RC DEEP BEAMS ANALYSIS CONSIDERING LOCALIZATION IN COMPRESSION

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ABSTRACT: It has been found that RC deep beams usually fail by the localized compressive failure of concrete. In this paper, the application of concept of localized compressive failure as the material model of concrete, based on the parameters such as the localized compressive failure length, L_p, and the compressive fracture energy, G_{Fc}, is performed. The analytical results using the lattice model and Mander's truss model considering the proposed material model show the satisfactory predictions in shear analysis of RC deep beams with and without transverse reinforcement.
KEYWORDS: RC deep beam, localized compressive failure, lattice model, Mander's truss model

1. INTRODUCTION

The RC deep beam is a RC structural member in which the ratio of the shear span to effective depth, *a/d*, is less than or equal to unity. At the failure stage of RC deep beams, along the diagonal cracks connecting the loading point and supports, crushing of concrete at the upper portion of the beams in the vicinity area of the loading point is usually observed, which is called localized compressive failure [1]. To obtain more accurate prediction of the shear behavior of RC deep beams, not only the concept of tension softening, but the concept of localized compressive failure of concrete should also be incorporated. In this study, the localized compressive failure length, L_p, and the compressive fracture energy, G_{Fc}, proposed by Lertsrisakulrat, et al. [1] have been applied to formulate the stress-strain relationship of the concrete in compression.

The analytical methods applied in this study are the lattice model and Mander's truss model. The lattice model is considered as a simplified analytical model to clarify the change in shear resisting mechanism of the RC beams [2]. Alternatively, Mander's truss model is the modified truss model with a smaller number of degrees of freedom compared with the lattice model. It can be used to assess the shear behavior of RC members by considering the interaction between the shear and flexure mechanisms [3].

In this paper, the lattice model and Mander's truss model considering the concept of localized compressive failure have been utilized to evaluate the shear test of RC deep beams with and without the transverse reinforcement carried out by Lertsrisakulrat, et al. [1].

2. LATTICE MODEL ANALYSIS

Figure 1 shows the schematic diagram of a RC deep beam which is modeled by the lattice model into an assembly of the truss components. The concrete and the reinforcements are modeled into the members as shown in **Fig. 1**. The modeling of concrete consists of flexural compression members, flexural tension members, diagonal compression members, diagonal tension members, vertical compression members and an arch member. The reinforcement is modeled into the horizontal and vertical members. The diagonal compression and the diagonal tension members have an inclination of 45° and 135°, respectively. By considering the concrete diagonal tension member, which is one of the major outstanding points of the lattice model, the shear behaviors of concrete beams before and after the initiation of the diagonal cracking can be captured appropriately.

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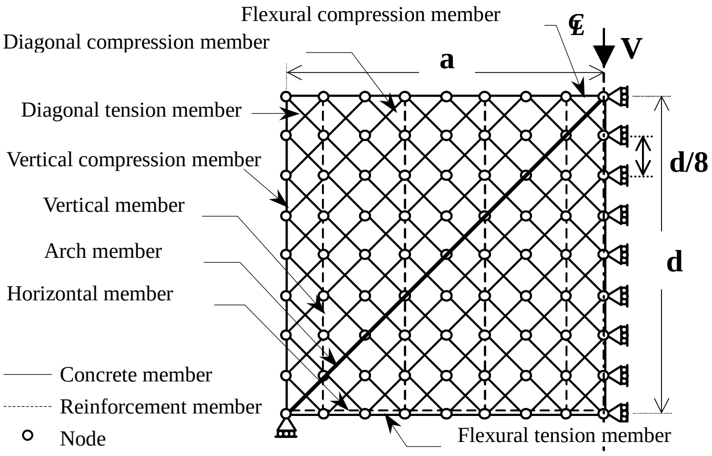

