コンクリート工学年次論文集，Vol. 23，No. 1，2001

論文 Overall Damage Investigation on Stucture using Laser Doppler Vibrometer (LDV) and Time-Domain System Identification Algorithm for the Analysis

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ABSTRACT: Major recent consideration about method for conducting overall investigation on structures for damage evaluation is being considered and solved. The device and method being used and developed respectively are a combination between *Laser Doppler Vibrometer* that can capture ambient vibration information of a structure and *Time-Domain System Identification Algorithm* that can outcome stiffness values of it. By combining these approaches, the part-by-part regular damage investigation of other NDT techniques is replaced by an overall investigation. As a case, damage investigation on tunnel is presented and some proven abilities of the method are highlighted.

KEYWORDS: Laser Doppler Vibrometer (LDV), ambient vibration, time-domain system identification algorithm, discretization of structure, stationary process.

1. INTRODUCTION

Civil engineering structures are undergoing damage in their ongoing life. That is the fact that civil engineers have to deal with when they are considering necessary maintenance procedure and/or retrofitting procedure and schedule for subsequent use of the structures. The need is even seriously upheld due to the recent increase in humankind's concern on safety.

Looking on the achieved development of non-destructive testing (NDT) in civil engineering, we are lacking technology to perform overall investigation of a structure, that is, a technology that enable us to have an overall view of remaining strength of the object structure. Most of the current NDT technologies only give us part-by-part "information" about a structure that can be related to its strength, such as position, pattern, and dimension of cracks, position of corroded reinforcement, etc, without having ability to say about the strength of a full structure.

That is the gap that I try to bridge; the gap between need for thorough/complete consideration about structure safety and the lack in NDT technology for that purpose. In order to do that, *Laser Doppler Vibrometer (LDV)* together with *Time-Domain System Identification Algorithm* are proposed here as the powerful device and method of investigation. It can give an overall consideration about a structure's strength in the form of stiffness matrix.

The following discussion will be about real application of LDV and Time-Domain system Identification Algorithm on a tunnel.

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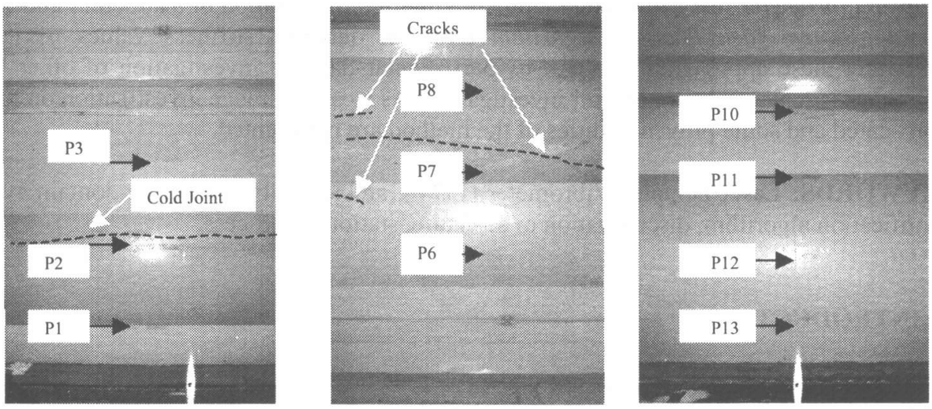
2. FIELD INVESTIGATION ON A TUNNEL

2.1 CHARACTERISTICS OF SPECIMEN

The "specimen" where we implemented the field investigation is a tunnel located in Kyobashi, Ginza, Tokyo. It is a concrete structure upon which an 8.5-meter width of roadway is passing through. Special attention is given to the middle part of the tunnel where most of the deteriorations are located. From bare-eye investigation, one can easily find cold joint separating the bottom and upper part of left-side wall of the tunnel. On the roof part of the tunnel, several cracks with different patterns exist that altogether make it good specimen for our study. Therefore, during the investigation, we isolated the middle part of the tunnel as the object of field investigation and excluded the left-right parts of it. Details about the crack patterns, cold joint and position of recorded points can be looked in Figure 1.



