NUMERICAL MODEL OF DRYING SHRINKAGE OF CONCRETE

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ABSTRACT: The overall time development of drying shrinkage of concrete can usually be predicted with reasonable accuracy by using the currently existing prediction formulas such as ACI, B3, CEB 1990, GL 2000 and JSCE models. However, they are not sufficient when the drying condition is complex and the local strain value is needed. Therefore, in this study, a numerical model approach is investigated. This numerical model includes some model parameters to be determined. These parameters are estimated by comparison between calculation results by prediction formulas and those by the present numerical model.
KEYWORDS: drying shrinkage, prediction model, simulation, modeling, concrete

1. INTRODUCTION

The current development of drying shrinkage prediction has reached a level at which a reasonably accurate prediction can be made quite easily by means of a simple mathematical formula. Some of these prediction formulas are ACI, B3, CEB 1990, GL 2000 and JSCE models. These models are very useful especially for design engineers by providing an estimation of how much a concrete body of a certain size will deform due to drying by merely inputting certain parameters such as the 28-day concrete strength and the cement type. These models, however, can only give the overall shrinkage strain estimation, so that for cases in which local shrinkage at a particular point in a concrete body is being concerned, they can no longer be utilized. Also for cases where the ambient humidity is variable, or the shape of the drying concrete is not simple, these models are no longer sufficient. To resolve this problem one needs to resort to a numerical simulation type analysis either by the finite element method or by other types of simulation method. However this approach requires some physically based mathematical model whose parameters are different from those used with the prediction formulas. These parameters might be the moisture diffusivity, the shrinkage coefficient, and so on. Unlike in the case of prediction formulas, these parameters are not generally available to design engineers, and need to be determined experimentally.

In the current study, an attempt has been made to estimate the model parameters by extensive comparison of prediction curves of shrinkage strain of cylindrical specimens against drying time calculated by prediction formulas and those by a numerical model. The resulted relationship between parameters of the prediction formulas and those of the numerical model are presented. Based on this result, applications for cases with cyclic ambient humidity and with different measurement points are presented.

2. NUMERICAL MODEL OF DRYING SHRINKAGE

The numerical simulation model using the finite element method with the nonlinear moisture diffusivity dependent on the local relative humidity and the concept of fictitious layer as proposed by the authors [1] is chosen for the current study.

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2.1 MOISTURE DIFFUSION

In this model, the humidity-dependent diffusivity D is represented by a tri-linear model, as shown in **Fig.1**. The humidity limit h_2 = 0.98 and the minimum diffusivity D_1 = 0.15D_2. For simplicity, the tri-linear relationship is further pre-determined by assigning h_1 = 0.60, thus leaving the maximum diffusivity D_2 as the only parameter which needs to be determined.

In this model it is also considered that there is a thin fictitious layer with thickness T connecting the surface of a specimen to the ambient air. For the current study the fictitious layer thickness is fixed as T = 0.1 cm. Moisture diffuses linearly in this layer so that a linear diffusion equation with diffusivity D_{fl} is used. D_{fl} is considered to be proportionally dependent on the ambient relative humidity h_a, as given by the following Eq. 1.

$$D_{fl} = C_{fl} h_a \quad (1)$$

where C_{fl} is the proportional constant, and is the second parameter to be determined in this model.

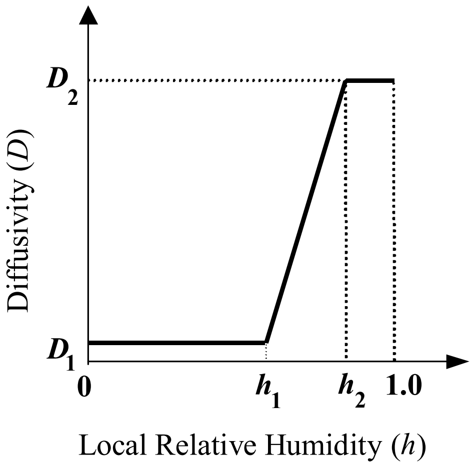

Fig.1 Humidity Dependence of Diffusivity

2.2 DRYING SHRINKAGE STRAIN

For strain calculation, a linear relationship between an incremental local relative humidity Δh and the unrestrained incremental change of shrinkage strain $\Delta\varepsilon$ is assumed, as given in Eq. 2.

$$\Delta\varepsilon_i = \alpha_{sh} \Delta h \quad (2)$$

where index i is used to stand for r, θ, z components in the cylindrical coordinate system and α_{sh} is a proportional constant. α_{sh} is the third parameter to be determined in the present model.