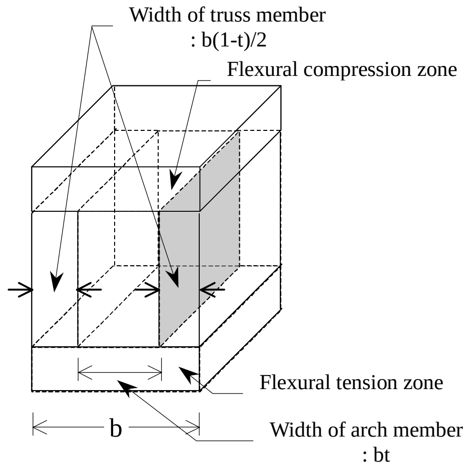

Fig. 1 Schematic diagram of RC deep beam in the lattice model (with transverse reinforcement)

Fig. 2 Cross section of a concrete beam in the lattice model

The web concrete is divided into truss member and arch member as shown in **Fig. 2**. By assuming the parameter t as the ratio of the width of the arch member to beam width, b, the width of the arch member and the truss member is equal to bt and $b(1-t)$, respectively, where $0<t<1$. The value of t is determined in such a way that it minimizes the total potential energy of the entire structure, Π. Π is calculated from the summation of the strain energy of each element and the work done by externally applied load based on the elastic analysis.

With the considerations of mesh discretization and the complexity of flow of internal stresses in the member, a RC deep beam has been suitably modeled in half of the specimen in which the horizontal spacing of each adjacent 2 nodes is equal to $d/8$ and the value of a/d is equal to 1.0 matching to the actual specimen as shown in **Fig. 1**. The arch member is connecting between the loading point and the support with the thickness assumed to be $(0.3d+r)\sin 45°$, where r is the bearing plate width.

The tension softening model named one-forth model and the tension stiffening model has been applied to the diagonal tension members and the flexural tension members, respectively. For the reinforcement members, the bilinear elasto-plastic model of steel is applied.

3. MANDER'S TRUSS MODEL ANALYSIS

In the shear mechanism analysis, a RC structural member may be considered as a structural element of combined mechanism between shear and flexure mechanisms [3]. Hence, the total deformation of the member can be expressed as:

$$\Delta = \Delta_u + \Delta_f \tag{1}$$

where Δ, Δ_u and Δ_f are total, shear and flexure deformations of a RC member, respectively.

And the shear resisting capacity should be the lesser of:

$$V_u = V_s + V_c \quad \text{and} \quad V_f = \frac{M_y}{L} \tag{2}$$

where V_u and V_f represent shear force resisted by shear and flexure mechanisms, correspondingly. V_s and V_c are the shear resistance due to the contributions of transverse reinforcement and concrete, respectively. M_y is the yielding moment of the RC member and L is the member's length. In the division of web concrete for V_s and V_c model, it was found that, similar to the lattice model, the ratio of the model width which minimizes the potential energy, gives a good prediction.

3.1 SHEAR MECHANISM

Figures 3(a) and **(b)** show the schematic diagram of one half of a RC deep beam, which is modeled by applying Gauss 2-point quadrature with the normalized coordinate parameter, x_1, to Mander's truss model, for evaluating V_s and V_c in shear mechanism, respectively [3]. In V_s model, the transverse reinforcements have been modeled perpendicularly to the beam axis. In V_c model, the inclined concrete ties have been modeled corresponding to the ineffective zones in which the effect of flexural cracking in these regions should be eliminated. The diagonal struts represent the concrete compression field stabilizing the truss model. In the traditional truss model, the shear resistance is assessed from the effects of transverse reinforcement and concrete tensile strength in which the shear resisted by the strut along the diagonal cracks is neglected. However, for RC deep beams, the arch action, which is created from the diagonal cracks and longitudinal reinforcement, becomes the significant resistance in governing the fracture mechanism after the diagonal crack occurred. Thus, for simplicity, the shear resistances by the struts in V_s model (V_{sd}) and in V_c model (V_{cd}) should be taken into the consideration.

