ANALYSIS OF BOX TYPE SHELL STRUCTURES UNDER CYCLIC LOADING BY LATTICE EQUIVALENT CONTINUUM MODEL

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ABSTRACT: The application of lattice equivalent continuum model in nonlinear FEM is presented here for the analysis of box shell structure under cyclic loading. The box type multilayer RC shell structure is formulated using the system of lattices. In this model, shear interlocking between cracked elements may also be simulated. In addition, each element in each layer can be governed by its own hysteretic rule, and constitutive equations are capable for loading and unloading path so that behavior of reinforced concrete can be satisfactorily predicted. Lattice equivalent continuum model is characterized by numerous numerical calculations.
KEYWORDS: reinforced concrete shell, constitutive equation, lattice equivalent continuum model, cyclic loading, finite element method

1. INTRODUCTION

The lattice equivalent continuum model renders constitutive equations for cracked reinforced concrete expressing cracked segments and reinforcement with lattices and those lattices then being replaced to equivalent continuum. The system of lattice equivalent continuum elements is used for simulating complex behavior of reinforced concrete structures such as walls, beams and columns in practice. The model has high possibility to be broadly applied to many other kinds of structures. The applications of lattice equivalent continuum model in the past three years to the basic element of concrete structures, for example, beams and columns were found very successful so far. Due to the flexible application of the lattice idea, the more complicated structural element such as plate and shell are investigated in the paper. The comparison of analysis and experimental results for a box type shell structure will be presented featuring basic characteristics of shear stiffness of cracked reinforced concrete shell element.

In a shell structure where multi-directional cracking is expected, rational modeling for shear rigidity affected by multi-directional cracking is exceedingly important. The paper treats the modeling of shear transfer mechanism at crack using the shear lattices. The fundamental concept for shear transfer at cracked section is as follows;

(1) Shear transfer is essential due to the aggregate inter-locking between two opposite surfaces of a crack.
(2) Aggregate inter-locking can be replaced with lattices that situate at interlocking point of the two surfaces defining the angle of stress transfer at the point.
(3) The shear lattices can also be defined as many as a number of crack orientations.

In this paper, analysis of shell structures using FEAP (a general purpose finite element analysis program which is designed for research and educational use written by R.L.Taylor) is presented. The constitutive equation in this finite element program is totally replaced using lattice equivalent model.

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Due to the sensitivity of thickness and its properties of shell structure, the system is adopted where multilayer reinforced concrete shell and prestressed concrete shell structures, which has different material properties as well as reinforcement alignment in each layer, are formulated. For cyclic loading, each layer and each material in each layer can have its own hysteretic rule. Finally, behavior of a box shell subjected to cyclic loading is examined analytically and experimentally.

2. SHELL FORMULATION

The shell structure that is analyzed in this paper is formulated in 3D space and is assembled by a number of flat 2D elements. The element type that is composing shell structure is quadrilateral isoparametric shell element with 6 degree of freedom (DOF) per each node. Those DOFs are translation and rotation at mid-surface in orthogonal x, y, and z direction, respectively, as shown in **Fig.1**.

When all elements meeting at a node are co-planar, difficulty will arise due to drilling DOF (rotational DOF about axis that perpendicular to element plane, q_z) which can causes distortion of elements on surface of the structure. This problem occurs because at the node that elements lying on the same plane are connecting to, is assigned by zero stiffness in the direction of q_z. The correction of the drilling DOF to elemental shape function is necessary [1].

Due to the requirement for continuity in the first derivative of translational displacements, C_1 continuity element type is necessary for element formulation to ensure that the element remains continuous all over the structure. In the integration for calculating stiffness matrix and internal forces on each element, two Gaussian quadrature points per each direction are necessary. In this study, 5-layers shell element with four Gaussian quadrature points in each layer is utilized. Each layer of 6-DOFs multi-layered shell element is formulated by using 3-DOFs discrete Kirchhoff quadrilateral plate bending element (2 translational and 1 rotational DOFs on local XY plane) with deep shell curvature corrections. This correction is necessary in order to compensate the out-of-plane strain due to shell element.

As mentioned above, displacements at shell element level corresponds to those 6-DOFs is reduced to 3-DOFs discrete Kirchhoff plate element at layer level, simply says

$\boldsymbol{u}_{\text{shell}}=\{u \quad v \quad w \quad q_x \quad q_y \quad q_z\}^T \longrightarrow \boldsymbol{u}_{\text{plate}}=\{u \quad v \quad q_{xy}\}^T$.