Left-Side Wall View Roof View Right-Side Wall View

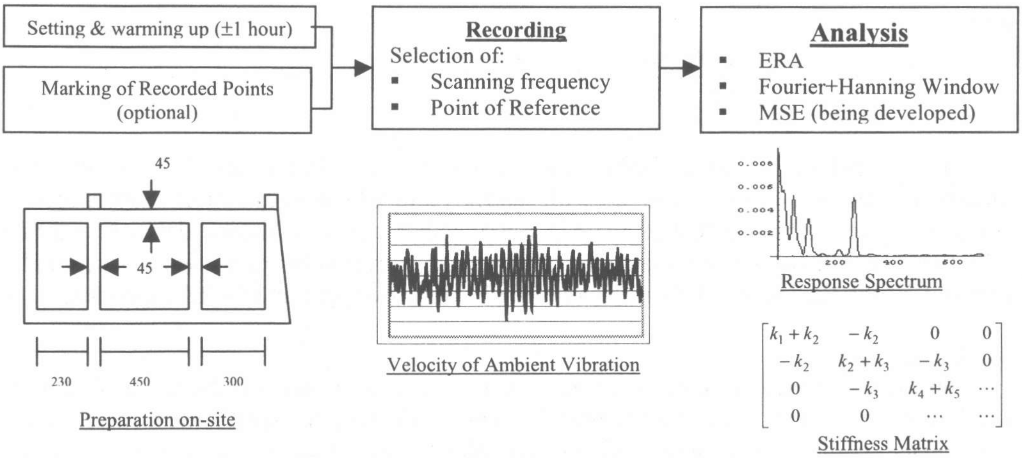
Figure 1 Deteriorations on the tunnel

Considering both total time for recording and number of laser devices we have, we decided to divide the wall, each side, into 5 spring-damper-interconnected lumped-masses and for the roof part 3 spring-damper-interconnected lumped-masses (see Figure 3).

2.2 PRINCIPLE OF MEASUREMENT

On the day of field investigation, in collaboration with Tokyo Metropolitan Expressway who is the owner of the property, we started the experiment at 4.00 pm after making our laser devices warmed up for 90 minutes. Meanwhile, we put some marks on the walls and roof as the positions of recorded points. Note that this action was only intended to increase degree of accuracy of our experiment so that when we deal with a bigger structure whose surface is unreachable, it is unnecessary to put marks for recorded points.

Ambient vibration of each point was recorded at 5000 Hz sampling rate for 10 minutes. For the recording purpose, several points of reference were selected, which are point number 1 for the left wall, point number 7 for the roof, and point number 13 for the right wall. Therefore, during the recording stage, the first laser device was fixedly pointed to the point of reference of each segment of structure and the other one (the 2nd laser device) was pointed to the other designated points in sequence. This was repeatedly done until all points' ambient vibration was fully recorded. Then, following it, an in-house analysis using our own developed application program was done.



$$\begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_4+k_5 & \cdots \\ 0 & 0 & \cdots & \cdots \end{bmatrix}$$

Figure 2: Principle of Measurement in Summary (Top), Result of Each Stage (Bottom)

3. THEORIES AND ANALYSIS

3.1 DAMAGE INVESTIGATION BASED ON STIFFNESS MATRIX – THE EIGENSYSTEM REALIZATION ALGORITHM (ERA)

When we try to consider strength of a structure, we always have to come up with stiffness matrix. So structure strength and structure stiffness are inseparable items in damage investigation. Based on this fact, the recorded information of velocity of ambient vibration is analyzed in such a manner that we will obtain stiffness matrix of the structure. The method is the so-called "Time Domain System Identification Analysis" taking the advantage of *Eigensystem Realization Algorithm (ERA)* that is so much powerful in analyzing vibration with unknown input excitation.

Equation of motion of a MDOF structure in vibration can be written in ordinary form and state-space form as follows:

$$\textit{Ordinary form}: \quad M\ddot{q}(t)+C\dot{q}(t)+Kq(t)=Ju(t) \tag{1}$$

$$\begin{aligned} \textit{State-space form}: \quad \dot{x}(t) &= F_c x(t)+H_c u(t) \\ y(t) &= D_c x(t)+v(t) \end{aligned} \tag{2}$$

,where *u(t)* is the unknown excitation; *M,C*, and *K* are mass, damping and stiffness matrix respectively; *q* is the lateral structural displacement and the dot over it representing degree of differentiation of *q* over time. In state-space form, $x(t)$, F_c, H_c, and D_c can be described more detail into:

$$x(t)=\begin{pmatrix} q(t) \\ \dot{q}(t) \end{pmatrix}, \quad F_c=\begin{bmatrix} 0_{mxm} & I_{mxm} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad H_c=\begin{bmatrix} 0_{mxm} \\ M^{-1}J \end{bmatrix}, \quad D_c=\begin{bmatrix} I_{mxm} & 0_{mxm} \end{bmatrix} \tag{3}$$