By utilizing the virtual work method to the model, the relationships between the deformation and each shear resistance component can be derived.

The crack angle is simply proposed to be equal to $\alpha = tan^{-1}(jd/a)$ for RC deep beams without transverse reinforcement. Whereas, for RC deep beams with transverse reinforcement, the crack angle can be determined from **Eq. 3** proposed by Mander, et al. [3].

$$\theta = tan^{-1}\left(\frac{r_w n + \zeta \dfrac{r_w A_v}{p_t A_g}}{1 + r_w n}\right)^{1/4} \qquad (3)$$

where $A_g = bd$, $A_v = b \times jd$, $p_t = A_{sh}/A_g$, $n = E_s/E_c$, ζ = boundary condition constant = 1.5704, A_{sh} = cross-sectional area of the longitudinal reinforcement, and r_w = transverse reinforcement ratio.

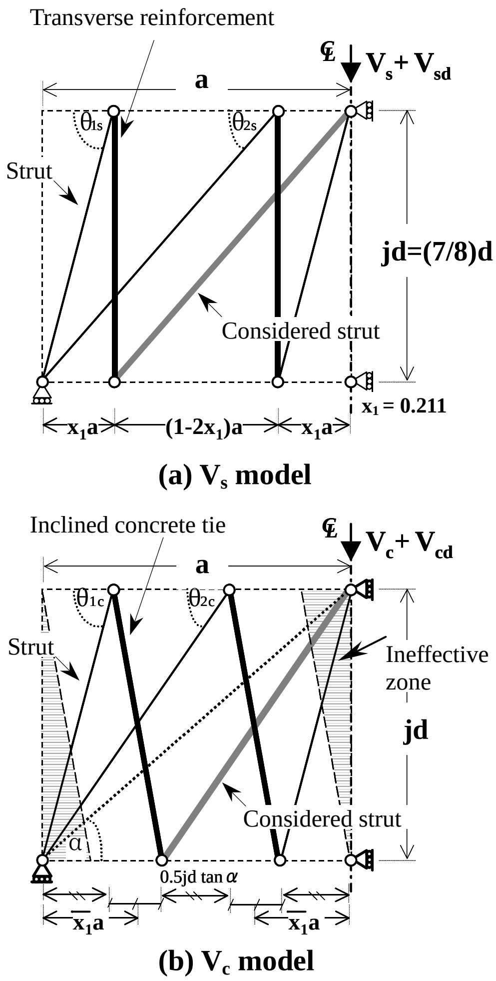

Fig. 3 Modeling of RC deep beam in Mander's truss model

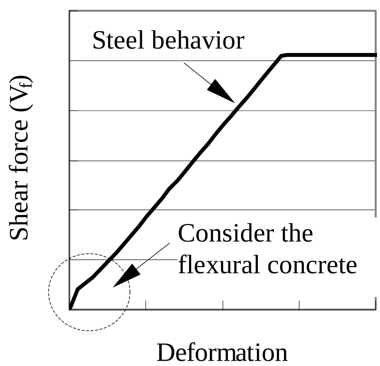

Fig. 4 Shear force – deformation relationship for V_f

3.2 FLEXURE MECHANISM

In order to evaluate the relationship between the shear force due to the flexure mechanism and the deformation, for simplicity, the following way of thinking was applied. As shown in **Fig. 4**, before the onset of crack, the rigidity due to the flexural concrete should be considered. With the increase in moment, the flexural rigidity of the section is reducing by cracking of concrete. The behavior of the section after crack is dependent mainly on the reinforcement content.

