論文

SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS WITH A SMALL AMOUNT OF WEB REINFORCEMENT

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ABSTRACT: The shear strength of reinforced concrete beams with a small amount of web reinforcement, which are 0.035%, 0.05%, 0.065%, and 0.08%, was tested. Estimated shear strength calculated by summing up the shear strength of concrete beams without web reinforcement (V_c) and web reinforcement (V_s) was compared to the tested shear strength (V_u). To explain the applicable of superposition method, clarify aggregate interlocking from the shear crack sliding and opening was necessary. As a result, the aggregate interlocking force is 80% of the shear crack load (V_{cr}). In conclusion, the superposition method (V_c+V_s) is applicable for estimating the shear strength of beams with a small amount of web reinforcement.
KEYWORDS: shear strength, superposition, beams, web reinforcement, aggregate interlocking

1. INTRODUCTION

In general infrastructures, the amount of web reinforcement, comparing to the cross-sectional area of members, is very small. One example is old Shinkansen viaduct structures, which was designed in 1970 and consisted of many large reinforced concrete columns. These columns were lightly reinforced with web reinforcement ratio about 0.08% as reported by JSCE [1]. According to available shear design methods in that time, the allowable shear strength of the concrete was always added with the shear resistance contributed by the web reinforcement, normally calculated by the truss mechanism, in order to make economic design. However, the estimated shear strength became overestimation because the allowable shear strength of concrete was not properly considered [2].

Recently, the shear strength of beams with very small web reinforcement ratio cannot be practically estimated and explained by the force resistant mechanisms. The plasticity method proposed by Nielson [3] does not concern the stress transfer across the shear crack. Vecchio and Collin [4] presented the modified compression field theory (MCFT), which includes a rationale for determining the stress transfer across the shear crack. However, Vecchio [5] reported that the MCFT showed inaccuracy results for the beams with web reinforcement ratio less than 0.10% due to the basic assumptions used in the model. Besides to the MCFT, the truss and crack friction model has recently used in the FIP design specification 1996. Nevertheless, it is also reported that the crack friction term for the beams using web reinforcement less than minimum requirement can not be reliably formulated [6]. That minimum requirement is the mechanical reinforcement ratio, which is about 0.12%. Therefore, the problem of all proposed shear strength estimation methods solely based on the stress transfer across the shear crack.

Escaping from the aforementioned problems, it can be said that the estimation method without implicitly calculating the stress transfer across the shear crack is actually simpler. For example, the superposition between the shear strength of beams without web reinforcement (V_c) proposed by Okamura [7] and the web reinforcement estimated by the 45 degrees truss mechanism (V_s). However, by using this method the contribution concrete after shear crack occurred is necessary to be investigated. Therefore, in this research the shear crack deformation was measured and then the aggregate interlocking force was calculated based on that measured deformation. Consequently, the applicability of superposition method (V_c+V_s) can be systematically explained by the shear crack deformation and aggregate interlocking force.

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2. EXPERIMENTS

2.1 TEST SPECIMENS

The layout of tested specimens and their cross-section are shown in **Fig. 1(a)**. The shear spans of tested beams were asymmetrically located which their left and right spans were 3.0 and 1.5, respectively. The cross-section was 400 mm. in both width and height. The tested beams were doubly reinforced by eight D22 bars as shown in **Fig. 1(b)**. Therefore, the tension reinforcement ratio equals to 1.02%. The amount of web reinforcement at the left span was differently reinforced as 0.035%, 0.05%, 0.065%, and 0.08% by D4 bars. To protect the unintentional shear failure at the right span of all tested beams, heavily reinforced by D10 bars with spacing 70 mm. was necessary.

