k_f - \alpha_f I_1)^2 + (k_f - \alpha_f \eta)^2 \tag{1}$$

Moreover, a similar expression for the plastic potential function is assumed as

$$g = J_2 - (k_g - \alpha_g I_1)^2 + (k_g - \alpha_g \eta)^2 \tag{2}$$

where $I_1 = \sigma_{kk}$ and $J_2 = \frac{1}{2} s_{ij} s_{ij}$ are the first invariant of stress tensor σ_{ij} and the second invariant of deviatoric tensor s_{ij} respectively and α_f, k_f, α_g and k_g are material constants. According to the cohesion c, the internal friction angle ϕ, α_f and k_f are defined as

$$k_f = \frac{6c\cos\phi}{\sqrt{3}(3 + y\sin\phi_l)} \tag{3}$$

$$\alpha_f = \frac{2\sin\phi}{\sqrt{3}(3 + y\sin\phi_l)} \tag{4}$$

where ϕ_l is constant ($\phi_l = 14^o$) and the function of y is defined as

$$y = \sqrt{a(\cos 3\theta + 1) + 0.01} - 1.10 \tag{5}$$

$$a = \frac{1}{2} r^2 + 2.1 r + 2.2 \quad \text{where}$$

$$r = \begin{cases} 3.14 & I_1 \le f_c' \\ 2.93\cos\left(\frac{I_1}{f_c'}\pi\right) + 6.07 & f_c' < I_1 \le f_t \\ 9.0 & I_1 > f_t \end{cases} \tag{6}$$

where $\cos 3\theta = 3\sqrt{3} J_3 \big/ 2 J_2^{\frac{3}{2}}$ and $J_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki}$ is the third invariant of the deviatoric tensor s_{ij}. The notation ϕ and c are two strength parameters in Mohr-Coulomb criterion namely so-called mobilized friction angle and mobilized cohesion which are not constant but depend on the plastic strain history through the damage parameter ω. Possible relations for hardening and softening model are suggested as follows.

$$c = c_0 \exp[-(m\omega)^2] \tag{7}$$

$$\phi = \begin{cases} \phi_0 + (\phi_f - \phi_0)\sqrt{2\omega - \omega^2} & \omega \le 1 \\ \phi_f & \omega > 1 \end{cases} \tag{8}$$

where m is material constant.

The notation c_0, ϕ_0 and ϕ_f denote the initial cohesion, the initial internal friction angle and the final internal friction angle of concrete respectively. In the similar manner k_g and α_g are defined as

$$k_g = \frac{6c\cos\psi}{\sqrt{3}(3 + y\sin\phi_l)} \tag{9}$$

$$\alpha_f = \frac{2\sin\psi}{\sqrt{3}(3 + y\sin\phi_l)} \tag{10}$$

$$\psi = \begin{cases} \phi_0 + (\psi_f - \phi_0)\sqrt{2\omega - \omega^2} & \omega \le 1 \\ \psi_f & \omega > 1 \end{cases} \tag{11}$$

with a defined mobilized dilatancy angle ψ. For $\phi = \psi$, we have $f = g$ and the classic associated flow rule is recovered while for $\phi \ne \psi$ non-associated flow rule is contacted. ϕ_l and ϕ_0 have fixed values that $\phi_l = 14^o$ and $\phi = 5^0$ have been suggested [1].The notation η in Eqs.1 and 2 is tension stiffening which has a softening nature is expressed as

$$\eta = \eta_0 \exp(-\frac{\omega}{b}) \tag{12}$$

in which b is a function of steel ratio and η_0 is the tensile strength on hydrostatic axis which is also close to the uniaxial tensile strength. The damage parameter ω define the damage of material accumulated due to progressive growth of the micro cracks and can be defined in the form of

$$\omega = \frac{\beta}{\sigma_e \varepsilon_0} \int dW^p \tag{13}$$

where σ_e is the effective stress, W^p denotes the plastic work accumulated after the initial failure, β is a material constant and ε_0 is a constant value which is fixed at $\varepsilon_0 = 0.002$ or is an experimental data (for more details see [1]). β is an important material constant in defining damage parameter which has effect on compressive strain as well as softening branch of tension behavior rather than compressive and tensile strength of concrete. By comparing with the Kupfer's experimental results $\beta = 0.4$ is proposed [1]. The notation b also can be defined based on reinforcement ratio as

Table 1: Material properties for specimens pv10,pv11,pv23 and pv25

Specimen	σ / τ	$\rho_\ell \%$	$f_{y\ell}$ (Mpa)	$\rho_t \%$	f_{yt}	ε_0	f'_c (Mpa)	Failure Mode
pv10	-0.00/1.0	1.785	276	0.999	276	0.0027	14.5	I
pv11	-0.00/1.0	1.785	235	1.306	235	0.0026	15.6	I
pv23	-0.39/1.0	1.785	518	1.785	518	0.0020	20.5	II
pv25	-0.69/1.0	1.785	466	1.785	466	0.0018	19.2	II

$$b = \begin{cases} 0.209\exp(-1.228\rho)+0.07 & \rho \geq \rho_{\min} \\ 0.06 & \rho < \rho_{\min} \end{cases} \qquad (14)$$

where $\rho = \rho_x \cos^2\theta_{cr} + \rho_y \sin^2\theta_{cr}$ and $\rho_{\min} = 0.4\%$. ρ_x, ρ_y are reinforcement ratios for the x and y direction respectively and θ_{cr} denotes the angle of initially formed crack to x axis.