For a given set of nodal displacement at node *i*, $\boldsymbol{a}_{\text{shell},i}=\{u_i \quad v_i \quad w_i \quad q_{xi} \quad q_{yi} \quad q_{zi}\}^T$, strain field following the small displacement theory with deep shell curvature corrections at each layer *l* can be expressed as follows.

$$e = \begin{Bmatrix} e_x \\ e_y \\ g_{xy} \end{Bmatrix} = \begin{Bmatrix} e_{01} + h(l)\cdot e_{04} \\ e_{02} + h(l)\cdot e_{05} \\ e_{03} + h(l)\cdot e_{06} \end{Bmatrix} \tag{1}$$

where

$$e_{01} = \sum_{i=1}^{4} \frac{\partial N_i}{\partial x}\cdot u_i - shpx_{1i}\cdot q_{zi}$$

$$e_{02} = \sum_{i=1}^{4} \frac{\partial N_i}{\partial y}\cdot v_i - shpy_{2i}\cdot q_{zi}$$

$$e_{03} = \sum_{i=1}^{4} \frac{\partial N_i}{\partial x} \cdot v_i + \frac{\partial N_i}{\partial y} \cdot u_i - \left(shpx_{2i} + shpy_{1i} \right) \cdot q_{zi}$$

shpx, shpy = drilling DOFs correction factor in local x, y direction, respectively

$$N_i = \frac{1}{4}(1 + x \cdot x_i)(1 + y \cdot y_i)$$

x_i, y_i = normalized local coordinate at node *i* in

$$h(l) = \frac{t}{2} \cdot sgt(l)$$

t = thickness of shell element

$sgt(l)$ = Abscissae quadrature at depth level *l* $= \left\{ -1 \quad -\sqrt{\frac{3}{7}} \quad 0 \quad \sqrt{\frac{3}{7}} \quad 1 \right\}^T$

e_{04}, e_{05}, e_{05} = deep shell curvature correction factors (tensor product between modified shape function and nodal displacements of all nodes in an element, $\boldsymbol{a}_{\text{shell}}$), which depend on geometry of shell element

In brief, the geometry and displacement components can be shown in the following figure.

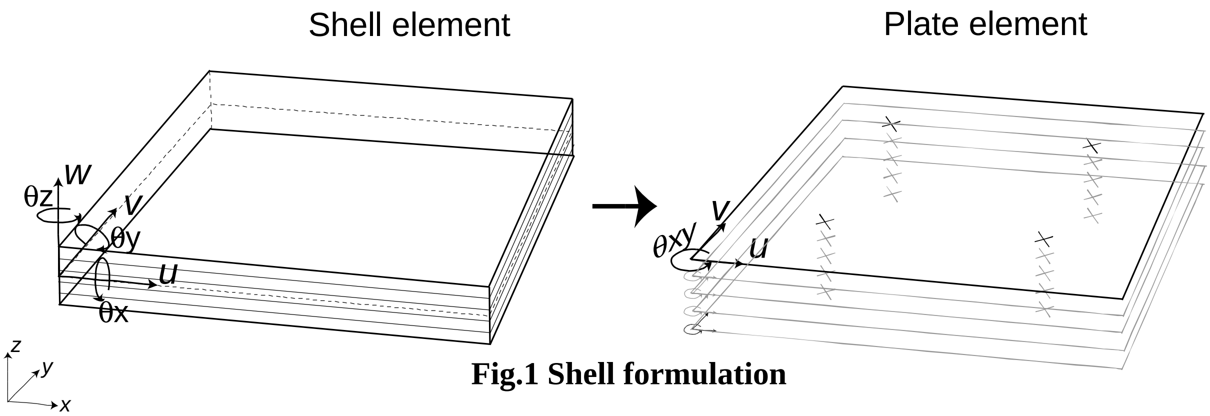


Fig.1 Shell formulation

The effect of reinforcements enclosed by concrete was considered by the assumption that reinforcing bars are uniformly distributed throughout an element in each direction, which leads to the concept of smeared crack, so that the stiffness of cracked element can be expressed by the summation of stiffness of reinforcements and concrete.

Once strains at each Gaussian point in each layer were obtained by the formulation described above, stress field shall be calculated according to the concept of lattice equivalent continuum which will be mentioned in the next section.