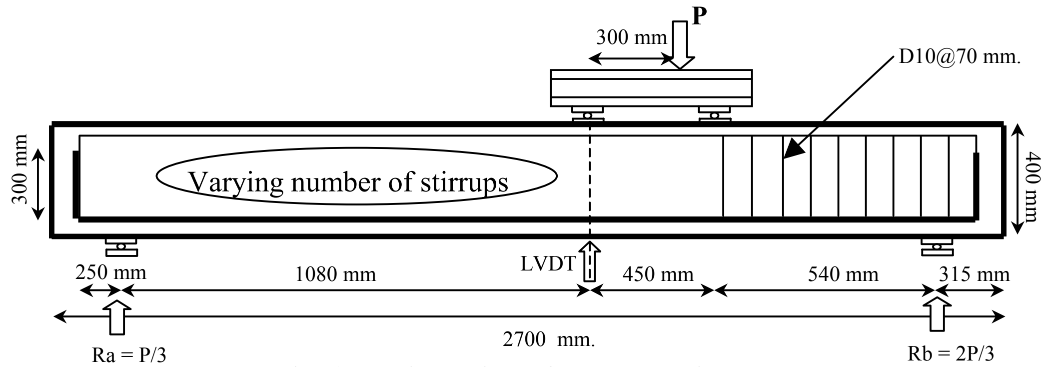

Fig. 1(a) Dimension of tested specimens

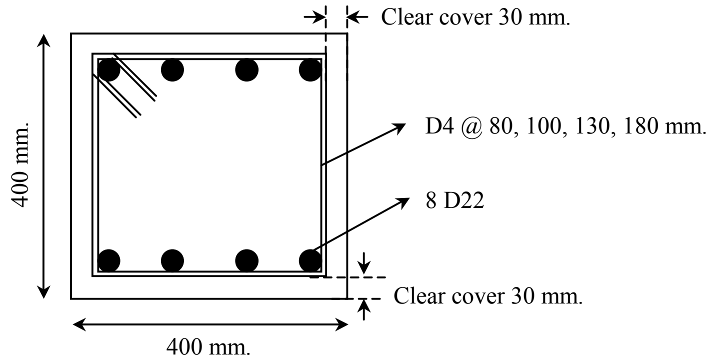

Fig. 1(b) Cross-section of tested beams

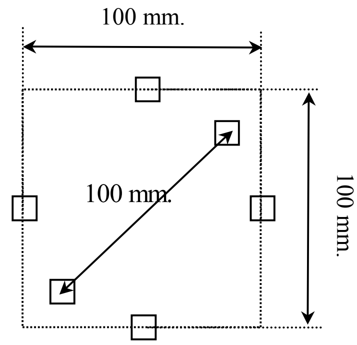

Fig. 2 One group of contact chips

Table 1 Cylindrical compressive strength (MPa)

	Specimens reinforced with web reinforcement ratio			
	0.035%	0.05%	0.065%	0.08%
Compressive strength (MPa)	41.5	43.5	41.7	46.2

According to the material tested results, the yield strength of reinforcing bars was 245 MPa and 715 MPa for the web and main reinforcing bars, respectively. **Table 1** illustrates the cylindrical compressive strength of the tested beams before tested.

2.2 MEASUREMENT

All tested beams were statically loaded by the four points bending method. The loading points were asymmetrically located in order to create zero shear force inside the maximum moment areas. For measuring the applied load, the load cell was located on the load transferred beam. In addition, two LVDTs were located under the tested beams corresponding to the left side of the load transferred beam. According to this set-up method, the applied shear force in the left span of all beams was calculated by dividing the applied load by 3.0. In this content, the applied shear force, defined as the divided value of the applied load, is used only.

The crack deformation along the shear crack plane of all beams was also measured by utilizing the extensometer and a set of contact chips as shown in **Fig. 2**. Three pairs of contact chips comprising one group of contact chips were adhered on the concrete surface with the angle of 0, 45, and 90 degrees against the beam axis (X-axis). Both surfaces of all tested beams were identically attached by these contact chips. Each group of contact chips was located in the blocks of gridlines shown in **Fig. 3**.

2.3 AGGREGATE INTERLOCKING FORCE

In this research, the aggregate interlocking force (V_{agg}) was calculated from the crack deformation along the shear crack plane. Firstly, the deformation data at each group of contact chips intersected with the shear crack was converted to the crack opening and sliding respect to the shear crack plane. Next, the stress transfer across the crack perpendicular to the shear crack plane can be calculated by using the crack opening and sliding as the input data into the later mentioned model. After that, that stress was converted to be the force perpendicular to the shear crack plane at each group of contact chips. Finally, the aggregate interlocking force can be obtained by integrating the Y-component of that force.