4. CONCEPT OF LOCALIZED COMPRESSIVE FAILURE OF CONCRETE

4.1 APPLICATION OF THE CONCEPT

In localized compressive failure of concrete, the localized compressive fracture length, L_p, can be determined by **Eq. 4** [1].

$$\frac{L_p}{D^*} = \begin{cases} 1.36 & ; D^* < 100 \\ -3.53\times10^{-5}D^* + 1.71 & ; 100 \le D^* \le 180 \\ 0.57 & ; D^* > 180 \end{cases} \quad (4)$$

where $D^* = \sqrt{A_c}$: the equivalent cross-sectional width (mm) and A_c is the cross-sectional area of the concrete member (mm^2). The localized compressive failure volume, V_p, can be derived by $L_p \times A_c$.

The compressive fracture energy, G_{Fc}, is defined as the energy required to cause compressive failure per unit volume of failure concrete. According to Lertsrisakulrat, et al. [1], G_{Fc} obtained from the RC deep beam tests is equivalent to G_{Fc} from the uniaxial compressive tests and can be calculated from the empirical equation in terms of f_c' as **Eq. 5**.

$$G_{Fc} = 0.086\, f_c'^{1/4} \quad \left(N/mm^2\right) \quad (5)$$

At this juncture, by assuming V_p the energy consumed by the failure portion, E_{net}, which is equivalent to the area under the load-deformation curve (P–Δ), can be derived by multiplying G_{Fc} with V_p. To apply this concept to the material model of concrete in compression, E_{net} should be transformed to energy per unit volume, e_{net}, which is equal to the area under the compressive stress-strain relationship (σ-ε) as expressed in **Eq. 6**. Here, it is noteworthy that the empirical factor K is introduced in order to take into account the transverse reinforcements and the effect of the energy consumed by the friction and assumed to be 1.14 and 1.82 for beam without and with transverse reinforcement, respectively.

$$e_{net} = K\int \sigma d\varepsilon = K\int_0^{L_c} \frac{P}{A_c}\frac{d\Delta}{L_c} = K\frac{E_{net}}{V_c} = K\frac{G_{Fc}V_P}{V_c} = KG_{Fc}\frac{L_p}{L_c} \quad (6)$$

4.2 FORMULATION OF MATERIAL MODEL

From the obtained parameters, the stress-strain relationship for compression members can be proposed as shown in **Fig. 5.** The formula of the material model based on concept of localized compressive failure of concrete has been simply proposed as summarized in **Eq. 7** [4]. For the ascending branch (pre-peak), the relationship proposed by Vecchio, et al. [5] has been applied as **Eq. 7a** and the consumed energy of this part is set to be e_1 (**Eq. 8a**). For the descending branch (post-peak), the bilinear model has been proposed. By subtracting e_{net} by e_1, the consumed energy in the post-peak region, e_2, is obtained (**Eq. 8b**). By applying the empirical factor m (**Eq. 8c**), the ε'_{last} can be derived (**Eq. 8d**). *Line A* can be obtained as **Eq. 7b**. Assuming the slope of *Line B* to be equal to $-E_c/100$, *Line B* can be derived as **Eq. 7c**. Finally the stress-strain relationship for compression members can be proposed.

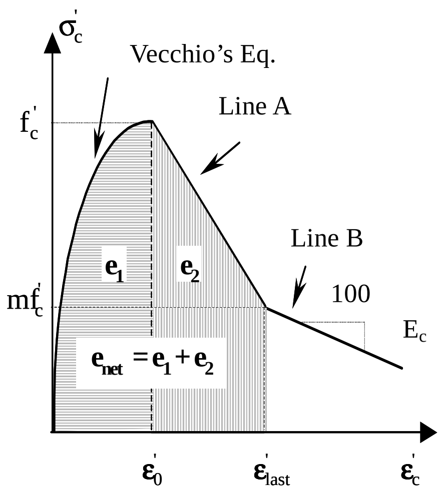

Fig. 5 Proposed stress-strain curve for concrete in compression

Vecchio's Equation,

$$\sigma'_c = f'_c\left[2\left(\frac{\varepsilon'_c}{\varepsilon'_0}\right) - \left(\frac{\varepsilon'_c}{\varepsilon'_0}\right)^2\right] \quad ; \quad \varepsilon'_c < \varepsilon'_0 \quad (7a)$$