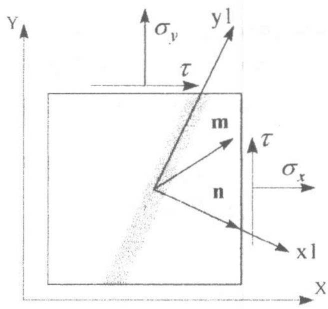

Fig.1 Localized damage band

3. MECHANISM OF LOCALIZATION AND SHEAR BAND FORMATION

Localization is a phenomenon that large strain accumulates inside a part of material without substantially affecting the strain in the surrounding materials as can be seen in Figs.1 and 6 . As it is known that material instability which can be possibly lead to the localization phenomenon in structures, occurs when acoustic tensor loss its positive definiteness. The physical mechanism for this phenomenon is that strain field across the damage band can possibly take a jump, while the equilibrium of the stress across the damage band remains to be satisfied (Fig.1). To derive under what condition this kind of localization is triggered, we will adopt the element level bifurcation analysis of Ortiz, *et al.*[3] The criteria for this kind of localization phenomenon can be expressed as

$$det.(\mathbf{A}(\mathbf{n}))=0 \qquad (15)$$

where $\mathbf{A}(\mathbf{n})$ is the acoustic tensor for composite material. Eq.15 can be rewritten as:

$$A_{ij}(n)m_k = (n_i D_{ijkl} n_l)m_k = 0 \qquad (16)$$

where D_{ijkl} is the tangential constitutive matrix of the material. If the material satisfies Eq.(16) ,then increments of strain can have a jump along the discontinuous surface(Fig.1).

It is notable that $\mathbf{n}^T\mathbf{m}$ can define the type of the failure. For a pure mode-I , $\mathbf{m}$ is aligned with $\mathbf{n}$ and $\mathbf{n}^T\mathbf{m} = 1$,on the other hand for a pure mode-II, $\mathbf{m}$ is perpendicular to $\mathbf{n}$ then $\mathbf{n}^T\mathbf{m} = 0$. Alternatively the first condition is related to the tension failure and the second one indicates shear failure. Likewise between two amounts zero and one is possible which shows mixed mode of failure in the element.

4. NUMERICAL INVESTIGATION

In order to evaluate the capability of the described model, four reinforced concrete panels tested by Vecchio and Collins are analyzed here and are checked to see if aforementioned kind of localization is detected or not. Analyses are in constitutive equation level rather than finite element formulation. It is obvious that this kind of localization is just detected in RC panels which failed in shear in mode-II. In mode-I, either the longitudinal or transverse reinforcements are yielded and direction of the final cracks are almost the same as initially formed cracks. For this mode, cracks direction can be easily calculated by principal stress directions. For mode-II, two types of failure have been observed.

1. sliding shear failure of the concrete after yielding of the transverse steel but prior to yielding of the longitudinal steel; or
2. sliding shear failure of concrete prior to yielding of either the longitudinal or transverse steel.

Panels pv10 and pv11 are analyzed here for mode-I along with pv23 and pv25 for mode-II.

All panels are analyzed using both associated and non-associated flow rule. As will be shown in the results, for all cases non-associated flow rule gives much more reasonable results comparing with associated flow rule. This has been mentioned with other researcher as well. It is notable that in the absence of compression, $\psi > \phi$ gives better results while $\psi < \phi$ yield closer result to experiment when compressive stress is applied. Material properties for specimens are given in Table1.

4.1 ANALYSIS OF PANELS PV10 AND PV11

These two panels failed in mode-I and II respectively without forming shear band reported by experiment and also found by analysis. Figs.2 and 3 show the response of shear stress in terms of shear strain for both non-associated and associated flow rule along with experimental results. Agreement with experiment is good in the case of using non-associated flow rule for both specimens. Since Eq.15 never satisfied for these panels, localization does not occur. That means shear bands are never formed for these specimens and the directions of cracks can be found through principal stress directions and they are 45° based on loading condition. Cracks directions reported by experiment are 45° but for pv10 at the last stage of loading, after yielding of transverse reinforcement, cracks shifted direction to become more acute to longitudinal direction.

