3D REINFORCED CONCRETE ANALYISIS BASED ON LATTICE EQUIVALENT CONTINUUM MODEL IN KINEMATIC CONSTRAINT

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ABSTRACT: In this paper, robust and engineering oriented three dimensional reinforced concrete constitutive laws are formulated from the concept of equivalent continuum of lattice system in known strain field i.e. kinematic constraint. Shear lattice is introduced in the system for transfer of interlocking shear stresses between the contact surfaces of adjacent concrete lattices. A beam-column-slab assembly is then investigated applying the developed model. The calculation results show good agreement with the experiment.
KEYWORDS: reinforced concrete, lattices, constitutive equations, three-dimensional models

1. INTRODUCTION

The lattice equivalent continuum model, hereafter *LECM*, has previously been developed and applied to two dimensional problems in reinforced concrete, and its analytical results showed good agreement with the experimental results for both static and time varying load [1]. However, three dimensional constitutive equation for concrete structures needs to be developed in order to analyze real behavior of concrete structures. In a previous paper [2], concept of three-dimensional approach to be taken in LECM was elaborated in detail from different constraints point of view and was applied to plain concrete. In this paper that concept is extended with the introduction of crack surface modeling by introducing shear lattices and its validity is tested for real three-dimensional reinforced concrete behavior by analyzing a beam-slab-column assembly.

2. FORMULATION OF THE CONSTITUTIVE EQUATION

In the formulation for constitutive laws of concrete in *LECM*, reinforced concrete is assumed as cracked and composed of steel and concrete lattices. The stresses and direction of the steel lattices are those of reinforcing steel itself, while crack directions in concrete, which in turn fixed by principal stresses, determine concrete lattices and their orientation. Although formulation is based on cracked reinforced concrete, it can easily be extended to un-cracked reinforced and plain concrete as well.

We begin assuming uniformly strained 3D continuum as shown in **Fig.1(a)**, strain of which read in the global coordinate,

$$\{e_g\}=[e_x \quad e_y \quad e_z \quad g_{xy} \quad g_{yz} \quad g_{zx}]^T \tag{1}$$

The strain in a lattice member in this strain field is assumed to be identical with the strain of Eqn.(1). Therefore, for an inclined member of lattice for which local coordinate (x, h, z) is taken such that x coordinate coincide with lattice axis, lattice strain e_x reads,

$$\{e_x\}=[l^2 \quad m^2 \quad n^2 \quad l\,m \quad m\,n \quad n\,l\,]\{e_g\} \tag{2}$$

Where l, m and n are directions cosines of the lattice direction with respect to x, y and z axis as shown in

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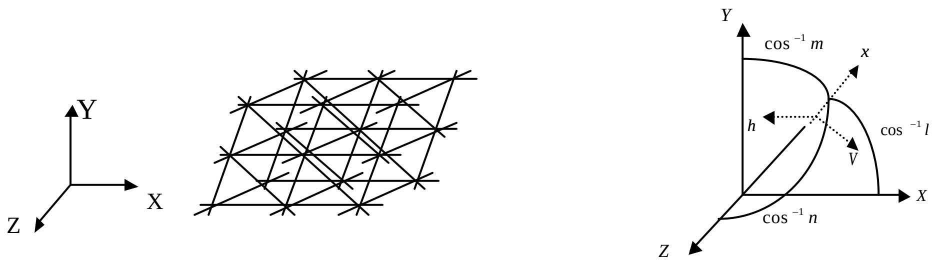


(a) Uniform Lattice (b) Local Coordinate in a Lattice Unit

Fig.1 Lattice in Uniform Strain Field

Fig.1(b). If *n* number of lattices exists in the continuum we can write,

$$\{e_l\} = [L_e]\{e_g\} \tag{3}$$

where, $[L_e]$ is the matrix to transform stress from global to local coordinate considering '*n*' number of lattices. Multiplying strains with stiffness of each lattice, the incremental stresses of the replaced continuum can be evaluated as,

$$\{\Delta s_l\} = \begin{Bmatrix} \Delta s_{l1} \\ \vdots \\ \Delta s_{li} \\ \vdots \\ \Delta s_{\ln} \end{Bmatrix} = \begin{bmatrix} r_1 & & & & \\ & \ddots & & 0 & \\ & & r_i & & \\ & 0 & & \ddots & \\ & & & & r_n \end{bmatrix} \begin{Bmatrix} \Delta e_{l1} \\ \vdots \\ \Delta e_{li} \\ \vdots \\ \Delta e_{\ln} \end{Bmatrix}$$