2.3 MECHANICAL PROPERTIES

For stress calculation which takes into account the effect of creep, the following effective modulus E_{eff}, a simplified version of the age-adjusted effective modulus, is used.

$$E_{eff}(t, t_o) = \frac{E(t_o)}{1 + \phi(t, t_o)} \quad (3)$$

where E is the modulus of elasticity, ϕ is the creep coefficient and, t stands for the current concrete age in days and t_o for the age of concrete at the start of drying. For simplicity the creep coefficient ϕ is considered constant for the current study, and is assigned a moderate value of ϕ=2.5. The creep Poisson's ratio v_{cr} is assumed the same as the elastic Poisson's ratio v, and is given a value of v =0.2.

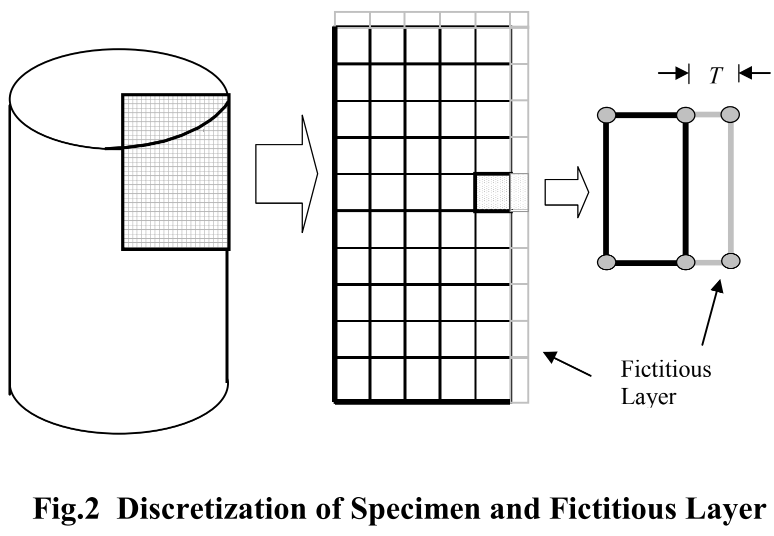

Fig.2 Discretization of Specimen and Fictitious Layer

2.4 AXISYMMETRIC FINITE ELEMENT MODELING

The present numerical model uses axisymmetric finite element modeling for both the diffusion analysis and the stress analysis. A four-node isoparametric axisymmetric linear element is considered appropriate to model the moisture flow as well as the shrinkage deformation of cylindrical specimens. The same element is used to model the fictitious layer.

The schematic finite element representation of a cylindrical specimen and the fictitious layer is shown in **Fig.2**. The figure represents the case for which all surfaces are exposed to drying. A fixed boundary condition is applied on the external surface of the fictitious layer.

3. DETERMINATION OF PARAMETERS IN THE MODEL

3.1 PREDICTION MODELS

The CEB 1990 prediction model [2] and the GL 2000 prediction model [3] as shown in **Table 1**, are chosen to determine the values of the parameters involved in the present model. These two prediction models are selected because the form of the shrinkage prediction formula is the product type. In the determination of parameters, this characteristic is convenient to make clear the influence of each parameter on the shrinkage strain. Both CEB 1990 and GL 2000 shrinkage prediction models require the 28-day mean compressive strength f_{cm} and the type of cement. The only difference of parameter appears when taking into account the effect of size. CEB 1990 model uses the notional size, whereas GL 2000 model uses the volume-surface ratio for this purpose.

Table 1 Prediction Models of Drying Shrinkage

Drying Shrinkage Strain $\varepsilon_{sh} = \varepsilon_0 \beta(h) \beta(t-t_0)$		
Functions	CEB 1990 model	GL2000 model
ε_0	$1.55[160+10\beta_{sc}(9-(f_{cm}/10))]\times 10^{-6}$	$1000K(30/f_{cm})^{0.5}\times 10^{-6}$
$\beta(h)$	$1-h^3$ $(0.40 \leqq h < 0.99)$	$1-1.18h^4$
$\beta(t-t_0)$	$[(t-t_0)/\{(t-t_0)+0.14(A_c/u)^2\}]^{0.5}$	$[(t-t_0)/\{(t-t_0)+0.15(V/S)^2\}]^{0.5}$