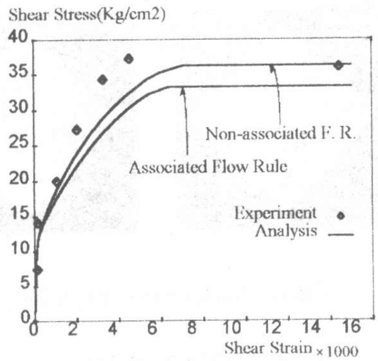

Fig. 2 Shear stress versus shear strain for pv10

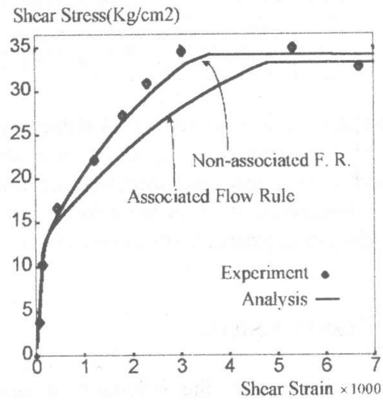

Fig. 3 Shear stress versus shear strain for pv11

4.2 ANALYSIS OF PANELS PV23 AND PV25

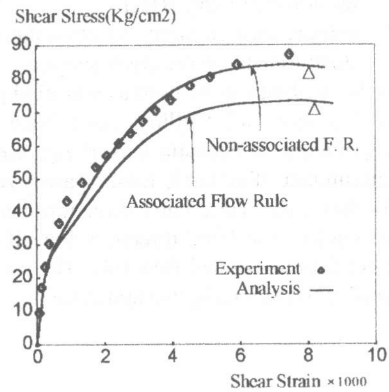

Fig. 4 Shear stress versus shear strain for pv23

To see the capability of the model in analyzing shear failure when shear band is formed, panels pv23 and pv25 are analyzed here. These two panels failed in shear (mode-II) prior to yielding of steel as reported by experiment and also obtained by analysis. Figs. 4 and 5 show the response of shear stress in terms of shear strain for both non-associated and associated flow rule along with experimental results. Agreement with experiment is good in the case of using non-associated flow rule for both specimens as before . In the points denoted by Δ , localization occurs and $det.\mathbf{A}(\mathbf{n})=0$ so each vector of **m** and **n** find an amount as are given in Table2. As can be seen and

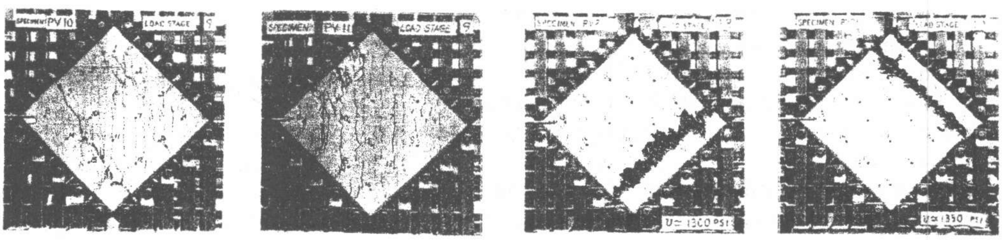

Fig. 6 Crack patterns at the last stage of loading for pv10,pv11,pv23 and pv25 respectively

Table 2: Shear band direction by analysis

Specimen	θ_m	θ_n or Shear band dir.
pv23	83.0	8.5
pv25	75.0	13.6

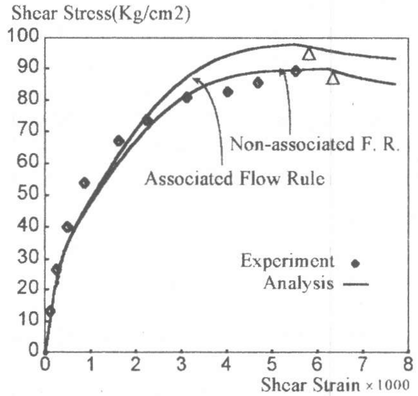

Fig. 5 Shear stress versus shear strain for pv25

compare with observed crack patterns (Fig.6, pv23 and pv25), agreement is good enough and reliable. It is notable that shear band direction given in Table 2 should be compared with shear band formed at the last stages of loading in experiment procedure (Fig.6, pv23 and pv25).

5. CONCLUSION

In this paper, the behavior of reinforced concrete panels based on modified Drucker-Prager model are analyzed. In this method three major nonlinear factors for concrete in reinforced concrete structures are considered as follows:

1. tension stiffening effect
2. compression strength reduction due to transverse tensile strain
3. shear transfer at the crack surface

so the method can be used as a unified plastic model for concrete . The analysis of four panels tested by Vecchio and Collins[2] have been done for non-associated and associated flow rule while determinant of acoustic tensor ($det.\mathbf{A}(\mathbf{n})$) signals localization near to peak points. As seen before, determinant of acoustic tensor passes zero point just when the failure is in mode-II or shear failure. For this case, shear band direction can be found with solving $det.\mathbf{A}(\mathbf{n})=0$ for **n** vector while **n** is normal to shear band direction. For all analyzed panels, non-associated flow rule gives much better result than associated flow rule. The method should be developed and checked for concentrated steel reinforcement like beams and columns.

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