$$= [R_n]\{\Delta e_l\} \tag{4}$$

where, $r_i = \partial s_{li} / \partial e_{li}$, denotes the tangential stiffness of Individual lattices. Continuum local stresses transformed to global coordinate has the following form:

$$\{\Delta s_g\} = \begin{Bmatrix} \Delta s_x \\ \Delta s_y \\ \Delta s_z \\ \Delta t_{xy} \\ \Delta t_{yz} \\ \Delta t_{zx} \end{Bmatrix} = \begin{bmatrix} l_1^2 & \cdots & l_i^2 & \cdots & l_n^2 \\ m_1^2 & \cdots & m_i^2 & \cdots & m_n^2 \\ n_1^2 & \cdots & n_i^2 & \cdots & n_n^2 \\ l_1 m_1 & \cdots & l_i m_i & \cdots & l_n m_n \\ m_1 n_1 & \cdots & m_i n_i & \cdots & m_n n_n \\ n_1 l_1 & \cdots & n_i l_i & \cdots & n_n l_n \end{bmatrix} \{\Delta s_l\} \tag{5}$$

which can be written as,

$$\{s_g\} = [L_e]^T \{s_l\} = [L_e]^T [R][L_e]\{e_g\}$$

$$\text{or} \quad , \{s_g\} = [D]\{e_g\} \tag{6}$$

where,

$$[D] = [L_e]^T [R][L_e] \tag{7}$$

Eq.7 is the general form of the constitutive matrix for lattice equivalent continuum based on kinematic constraint. For the implementation of the form to reinforced concrete, it is necessary to evaluate crack occurrence. Once the crack is detected, continuum constitutive equation is transformed into the form of **Eq.7**. In this study, failure surface proposed by Kang and Willam [3] is used as the criteria for crack occurrence. The curvilinear failure envelope, according to their proposed formulation, is a function of the three stress invariants I_1,J_2 and J_3 and is expressed as:

$$F(x, r, q)_{fail} = \frac{r r(q,e)}{f_c^{'}} - \frac{r_1}{f_c^{'}}\left[\frac{x - x_0}{x_1 - x_0}\right]^a = 0 \qquad (8)$$

In this work, eventual failure stress of any principal stress or lattice direction is taken as the one, which is achieved by decreasing (for tension, increasing) it up to failure surface (i.e. **Eq. 8**) while keeping other two stresses fixed. If (s_1, s_2, s_3) represent stress state of the current loading step in numerical computation. (s_1, s_2, s_{p3}) which lies in the failure surface, is achieved by changing stress parallel to s_3 direction. In the following step of numerical computation, s_{p3} represent the failure stress to compare with for cracking in that direction. Same procedure is applied for evaluating eventual peak stresses in other two directions. Upon detection of cracking, continuum equation transfers from elastic to lattice equivalent continuum with **Eq. 7** being the governing equation.