[Note] h: Humidity (decimal); f_{cm}: Comp. strength of concrete at 28 days (MPa); β_{sc}: 5(N, R cement), 4(SL cement), 8(RS cement); K: 1(Type I cement), 0.70(Type II cement), 1.15(Type III cement); t: Age of concrete (days); t_0: Age at drying (days); A_c: Cross-sectional area (mm^2); u: Perimeter of cross section under drying (mm); V/S: Volume-surface ratio (mm)

3.2 NUMERICAL VALUE DATA SETS

To compare the present model to the prediction models, a data set of numerical values is prepared by using prediction formulas and the present model. Cylindrical model specimens of 50-mm diameter and 200-mm height are chosen for the data set of the present model. This size is considered sufficient to examine the difference between shrinkage strain along the centerline and that at the surface. An axisymmetric element with 2.5 mm x 2.5 mm size is used to discretize the model specimen. The model specimen is assumed unsealed, so that the fictitious layer is attached to the whole surface. The time step is 0.25 day and the drying duration is 50 days. Data points are stored into the data set every 5-day drying time, giving 20 data-points for each interval. The ambient humidity is set to 40, 60 and 80% RH. The temperature is set constant, 20°C. The values of the three parameters in the present model are shown in **Table 2**. Simulations are done for all combinations of these parameter values.

Table 2 Model Parameters for Simulation

D_2 (cm^2/day)	C_{fl} (cm^2/day)	α_{sh}
0.1	0.005	0.0005
0.25	0.015	0.0010
0.50	0.025	0.0015
0.75	0.050	0.0025
1.00	0.075	0.0050
1.50		0.0100

Table 3 Parameters of Prediction Formulas

Type of cement	f_{cm} (MPa)
Slow hardening (Type II)	18
Normal (Type I)	28
Rapid hardening (Type III)	38
	48
	58
	68

3.3 METHOD OF INTERPOLATION

Drying shrinkage strains by the CEB 1990 and GL 2000 prediction formulas are calculated for all the combinations of parameters given in **Table 3**. It is considered that most concrete conditions fall within the range of parameters given in **Table 3**. Each shrinkage strain curve resulted from a certain combination of prediction model parameters is compared to the simulation curves in the data sets. A linear interpolation scheme is implemented with the simulation curve data set to find a combination of simulation model parameters whose interpolated curve matches most closely with each of the curves of the prediction model. The closest interpolated curve is searched with accuracy in D_2 to 0.01 cm^2/day, in C_{fl} to 0.001 cm^2/day, and α_{sh} to 0.00001. The solution is obtained by the minimum closeness index, which is defined as the average difference between the corresponding set of data points.

3.4 COMPARISON RESULT

A match to each prediction curve is found as the interpolated simulation curve, which has the smallest closeness index. The comparison results show that all matching curves are found with closeness index less than 10^{-6}. The strains along the centerline were compared.

Through the comparison results with both prediction formulas, the dependence of the fictitious layer coefficient C_{fl} , the maximum diffusivity of concrete D_2, and the shrinkage coefficient α_{sh} on the parameters used in the prediction formulas, i.e., the type of cement and the compressive strength of concrete was determined. It was assumed that the fictitious layer coefficient C_{fl} is influenced by the ambient relative humidity h_a only. It was also assumed that the maximum diffusivity of concrete D_2 and the shrinkage coefficient α_{sh} are influenced by the ambient relative humidity h_a. This assumption was needed to model the effect of cyclic humidity change on the drying shrinkage with a limited number of model parameters. The effect of the type of cement and the compressive strength of concrete was implemented into the shrinkage coefficient α_{sh} only, due to limited experimental data of the moisture transfer in concrete. The shrinkage coefficient α_{sh} increases with more rapid hardening cement and with decreasing compressive strength of concrete.

The result of comparison study is represented in the following equations. The forms of the equations were chosen to best fit the parameters' behavior as indicated by the comparison results. The fitting is performed with correlation coefficient R^2 of greater than 0.95.

For comparison with CEB 1990 prediction formula:

$$D_2 = -0.0133\,h_a + 1.38 \tag{4}$$

$$C_{fl} = 0.0038\,h_a + 0.1 \tag{5}$$

$$\alpha_{sh} = \begin{cases} (-0.0097\,h_a f_{cm} - 0.476\,f_{cm} + 1.33\,h_a + 58)\times 10^{-5} & \text{for slow hardening cement} \\ (-0.0125\,h_a f_{cm} - 0.580\,f_{cm} + 1.55\,h_a + 69)\times 10^{-5} & \text{for normal cement} \\ (-0.0205\,h_a f_{cm} - 0.880\,f_{cm} + 2.25\,h_a + 98)\times 10^{-5} & \text{for rapid hardening cement} \end{cases} \tag{6}$$