"Universal model for stress transfer across cracks in concrete" proposed by Bujadham and Maekawa [8] was adopted here because it was obtained from the coupling deformation between crack opening and sliding simultaneously. Therefore, it can definitely suit with the shear crack deformation in the reinforced concrete beams.

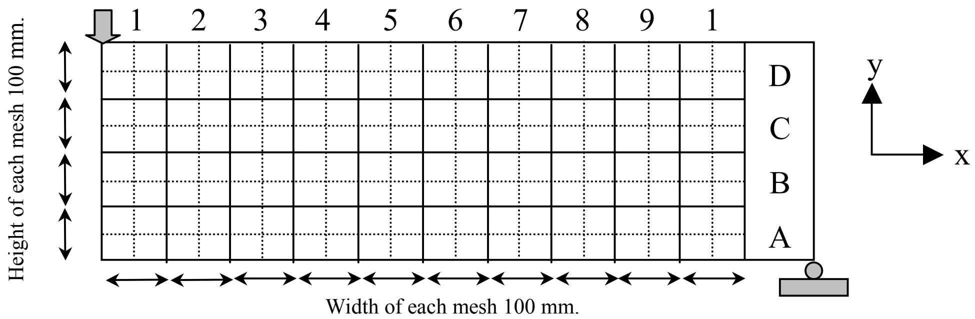

Fig. 3 Location of contact chips

Table 2 Load comparison of all tested beams

Web reinforcement ratio / Categories	0.035%	0.05%	0.065%	0.08%
1.) V_s^* (kN)	17.8	24.6	32.0	40.0
2.) Shear crack (estimation), V_c, (kN)	160.8	163.4	161.1	166.6
3.) Shear crack (test), V_{cr}, (kN)	156.3	145.6	157.7	170.2
4.) Shear strength, V_u, (kN)	**187.5**	**190.8**	**187.8**	**226.6**
5.) V_u / V_c+V_s	1.05	1.01	0.97	1.1
6.) V_u / V_{cr}+V_s	1.08	1.12	0.99	1.08

* The value was calculated by assuming 45 degrees truss model.

3. EXPERIMENTAL RESULTS

3.1 FAILURE OF TESTED BEAMS AND THEIR SHEAR STRENGTH

During the experiment, the crack form and shear crack load were carefully observed. The crack form of all beams can be representatively illustrated in **Fig. 4**. The large crack at the mid-depth of beams generated the shear failure in all tested beams. At the failure, the web reinforcement intersected with the shear crack cut off and then the width of shear crack suddenly became very wide. From the measurement, the load-deflection curves of all tested beams are shown in **Fig. 5**. The applied shear force (V) of all beams is normalized by $0.2f_c^{1/3}$ which adopted from the shear strength equation of beams without web reinforcement proposed by Okamura [7]. In this figure, the load-deflection curve of beam with web reinforcement 0.08% does not clearly show the drop in load-deflection curve since the development of crack was slow. In contrast, the beams with web reinforcement 0.035%, 0.05%, and 0.065% clearly show the drop in load-deflection curve since the shear crack suddenly appeared and immediately developed.