3. MODELING CRACK SURFACE IN TERMS OF SHEAR LATTICE

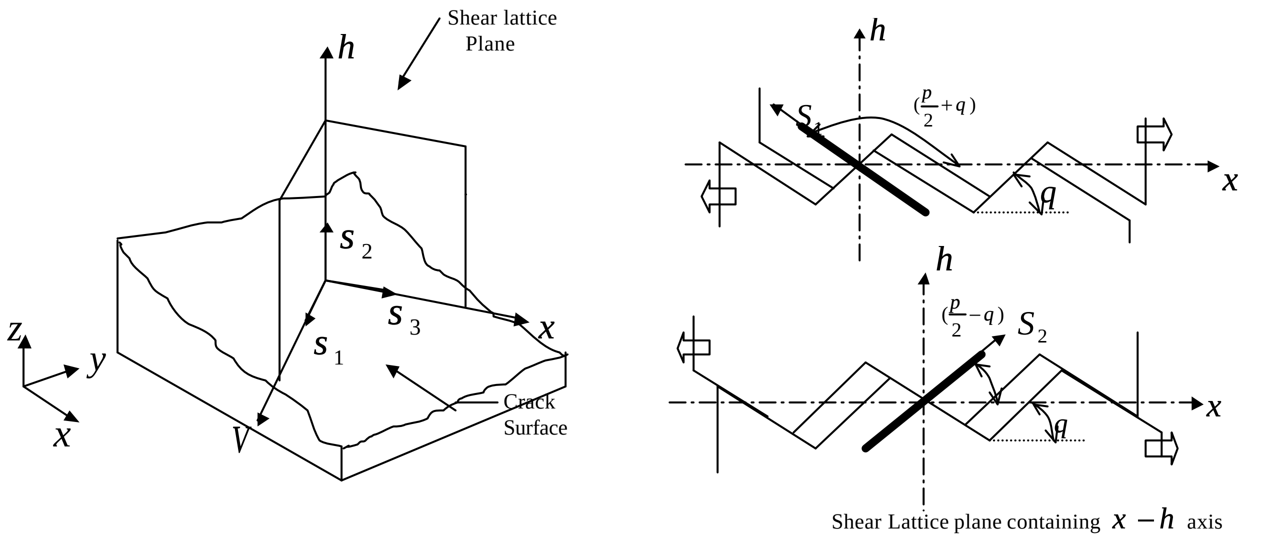


Fig.2 Crack Surface and Shear Lattice Modeling

The 3D formulation developed herein decomposes the incremental strain $\{\Delta e\}$ into the uncracked or elastic strains $\{\Delta e_e\}$ and the crack strain $\{\Delta e_{cr}\}$. Four lattices are introduced for one cracked plane surface at a sampling point to transfer the cracked portion of the total strain in the concept of interlocking between adjacent faces. However, due to the limited volumetric characteristics in interlocking region in a cracked face, a modification factor, in the name of shear controlling matrix $[\Omega]$ is introduced in the system. Therefore, we get, from crack strain,

$$\{\Delta e_{cr}\} = [\Omega]\{\Delta e_x \quad \Delta(e_h - e_e) \quad \Delta e_V \quad \Delta g_{xh} \quad \Delta g_{hV} \quad \Delta g_{Vx}\}^T = \{0 \quad \Delta e_{hcr} \quad 0 \quad \Delta g_{xh} \quad \Delta g_{hV} \quad 0\}^T \quad (9)$$

Two lattices are introduced in each of the local coordinate direction parallel to the cracked plane as shown in **Fig. 2**. Among these two, the lattice that is activated to bear the stresses depends on the negative or positive traction direction as shown in Fig 2. Now, incremental stresses in the shear lattice direction of *S1* and *S2* (in plane $x-h$) and *S3* and *S4* (in plane $h-V$, not shown in the figure) can be evaluated using following relation:

$$\{\Delta s_{sl}\} = \begin{bmatrix} E_{S1} & 0 & 0 & 0 \\ 0 & E_{S2} & 0 & 0 \\ 0 & 0 & E_{S3} & 0 \\ 0 & 0 & 0 & E_{s4} \end{bmatrix} [T_{e,S1,S2,S3,S4}]\{\Delta e_{cr}\} = [D_{shear,uni}][T_{e,S1,S2,S3,S4}]\{\Delta e_{cr}\} \qquad (10)$$

where, $[T_{e,S1,S2,S3,S4}]$ is the matrix to transform incremental strain from local direction to shear lattice direction. Multiplying that incremental strain with tangent stiffness modulus of individual lattices, we derive the stress increment in shear lattices. Direction of shear lattice depends the angle of the protruding elements of the rough surface i.e. q , as shown in **Fig.2**. and $E_{si} = \partial s_{s1} / \partial e_{s1}$ is the tangent elastic modulus from uniaxial stress strain relation of individual shear lattices. Local stress increment can be evaluated using the following relation:

$$[s_l] = [\Omega][T_{s,S1,S2,S3,S4}]^{-1}\{\Delta s_{sl}\} \qquad (11)$$

Therefore, we get, constitutive matrix for shear lattice to be:

$$[D_{shear}]_{XYZ} = [T_s]^{-1}[D_{shear}]_{xhz}[T_e] \qquad (12)$$

where $[T_s]$ and $[T_e]$ are transformation matrix, and

$$[D_{shear}]_{xhV} = [\Omega][T_{s,S1,S2,S3,S4}]^{-1}[D_{shear,uni}][T_{e,S1,S2,S3,S4}][\Omega] \qquad (13)$$