When we are dealing with time history record of ambient vibration, the recorded data will be discretized by a fixed interval of time so that the continuous-time state-space form of equation (2) can be rewritten in discrete-time form for the *k*th time step as follows:

$$\begin{aligned} x_{k+1} &= Ax_k+Bu_k = Ax_k+w_k \\ y_k &= Dx_k+v_k \end{aligned} \tag{4}$$

and

$$A = e^{F_c \Delta t} = \sum_{j=0}^{\infty} \frac{(F_c \Delta t)^j}{j!}, \qquad B = -\int_0^{\Delta t} e^{F_c \tau} H_c d\tau = F_c^{-1}(A-I)H_c \tag{5}$$

Eq.(3) and Eq.(4) are the equations relating discrete time series data of (ambient) vibration of the structure (y_k and x_k) with matrix F_c in which information about structure stiffness is retractable so that if we can put together the time-history data of structure vibration and transform it into matrix F_c, then we can extract submatrix ($-M^1 K$) and finally matrix K as the final result. This is the key idea behind development of ERA algorithm.

3.2 ANALYSIS

In total, there are 10 pairs of recorded data, which are 4 pairs for the left wall, 4 pairs for the right wall, and 2 pairs for the roof. Looking to the fact that until now, *Laser Doppler Vibrometer* could not capture information about joint rotation degree of freedom, simplification on the structure modeling was done, that is, by neglecting any joint rotation degree of freedom between 2 adjacent balls and that between wall and roof on the two corner parts of the tunnel. As a result in the modeling, the restriction leads to separation of the tunnel into 2 walls supporting 1 roof on top. Therefore, in total we have 3 isolated sub-structures for the analysis, as depicted in Figure 3.

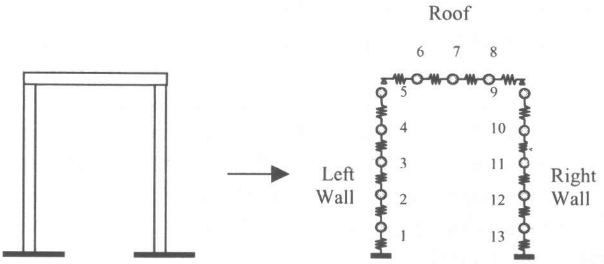


Figure 3: Discretization of the tunnel.

Then, analysis of the data was done in sequence for each segment of the tunnel by first compiling the 4 pairs data of the left wall, which were obtained from 4 sequence recordings, into one group of 5-lumped-mass time history data. In order to do that, we have to assume that the ambient vibration is stationary in nature, that is, the statistics of the vibration are not affected by a shift in the time origin. Put it in more detail: $x(t)$ *and* $x(t+\varepsilon)$ will have the same statistics for any ε. By assuming this, we can directly combine all vibration records of all the 5 points regardless the difference in recording time origin.

Afterwards, the compiled data of 5 lumped-masses was used as an input to *ERA* analysis performed with our developed application program, built in *Mathematica* environment. Construction of Hankel matrix from our compiled data of 5 lumped-masses was done first and followed by subsequent flow of analysis. After having the matrix F_c following the form as described in Eq.(3), the submatrix ($-M^1$ K) was extracted. Nevertheless, we could not directly compute the stiffness K from that submatrix since we have not known the system mass M yet. Here lays some incorrectness due to inadequate information about the structure mass. However, we decided to see this matter as challenge instead of barrier that hindered us to come to the solution that, as precise as possible, we calculated the tunnel system mass and used it as the input to calculate stiffness K. As a result, the stiffness value of each lumped mass could be obtained and this is the end of analysis for the left wall.

Following the method previously explained, analysis of recorded data for the 5-lumped-masses on the right wall and that for the 3-lumped-masses on the roof could be executed similarly as the foregoing. After performing back substitution, stiffness values of all points were extracted and listed below in Table 1.