3. CONSTITUTIVE EQUATIONS

As stated in the first section, stiffness of any element is replaced by systems of lattices in which its behavior can be expressed by uniaxial properties of concrete and reinforcement. By using of uniaxial properties of concrete and reinforcement, even complicated hysteretic rule (loading, unloading and reloading) can be assigned independently to each material. As shown in **Fig.2**, cracked concrete is replaced by system of concrete lattices where a_i is concrete lattice direction of lattice *i*, in which parallel to the direction of crack computed by principle stress theory. Similarly, reinforcements are also replaced by the system of lattices that lie in the same direction as alignment of reinforcement. Summation of concrete and reinforcement lattice systems yields representation of RC element, so called main lattice system.

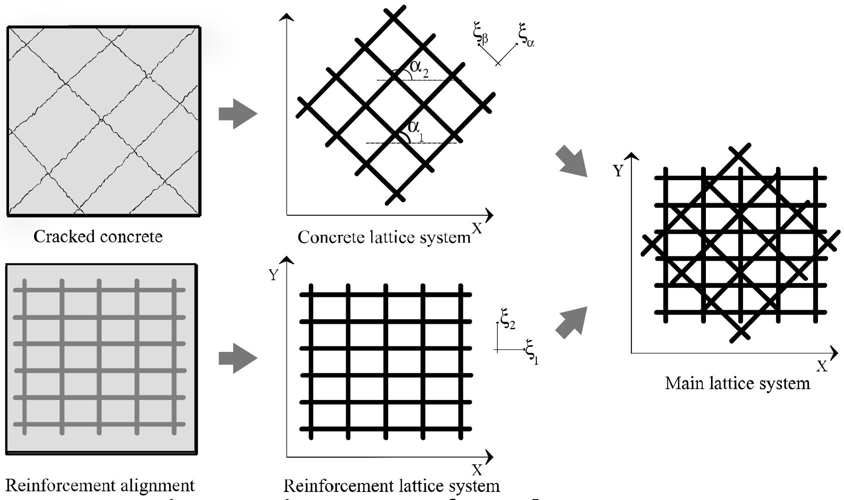


Fig.2 Lattice system of RC element

Prior to cracking of concrete, element will behaves elastically with the stiffness of the summation of both concrete and reinforcement. The following equation shows the stiffness matrix of reinforced concrete element before cracking.

$$[D]_{uncrack} = \frac{E_c}{1-n^2}\begin{bmatrix} 1 & n & 0 \\ n & 1 & 0 \\ 0 & 0 & \frac{1-n}{2} \end{bmatrix} + \begin{bmatrix} r_x E_{sx} & 0 & 0 \\ 0 & r_y E_{sy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (2)$$

Where E_c is Young's modulus of concrete. E_{sx} and E_{sy} are Young's moduli of reinforcement in x and y direction, respectively. n is Poisson ratio of concrete, r_x and r_y are reinforcement ratios in x and y direction.

Once crack occurred, each concrete and reinforcement lattice will be controlled by its uniaxial constitutive equations. At any load step, incremental strain, Δe = $\Delta\{e_x \quad e_y \quad g_{xy}\}^T$ will be transformed to normal strain in any lattice direction, a_i, by transformation matrix $[L_e]_i$ = $\{\cos^2 a_i \quad \sin^2 a_i \quad \cos a_i \sin a_i\}$. Consequently, $\Delta e_{li} = [L_e]_i \Delta e$, where subscript *l* indicates local lattice coordinate system (ξ-η coordinate). Normal strain in each element can be written as a summation of Δe_{li} in each lattice direction *i*, as $\Delta e_l = [L_e]\Delta e$, where

$$[L_e] = \begin{Bmatrix} \cos^2 a_1 & \sin^2 a_1 & \cos a_1 \sin a_1 \\ & \vdots & \\ \cos^2 a_n & \sin^2 a_n & \cos a_n \sin a_n \end{Bmatrix},$$

n is total number of lattices in any element

Normal stress vector corresponding to Δe_l can be calculated by the uniaxial stress-strain relationship for each lattice, Δs_{li} =

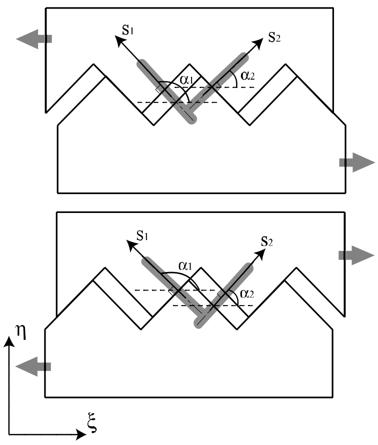


Fig.3 Shear-interlock

$E_i t_i \cdot \Delta e_{li}$. Where $E_i = \dfrac{\partial s_{li}}{\partial e_{li}}$ (lattice stiffness of either concrete or reinforcement), t_i is thickness ratio of any lattice i ($\sum_{i=1}^{n} t_i = 1.0$). Stress components in local element coordinate system (x-y coordinate), Δs will be converted back by process similar to case of strain components. Finally relation between Δe and Δs can be expressed as $\Delta s = [D]\Delta e$.