Therefore, the completed constitutive law for single crack in a sampling point,

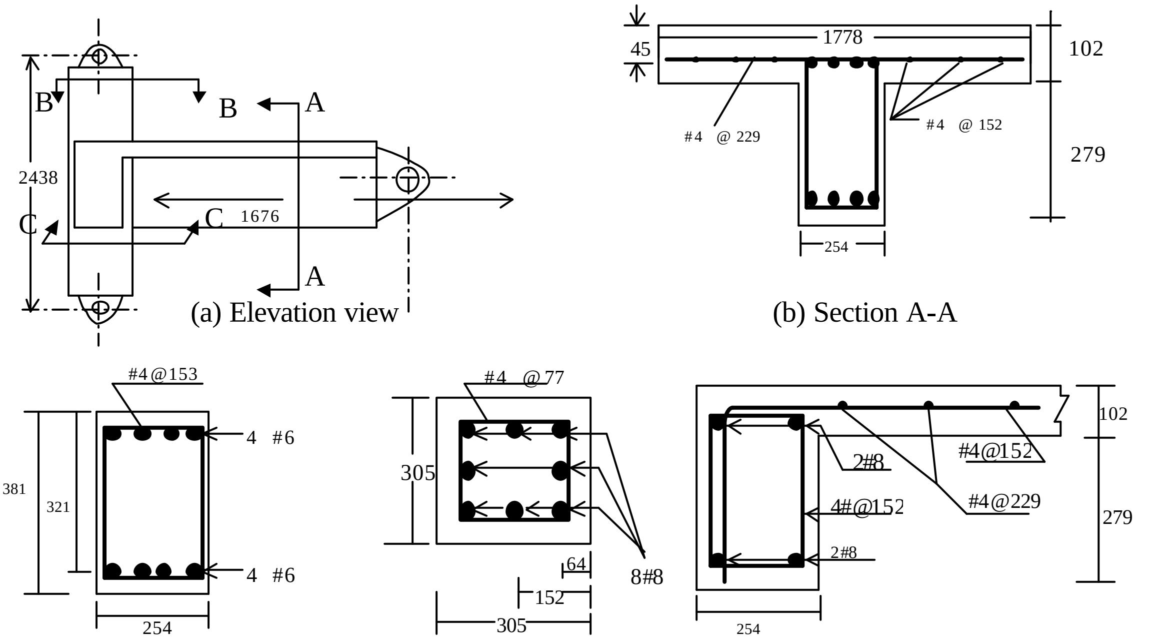
$$[D_{total}] = [D_{main}] + [T_s]^{-1}[D_{shear}]_{xhz}[T_e] \qquad (14)$$

For n number of cracks in a sampling point, we get, the final form of constitutive matrix in LECM for reinforced concrete:

$$[D_{total}] = [D_{main}] + \sum_i^n [T_s]_i^{-1}[D_{shear\,i}]_{xhz}[T_e]_i \qquad (15)$$

Eq.15 denotes the completed constitutive matrix for reinforced concrete in *LECM*.

4. NUMERICAL INVESTIGATION



(a) Elevation view (b) Section A-A

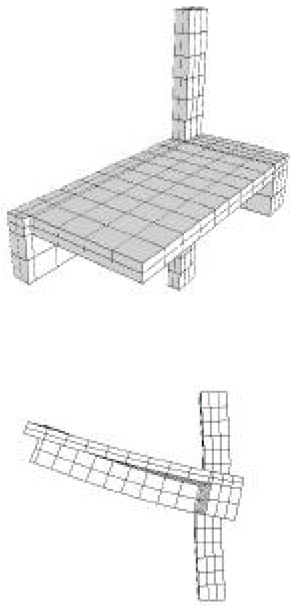
(c) Section A-A (d) Section B-B (e) Section C-C

Fig.3 Beam-Column-Slab Specimen

Table 1 : **Steel Material Properties**

Bar Size	f_{sy} (MPa)	e_{su}	f_{su} (MPa)	e_{su}
#4	531	.00256	752	.0146
#6	414	.00214	635	.0121
#8	483	.00241	717	.0124