For comparison with GL 2000 prediction formula:

$$D_2 = -0.0148\,h_a + 1.54 \tag{7}$$

$$C_{fl} = 0.0013\,h_a + 0.3 \tag{8}$$

$$\alpha_{sh} = \begin{cases} (7.5\times 10^{-5}\,h_a + 2.4\times 10^{-3})\,f_{cm}^{-1/2} & \text{for slow hardening cement} \\ (10.8\times 10^{-5}\,h_a + 3.5\times 10^{-3})\,f_{cm}^{-1/2} & \text{for normal cement} \\ (12.3\times 10^{-5}\,h_a + 4.0\times 10^{-3})\,f_{cm}^{-1/2} & \text{for rapid hardening cement} \end{cases} \tag{9}$$

The 28-day mean compressive strength f_{cm} is in MPa and the ambient relative humidity h_a is in percent RH.

4. CASE STUDIES

The simulation model using the obtained relationships are applied for two cases: case 1 is concerning a cyclic ambient humidity condition and case 2 is concerning different measuring point locations.

Table 4 Parameters Used in Case Study Simulations

Case	Model by CEB 1990			Model by GL 2000		
	D_2 (cm^2/day)	C_{fl} (cm^2/day)	α_{sh}	D_2 (cm^2/day)	C_{fl} (cm^2/day)	α_{sh}
1	0.52	0.347	0.99x10^{-3}	0.58	0.384	1.47x10^{-3}
2	0.78	0.271	0.94x10^{-3}	0.87	0.358	1.34x10^{-3}

For case 1 the experimental data of Muller *et al.*[4] is used. In this experiment, cylindrical concrete specimens with the diameter (d) to height (h) ratio d/h = 50 mm/200 mm were exposed to either a constant ambient humidity of 65% RH or a cyclic ambient humidity started with 90% RH for 7 days then followed by 40% RH for 7 days. The temperature was kept constant at 20°C and strain measurements were done at the drying surface. The modulus of elasticity at the beginning of drying at age 8 days was 29840 MPa. The cement used was normal cement.

For case 2 the experimental data of Ayano *et al.* [5] is used. In this experiment a cylindrical mortar specimen with the diameter (d) to height (h) ratio d/h = 150 mm/150 mm was allowed to dry under a constant ambient humidity of 45% RH. The top and bottom surfaces were sealed to prevent moisture transfer. The mean compressive strength at age 14 days was 33.6 MPa, and the Young's modulus was 25.8 GPa. The cement used was normal cement.

Using Eqs. 4, 5, 7 and 8 to obtain appropriate values of D_2 and C_{fl} according to the comparison study for both of the investigated prediction formulas, the values shown in **Table 4** are obtained. The compressive strength at age 28 days is estimated by using the ACI formula [3] from either the mean modulus of elasticity or the mean compressive strength at any arbitrary age, and the resulting compressive strengths are f_{cm} = 50.9 MPa for case 1, and f_{cm} = 38.9 MPa for case 2. Then α_{sh} is obtained form Eqs. 6 and 9 for normal cement.

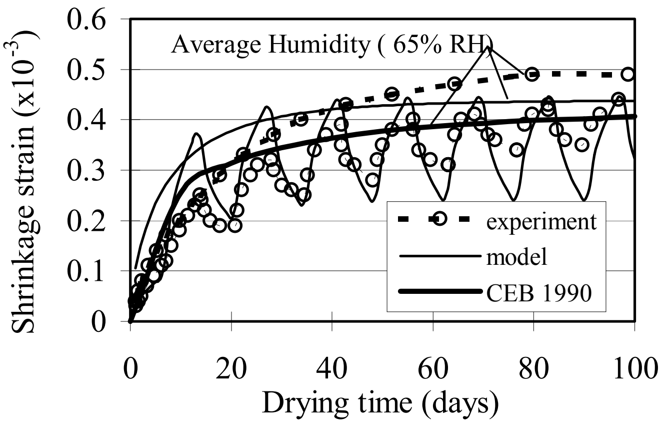

Fig.3 Simulation for Case 1 (CEB 1990 model)