In this model, shear transfer between crack surfaces of cracked element might be able to simulate using different system of lattice, so called shear lattice system. At crack surface, shear lattice will be inserted perpendicular to crack direction, as shown in **Fig.3**.

Shear interlocking can be divided into 2 modes as shown in the figure. Axis ξ and η are corresponding to crack surface of cracked concrete. New local coordinate has to be established in the direction of shear lattice, namely S_1, S_2. Transformation of strain twice from x-y system to ξ-η system and then to S_1-S_2 system are performed. To limit only the strain components that affect shear interlock at crack surface, matrix $[\Omega]$ will be multiplied to Δe_l. Where $[\Omega]$ is 3x3 matrix whose components are equal to 1.0 at (2,2) and (3,3) , and equal to 0 at elsewhere. By the process similar to that of the main lattice system, finally shear stiffness matrix can be expressed as $[D_{shear}] = [T_1][\Omega][D_{shear,uni}][\Omega]^T[T_2]$, where $[T_1]$, $[T_2]$ are transformation matrices, $[D_{shear,uni}]$ is stiffness matrix of shear lattices in S_1-S_2 system [2]. After all, total stiffness matrix is a summation between $[D]$ and $[D_{shear,uni}]$. The material models used in this study are nonlinear uniaxial and are briefly shown in the following figure.

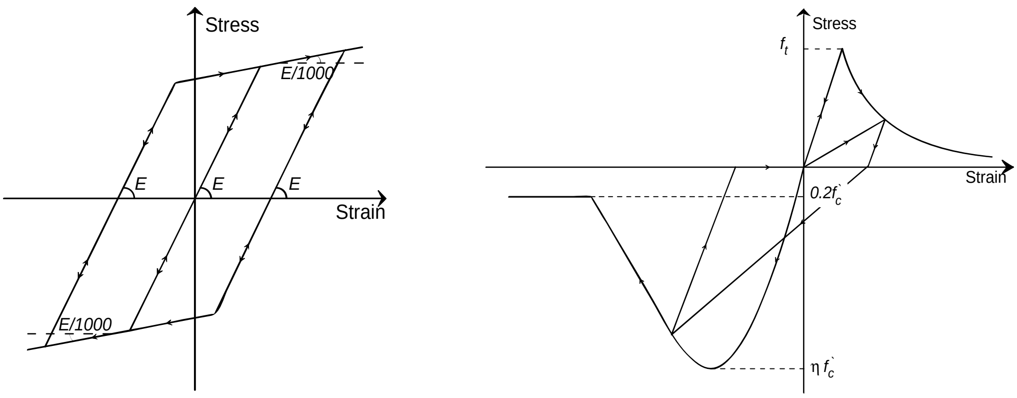
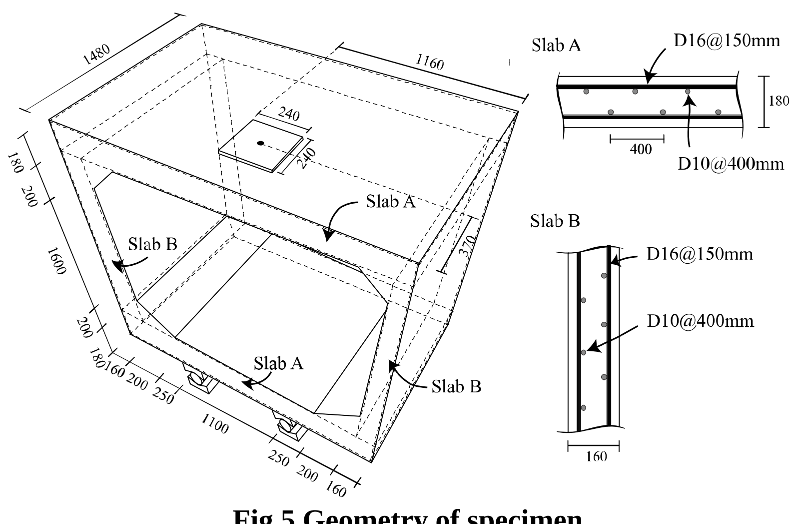