Line A;

$$\sigma_c^{'} = A_1\varepsilon_c^{'} + A_2 \quad ; \quad \varepsilon_0^{'} < \varepsilon_c^{'} < \varepsilon_{last}^{'} \quad : \quad A_1 = \frac{\left(m^2-1\right)f_c^{'2}}{2e_2} \quad ; \quad A_2 = f_c^{'} - \frac{\left(m^2-1\right)f_c^{'2}}{2e_2}\varepsilon_0^{'} \quad \text{(7b)}$$

Line B;

$$\sigma_c^{'} = B_1\varepsilon_c^{'} + B_2 \quad ; \quad \varepsilon_{last}^{'} < \varepsilon_c^{'} \quad : \quad B_1 = -\frac{E_c}{100} \quad ; \quad B_2 = mf_c^{'} + \frac{E_c}{100}\varepsilon_{last}^{'} \quad \text{(7c)}$$

The important factors in the calculations are recapitulated in **Eq. 8** as

$$e_1 = \int_0^{\varepsilon_0^{'}} [\,Eq.(7a)\,]\, d\varepsilon_c^{'} = \frac{2}{3}\varepsilon_c^{'} f_c^{'} \quad \text{(8a)} \qquad e_2 = K\frac{G_{Fc}L_p}{L_c} - \frac{2}{3}\varepsilon_c^{'} f_c^{'} \quad \text{(8b)}$$

$$m = -14\left(\frac{1200}{d}-1\right)^2 r_w^{\,2} + 30\left(\frac{1200}{d}-1\right)r_w + 30 \quad (\%) \quad \text{(8c)} \qquad \varepsilon_{last}^{'} = \varepsilon_0^{'} + \frac{2e_2}{1+mf_c^{'}} \quad \text{(8d)}$$

5. ANALYTICAL RESULTS AND DISCUSSION

The experimental data of 6 RC deep beams, tested by Lertsrisakulrat, et al. [1] are adopted and compared with the analytical results using the lattice model and Mander's truss model as tabulated in **Table 1**. It is noted that all cases of the specimens failed in the shear compressive mode.

Figures 6 and **7** show the comparisons between the experimental and the analytical results in cases where the effective depths are, respectively, 400 and 600 mm with r_w of 0%, 0.42% and 0.84%. The solid circles represent the experimental results (*Exp.*). For the lattice model analysis, the black thin lines represent the analytical results when the original equation proposed by Vecchio (*Vecchio*) has been incorporated to the compression member. While the black bold lines represent the results incorporating the proposed material model (*Lattice*). The gray bold lines represent the results using Mander's truss model applying the proposed material model (*Mander*).

In the lattice model analysis, it becomes apparent that analytical results in pre-peak region give the perfect predictions in most cases. For the post-peak region, the results applying the proposed material model show the better predictions of load-deformation relationship compared with ones in which Vecchio's equation was used.

Similarly, by incorporating the concept of localized compressive failure to Mander's truss model analysis, the analytical results show the acceptable tendency to the experimental results. However, the analytical results show somewhat difference. Since the flow of internal stresses in RC deep beams is comparatively complicated, it is difficult for the simplified model with a small number of members such as Mander's truss model to predict the shear behavior accurately as the lattice model. Nevertheless, it is shown that Mander's truss model can be used to evaluate the shear resisting capacity and its deformation in an acceptable degree.