An exterior beam-column-slab assembly [4] is analyzed using the proposed formulation. Details of the geometry and reinforcement are shown in **Fig.3**. During testing, an axial compression of 178KN was maintained on the column, and the free end of the longitudinal beam was subjected to reversed cyclic of increasing amplitude up to failure to simulate earthquake -type loading. The FE mesh used in the analysis is shown in **Fig 4** and is composed of 344 eight-noded solid elements totaling 619 nodes to model half of the test specimen. **Fig 4** also shows the upward exaggerated deflection of the pattern of the slab assembly. An extra layer of elements with artificially assigned high strength was added to the end of the longitudinal beam to facilitate the application of displacements (loading) during analyses. Restraining the vertical displacements of the column centerline nodes mechanical hinges were simulated at the top and bottom of the column. For concrete, f_c and f_t values are taken as 45.2 and 4.97 MPa, while, E and G_f values are taken as 38.3Gpa and 300 N/m, respectively. Steel material properties are shown on **Table 1**. One fourth model is proposed by Rokugo et al. is used to model tension behavior of concrete except in heavily reinforced region, where tension stiffening model is used. Two analyses were conducted considering upward and download monotonic displacement of the free end of the longitudinal beam while the column was subjected to an axial compression of 178 KN. The column load, modeled as uniformly distributed over the elements of its top end, was applied first. Then the top nodes of the protruded elements at the free end of the beam were displaced incrementally and the corresponding reactions were computed as the applied force. The result of the slab end load displacements for the upward loading (UL) and downward loading (DL) cases are compared with the envelopes of the experimental results along with the magnified deflected shape is shown in Fig.5. As can be



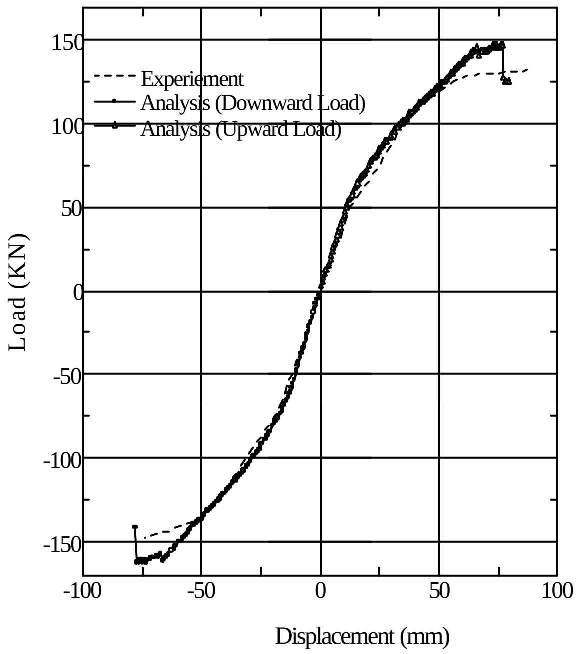


Fig.4 Finite element mesh with deflection pattern for upward load

Fig. 5 Comparison of Slab End Load vs. Disp.

seen from the figure, analyses results show good agreement with the experimental ones. For upward loading case, it can be seen that analysis result is around 10% higher than that of experimental results. This higher strength may be due degradation of concrete under the actual cyclic load which was not considered in the analysis.

5. CONCLUSION

The three dimensional constitutive laws for reinforced concrete based on *LECM* are formulated and presented in this paper. From the example shown in this paper, it can be concluded that lattice equivalent continuum method can effective predict the behavior of structures in three dimension.

REFERENCES

(1) Tanabe, Tada-aki and Ahmad, Syed Ishtiaq, “Development of Lattice Equivalent Continuum model for Analysis of Cyclic Behavior of Reinforced Concrete”, ASCE special publication on Modeling of Inelastic Behavior of RC structures under Seismic Loads, pp.297-314, 2001.

(2) Ahmad, Syed I. and Tanabe, Tada-aki, “Development of Concrete Constitutive Laws Based on 3D Lattice Equivalent Continuum” Preliminary accepted of publication, Journal of Materials, Concrete Structures and Pavements.

(3) Kang, H.D., “Triaxial Constitutive Model for Plain and Reinforced Concrete Behavior,” PhD Dissertation, University of Colorado Boulder, 1997.

(4) Zerbr,H.E., and Durrani, A.J., “Effect of slab on the behavior of exterior beam to column connections.” Rep. No. 30, Struct. Res. at Rice Univ., Houston, Tx, 1985.