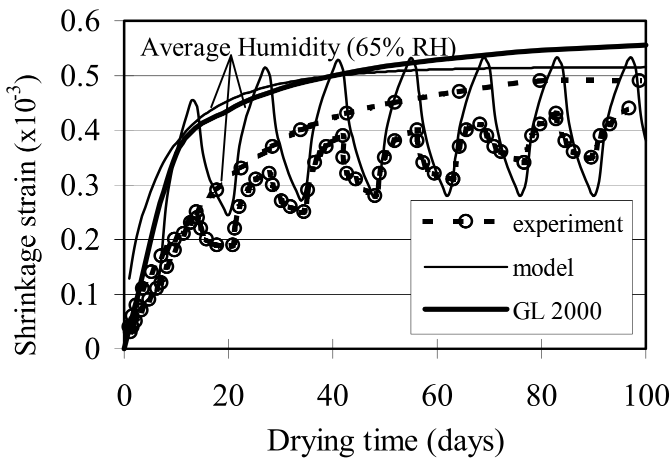

Fig.4 Simulation for Case 1 (GL 2000 model)

The simulation results for case 1 are shown in **Fig. 3** for the case with CEB 1990 model and in **Fig. 4** for the case with GL 2000 model. Though the general shrinkage behavior is predicted by both simulations, the model based on CEB1990 model gives closer agreement with the experimental data for the cyclic humidity change. The thick solid lines in **Fig. 3** and **Fig. 4** indicate the predictions by CEB 1990 model and GL 2000 model for the average constant humidity. Sufficient agreement between the prediction formulas and the present model is confirmed.

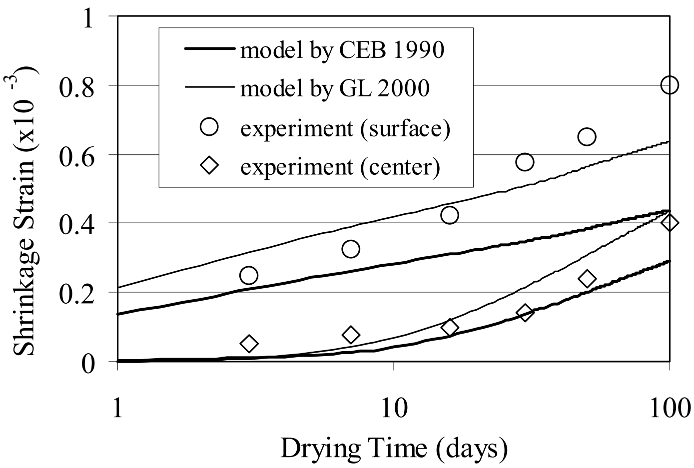

Fig 5 Simulation for Case 2

The simulation results for case 2 are shown in **Fig. 5**. The general shrinkage behaviors at the centerline and at the surface are predicted by the simulations based on both models. In this case the simulation based on GL 2000 model gives closer agreement with the experimental data.

5. CONCLUSIONS

A comparison study between the drying shrinkage prediction by CEB 1990 as well as by GL 2000 prediction formulas and the simulation results of the present numerical model was done. The characteristics of the present numerical model and the result of the simulation case studies are summarized as follows.

(1) The moisture transfer property at the drying surface represented by the fictitious layer was assumed seems to be influenced by the ambient relative humidity.
(2) The moisture diffusivity and the shrinkage coefficient, a function of both the type of cement and the mean compressive strength, are assumed to be influenced by the ambient humidity to take into account the effect of cyclic humidity change.
(3) It is confirmed from the simulation case studies that the drying shrinkage phenomenon is influenced by the complex phenomena such as the moisture flow, the internal deformation and the drying environment.

REFERENCES

1. Putro, H.S. and Tsubaki, T., "Simulation of Drying Shrinkage under Cyclic Ambient Humidity Condition," Proc. of the JCI, Vol.23, No.2, 2001, pp.757-762.
2. CEB : CEB-FIP Model Code 1990, Thomas Telford, 1993, pp. 53-58, 61-65.
3. Gardner, N.J., and Lockman, M.J., "Design Provisions for Drying Shrinkage and Creep of Normal-Strength Concrete," ACI Materials Journal, Vol.98, No.2, 2001, pp.159-167.
4. Muller, H.S. and Pristl, M., "Creep and Shrinkage of Concrete at Variable Ambient Conditions," Creep and Shrinkage of Concrete, Proc. of the Fifth International RILEM Symposium, 1993, pp.15-26.
5. Ayano, T., Sakata, K. and Wittmann, F.H., "Moisture Distribution, Diffusion Coefficient and Shrinkage of Cement-Based Materials," J. Materials, Conc. Struct., Pavements, JSCE, Vol.45, No. 634, 1999, pp. 387-401.