Table 1 Outline of the experimental data carried out by Lertsrisakulrat, et al. [1]

Specimen	d (mm)	Beam height (mm)	r_w (%)	f_c' (MPa)	r (mm)	Longitudinal reinforcement	f_y (MPa)	Transverse reinforcement	f_{wy} (MPa)
D400	400	450	0.00	35.5	100	PC-ϕ 25	1004	D6	331
D404			0.42	27.5					
D408			0.84	38.4					
D600	600	650	0.00	40.8	150	PC-ϕ32	1006		
D604			0.42	34.2					
D608			0.84	35.3					

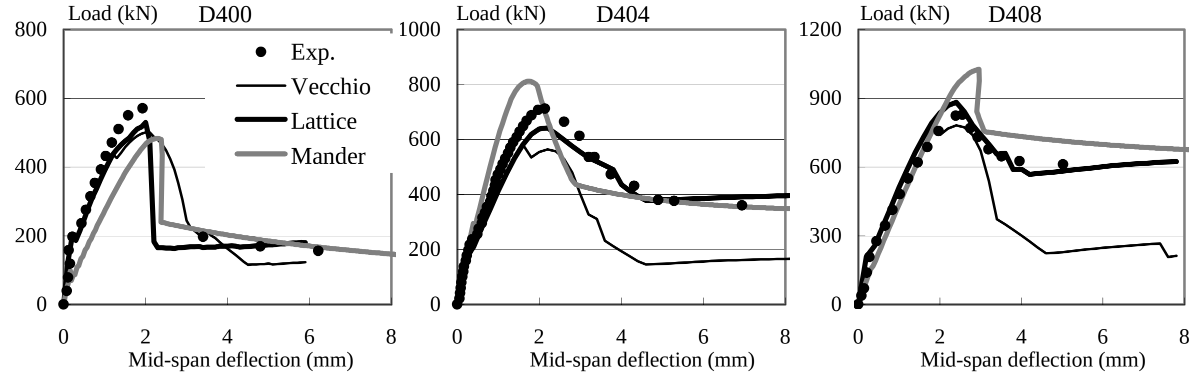

Fig. 6 Load-midspan deflection (d = 400 mm)

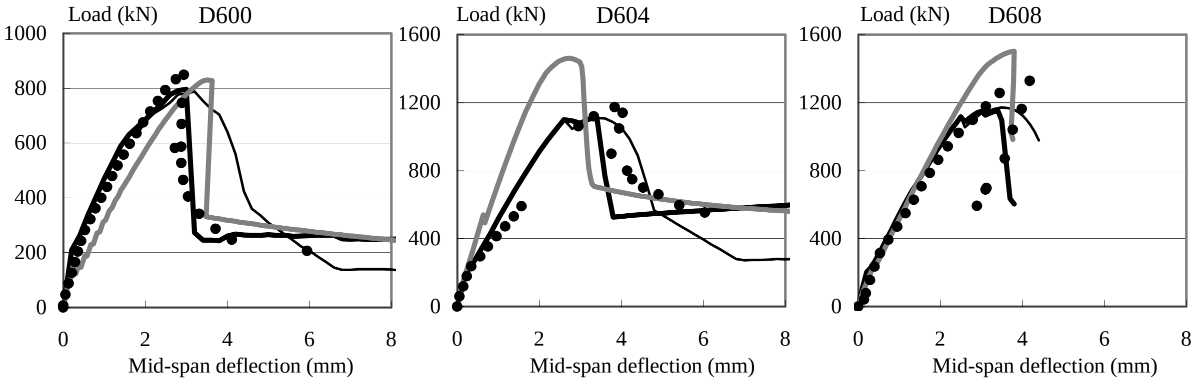

Fig. 7 Load-midspan deflection (d = 600 mm)

6. CONCLUSIONS

(1) The stress-strain relationship of concrete in compression considering the concept of localized compressive failure of concrete has been proposed based on the localized compressive failure length, L_p, and the compressive fracture energy, G_{Fc}.

(2) For RC deep beams with and without transverse reinforcement failed by the localized compressive failure of concrete, the lattice model incorporating the proposed material model provides the high accurate prediction of shear behavior until the ultimate stage.

(3) By comparing with the lattice model, Mander's truss model is simpler but yields an acceptable prediction on the load-deformation relationship, however, with a lower accuracy.

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