Fig. 4 Crack form

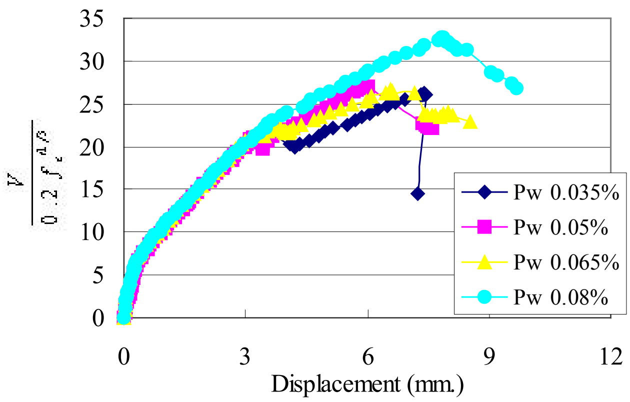

Fig. 5 Load deflection curves

The estimated values and tested results of each beam are shown in **Table 2**. The estimated shear strength of beam without web reinforcement, tested shear crack load, and shear strength from all beams are defined as V_c, V_{cr}, and V_u, respectively. The V_{cr} is defined as the load corresponding to the drop of load-deflection curve. In addition, the contribution of web reinforcement (V_s) was estimated by the 45 degrees truss mechanism. In the table, V_{cr} is less than V_c in all tested beams, especially the beam with web reinforcement 0.08%. This is possibly affected by the restraining force from web reinforcement on the shear crack. Furthermore, the summation of the V_c and V_s compare to V_u (No.5) shows the conservative results. Similarly, the summation between the V_{cr} and V_s compare to V_u (No.6) shows the accurate results. In conclusion, the superposition method between the shear strength of beams without web reinforcement or tested shear crack load and the 45 degrees truss mechanism is able to estimate the shear strength of reinforced concrete beams with a small amount of web reinforcement.

3.2 CRACK DEFORMATION

To clarify the applicability of the superposition method, it is now necessary to explain and understand more in the details of the shear crack deformation. Two groups of contact chips along the shear crack plane are representatively selected. The first and second groups are specifically located at the mesh C6 and mesh B7 as shown in **Fig. 3**. Furthermore, the definition of crack opening and sliding is the crack deformation in the normal and tangential directions against to the shear crack plane, respectively.

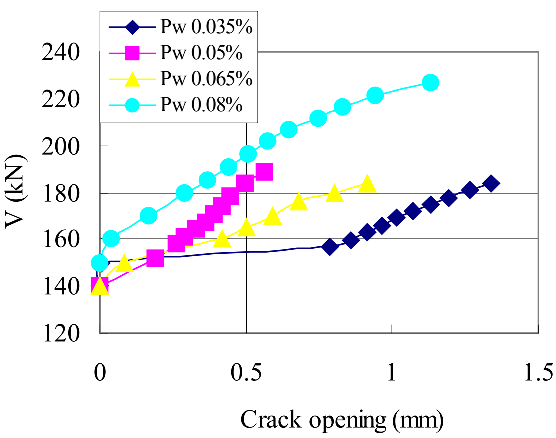

Fig. 6(a) Crack opening at C6

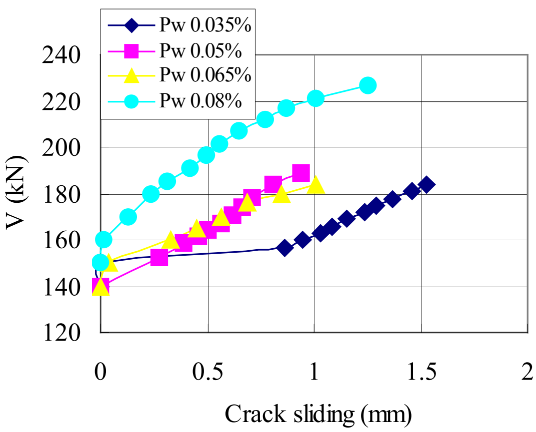

Fig. 6(b) Crack sliding at C6

In **Fig. 6 and 7**, the average crack opening and sliding of each beam surfaces at the upper side (mesh C6) and lower side (mesh B7) of shear crack are respectively plotted against the applied shear force (V). It can be noticed that the smaller crack opening and sliding will be achieved if the larger web reinforcement ratio is used, except the beams with web reinforcement 0.05% and 0.065%. However, it can be reasonably explained by the general fact that at the same loading level the smaller amount of reinforcement results in the larger crack opening and then the larger crack opening also generates larger crack sliding.