Fig.4 Material model (left: reinforcement, right: concrete)

4. EXPERIMENT

A box type RC shell from University of Tokyo [3] was selected for verification to the lattice model. Specimen geometry, loading position, reinforcement data and discretized FEM model details were shown in the **Fig.5**, **Fig 6**, with a unit of millimeters. Cyclic load in one direction was applied at center of steel



Unit: mm

Fig.5 Geometry of specimen

plate with a size of 24x24cm attached at the top slab of specimen. Both sides of wall and top/bottom slab have different thickness and reinforcement ratio, therefore it is divided in two types. Haunches at corner of box shell are discretized as plain concrete shell element. Since this specimen is in symmetry, only half of main structure was used in FEM. Compressive strength of concrete is 50 MPa and yield stress of reinforcement in both directions is 400MPa.

Fig.7 shows the load-displacement relation at the load point in the direction of loading (upper-left), in which satisfactory agreement can be obtained. Load-displacements at the other points (a, b, and c) are also obtained and shows in **Fig.7** (upper-right, lower-left and lower-right, respectively). But at the other points, analysis results seem to be stiffer than those of experiments.

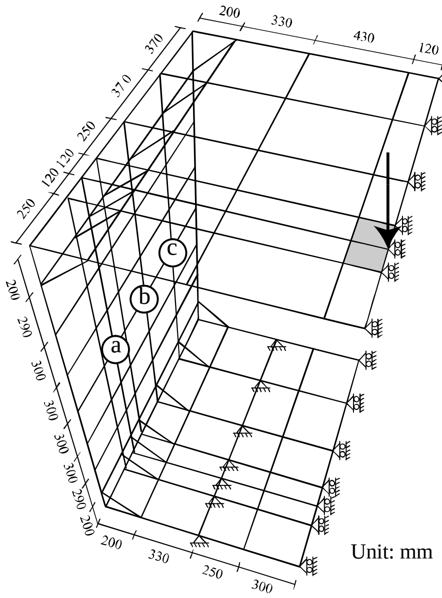


Fig.6 FEM model

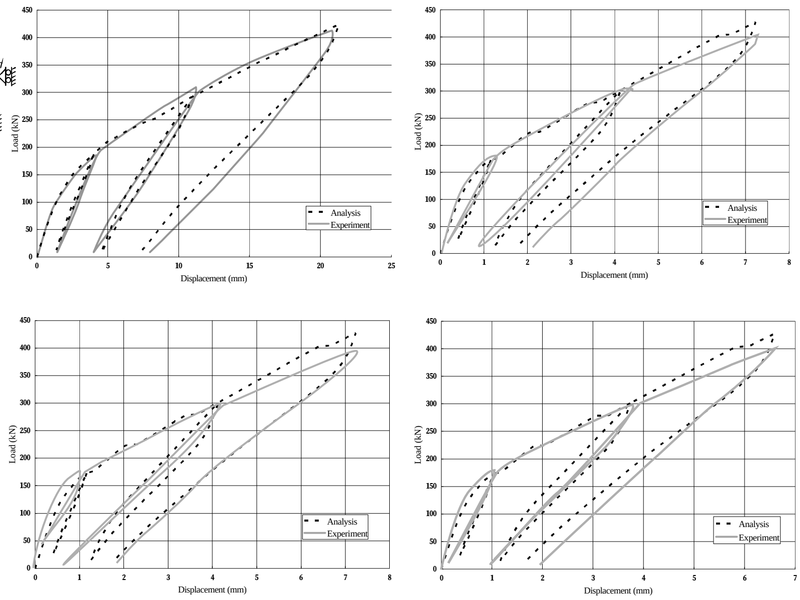


Fig.7 Load-displacement diagram

5. CONCLUSIONS

The box type RC shell structure in this study was modeled by using 6-DOFs shell elements in which each layer is represented by 3-DOFs plate elements. In addition, each element in each layer is governed by its own hysteretic rule in which the complicated behavior of whole structure can be predicted. By properly defining shear transfer rigidity at multi-directional cracking the behavior of reinforced concrete shell structures under cyclic loading in both loading and unloading can be simulated with good agreement to the experimental results using the concept of lattice equivalent continuum model.

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