Table 1 Stiffness Values of all nodes

Point No.	**Position**	**Stiffness Value ($\times 10^{11}$ N/m)**
1	L Wall	1.34062
2	L Wall	1.33649
3	L Wall	1.27142
4	L Wall	1.21674
5	L Corner	1.32668
9	R Corner	1.26033
10	R Wall	1.20912
11	R Wall	1.26114
12	R Wall	1.27665
13	R Wall	1.32592

Point No.	**Position**	**Stiffness Value ($\times 10^{11}$ N/m)**
6	Roof	1.25177
7	Roof	1.28656
8	Roof	1.23489
8_{end}	Roof	1.25146

3.3 DISCUSSION ON RESULT OF ANALYSIS

Stiffness values of all nodes have been obtained and now those values can be used to recalculate the frequencies of vibration of each segment of the tunnel. Reconstruction of sub-structure stiffness matrixes for each substructure (2 walls and 1 roof) was first done and then all frequencies of vibration can be obtained by solving the ordinary equation:

$$\det\left\lfloor K-\omega^2 M\right\rfloor=0$$

As a result, 5 frequencies of vibration for each wall together with 3 frequencies for the roof are listed in Table 2.

Table 2 Frequencies of vibration of tunnel substructures

Left Wall

Mode	**Freq. (Hz)**
1	473.291
2	1368.01
3	2177.44
4	2792.73
5	3152.98

Right Wall

Mode	**Freq. (Hz)**
1	468.248
2	1355.59
3	2148.06
4	2745.20
5	3119.42

Roof

Mode	**Freq. (Hz)**
1	1030.69
2	1906.55
3	2495.07

Several remarks need to be clarified here. First, as we can see from Table 2, the obtainable frequencies of vibration are in the range of high frequencies. Put it more straightly, we could not find low frequencies of vibration, which represent low mode shapes of vibration. The reason behind this fact is on our decision to discretize the tunnel into 5-lumped-masses for each wall and 3-lumped-masses for the roof. This led to an incomplete determination of mode shapes and corresponding frequencies of vibration. If we discretize the tunnel into higher number of masses, then we will find the frequencies related to lower modes of vibration. Nevertheless, this is not a substantial misfortune since every finite element analysis requires a sacrifice in incompleteness since meshing will not be able to cover all degree of freedom inside the object. Moreover, strength consideration neither need a full picture of structure's mode-shape nor vibration frequencies since the constructed stiffness matrix already gives us information about strength capacity of the object structure, stress-strain relationship, flexural deflection, and (hidden) damages.

Graphical description about the phenomenon of unobtainable frequencies of vibration can be described in detail using Figure 4. As the lumped masses are in the line of a certain mode shape (for instance, the full straight line for defining 1st vibration mode), we can never know the exact vibration mode of it since the parts in between 2 adjacent masses are left unobserved.

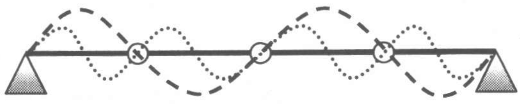


Figure 4 An Ambiguity in defining Mode Shape

Therefore, a trade-off between completeness and efficiency will always be present before determining a number of degree of freedom of a certain structure. Pursuing the former will increase the number of identified frequencies of vibration using ERA and reducing the efficiency of work, *vice versa.*

Secondly, up to this stage of our research progress, method for detecting and localizing damaged elements on structure is still being developed. The principle is that as a structure is deteriorated, the associated damage causes a loss of stiffness in one or more elements of the structure which, at the end, changing the vibration mode shape of it. This phenomenon can be examined by calculating Modal Strain Energy (MSE) of damaged and undamaged elements. Then, comparison between those 2 values will enable us to do the damage detection and localization. We leave this topic for future discussion.

4. CONCLUSION

Damage investigation on structure aiming to have overall picture about strength of a tunnel structure has been fully performed, analyzed, and presented in this paper. The device utilized here is *Laser Doppler Vibrometer* that can capture information of ambient vibration of a structure. Recorded data is further analyzed by taking the advantage of *Time-Domain System Identification Algorithm*, which is written as a routine application program developed in *Mathematica* environment. The final result is in the form of structure stiffness values. Several remarks have been clarified about incomplete acquired frequencies of vibration and future development of the system that enables us to detect and localize damaged elements.

ACKNOWLEDGEMENT

Special thanks are addressed to Mr. Takayoshi Hirata from Obayashi Corporation and K.Okuyama from Shibaura Institute of Technology who have helped me a lot during the field investigation.

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