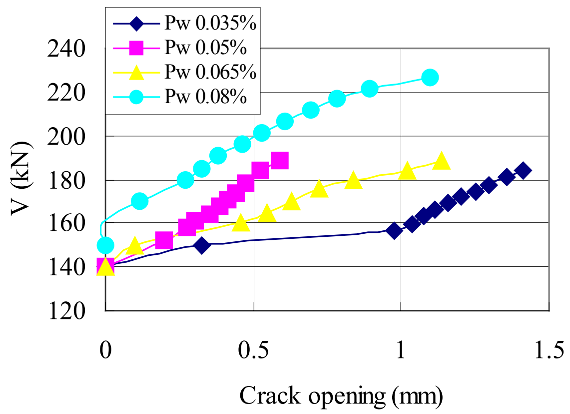

Fig. 7(a) Crack opening at B7

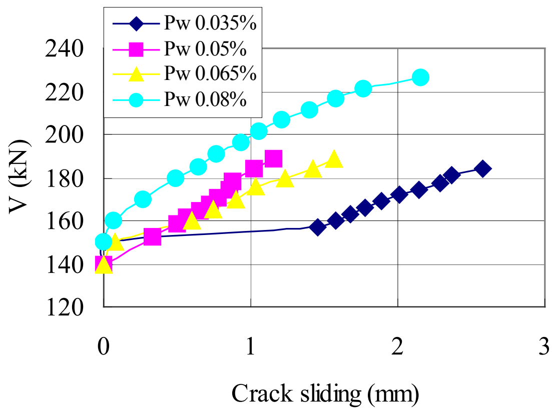

Fig. 7(b) Crack sliding at B7

In **Fig. 8**, the ratio of crack sliding to crack opening at the mesh C6 is representatively plotted against the ratio of applied shear force to the maximum shear strength among the tested beams ($V/V_{u,max}$). It can be also seen that all tested beams show the same maximum ratio of crack sliding to crack opening, about 1.0. Nevertheless, the disagreement is found from the beam with web reinforcement ratio 0.05% due to the different crack angle compare to other beams. According to the relation of those ratios, it can be observed that after shear crack took place the ratio of crack sliding to opening suddenly increases to the maximum in case of beams with smaller web reinforcement ratio. In contrast, after shear crack took place the ratio of crack sliding to opening gradually increases to the maximum in case of beams with larger web reinforcement ratio.

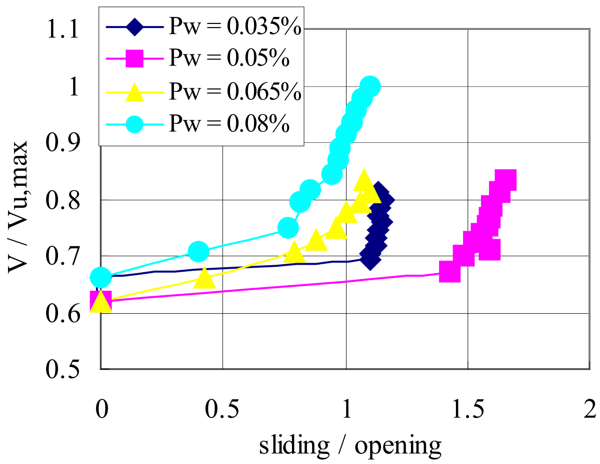

Fig. 8 Ratio of crack sliding to opening

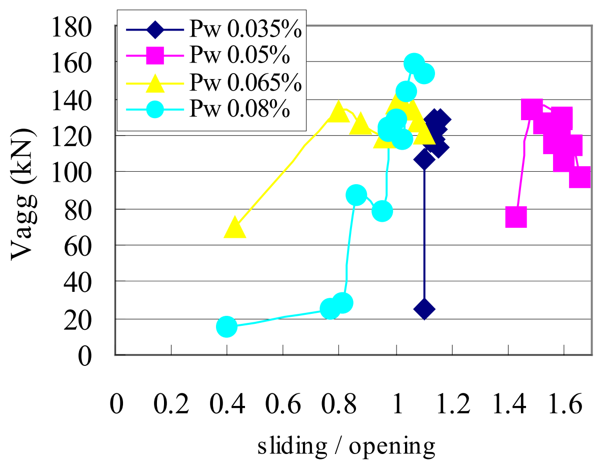

Fig. 9 Aggregate interlocking

4. SHEAR RESISTANCE CONTRIBUTED BY AGGREGATE INTERLOCKING

According to the crack deformation data, the aggregate interlocking force can be calculated as mentioned in **3**. From the **Fig. 9**, the aggregate interlocking force (V_{agg}) of each beam is plotted against the ratio of crack sliding to opening. The aggregate interlocking force of beams with web reinforcement 0.035, 0.05, and 0.065% are averagely 120 kN and that of beam with web reinforcement 0.08% is 160 kN. As shown in the figure, the aggregate interlocking force becomes large when the ratio of crack sliding to

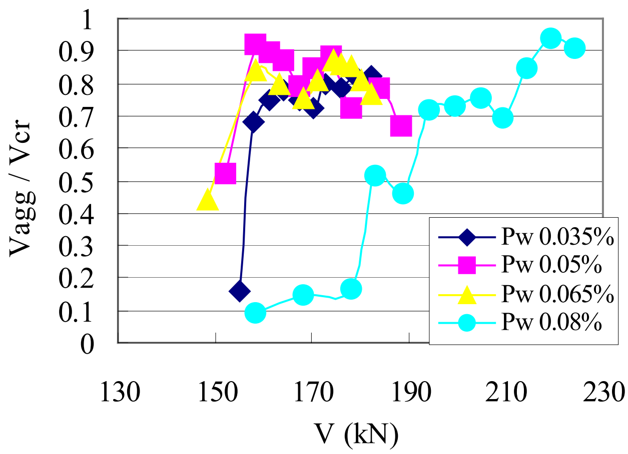

Fig. 10 Contribution of aggregate interlocking

opening becomes remarkably large. The ratio of aggregate interlocking force to shear crack load (V_{agg}/V_{cr}), which plotted against the applied shear force (V), is illustrated in **Fig. 10**. In the figure, the beams with web reinforcement ratio 0.035%, 0.05%, and 0.065% approximately show the same ratio of aggregate interlocking force to shear crack load. In contrast, the beam with web reinforcement 0.08% shows the gradual development of aggregate interlocking force. It is noticed from the figure that the 80% of shear crack load is largely contributed by the aggregate interlocking force even if the amount of web reinforcement is very small, and the remainder is basically contributed by the shear resistance of dowel action and compression zone.

5. CONCLUSIONS

This study intends to solve the lack of shear strength estimation equation and clarify the shear resistant mechanism for the beams with web reinforcement less than 0.10%. In the experiment, the shear strength and shear crack deformation were measured. The aggregate interlocking force was then calculated according to the shear crack deformation. Consequently, the applicability of the superposition method can be explained from the experimental results as following;

1) At the same applied shear force, the reinforced concrete beams with smaller web reinforcement ratio showed the larger shear crack opening and sliding.
2) After the shear crack had occurred, the ratio of crack sliding to opening of beams with smaller web reinforcement ratio increased faster than that of beams with larger web reinforcement ratio.
3) When the larger shear crack opening had taken place, the larger crack sliding also took place. Therefore, the aggregate interlocking force became large and was nearly the same as V_{cr} even if the web reinforcement was less than 0.08%.
4) Since the aggregate interlocking force was almost the same as V_{cr}, the shear strength of reinforced concrete beams with a small amount of web reinforcement can be estimated by the superposition method (V_c+V_s).

REFERENCES

1) JSCE, "Investigation of Causes of Damage to Civil Engineering Structure, Bridge Structure," Report on The Hanshin-Awaji Earthquake Disaster, V.1, 1996.
2) Okamura, H., and Kim, I.H., "Size Effect and Failure of RC Structures under Earthquake," Proceedings FRAMCOS-3, Gifu, Japan, 1998, 1263-1278.
3) Nielson, M.P., "Limit Analysis and Concrete Plasticity," Prentice-Hall Inc., 1984, pp.205-234.
4) Vecchio, F.J., and Collins, M.P., "Predicting the Response of Reinforced Concrete Beams Subjected to Shear Using the Modified Compression Field Theory," ACI Structural Journal, V.85, No.3, May-June, 1988, pp.258-268.
5) Vecchio, F.J., "Analysis of Shear-Critical Reinforced Concrete Beams, " ACI Structural Journal, V.97, No.1, Jan.-Feb., 2000, pp.102-110
6) FIB, "Design Examples for the 1996 FIP recommendations 'Practical design for structural concrete'," Bulletin 16, 2002, pp.165-186.
7) Okamura, H., and Higai, T., "Proposed Design Equation for Shear Strength of Reinforced Concrete Beams without Web Reinforcement, " Concrete Library, JSCE, V.1, July, 1983, pp.71-106
8) Bujadham, B., and Maekawa, K., "The Universal Model for Stress Transfer across Cracks in Concrete," Proc. of JSCE, V.17, No.451, Aug., 1992, pp.277-287.