ABSTRACT: This paper discusses the problem of predicting the life of reinforced concrete flexural member. Series of unstrengthened and CFS strengthened beams were tested in the lab to represent the highway flexural members. The beams were monitored closely during their degradation process by measuring their change in mid-span deflection, strain in steel reinforcement and natural frequency. Change in natural frequency is used as the indicator of beam degradation in stiffness. The degradation model is based on experimental observations. The proposed model can predict the life of undamaged specimens as well as specimens that have been strengthened.

KEYWORDS: Cumulative fatigue damage, Fatigue Life prediction, Carbon-fiber-Sheet (CFS), Reinforced Concrete Beam.

1. INTRODUCTION

In order to successfully extend the service life of a structure, long term performance of a strengthening technique is one of the most important aspects. The composite material itself is excellent in long term performance since it does not have chemical reactions with most substances and has an outstanding fatigue performance. However, when it is used to strengthen RC members, the structure becomes a composite structure and results can be different. The fatigue behavior of a composite structure needs to be clarified.

Despite many research documents on the fatigue behavior of concrete member strengthened by external reinforcement, up to the present there is still no method to predict the life of the damaged RC flexural members after being strengthened with this method. The objective of this research is an attempt to predict a service life under a repeated load of a RC flexural member both before and after being strengthened with Carbon Fiber Sheet (CFS).

2. OUTLINE OF EXPERIMENTS

2.1 MATERIALS

All beams were made using a normal strength coarse aggregate concrete of 20 mm maximum nominal size, w/c of 0.55. The mean concrete compressive strength after 28 days is 43 MPa. The steel reinforcement properties are listed in Table 1.

The external reinforcement was a carbon fiber sheet (CFS). CFS is available in rolled laminate of 0.167 mm effective thickness, 500 mm width and 100 m length. The effective thickness gives the section of the fibers in each single ply. The properties of CFS are shown in Table 2.
were tested under a four-point bending test over a span of 1400 mm. A shear span to depth ratio, a/d, equals 3.5. All beams were cast at the same time with the same reinforcement and the same load setup as shown in Fig. 1(a).

**Table 1. Properties of steel reinforcements**

<table>
<thead>
<tr>
<th>Type</th>
<th>Cross section Area (cm²)</th>
<th>Yield Strength fₓ, (MPa)</th>
<th>Yield strain (×10⁻⁶)</th>
<th>Rupture Strength fᵤ, (MPa)</th>
<th>Elastic Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D9</td>
<td>0.60</td>
<td>300</td>
<td>1500</td>
<td>440</td>
<td>200</td>
</tr>
<tr>
<td>D13</td>
<td>1.27</td>
<td>394</td>
<td>2120</td>
<td>480</td>
<td>185</td>
</tr>
</tbody>
</table>

**Table 2. Properties of carbon fiber sheet**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber Density (g/m²)</td>
<td>300</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>0.167</td>
</tr>
<tr>
<td>Tensile Strength (MPa)</td>
<td>3,400</td>
</tr>
<tr>
<td>Rupture strain (×10⁻⁶)</td>
<td>14,900</td>
</tr>
<tr>
<td>Tension Modulus of Elasticity (MPa)</td>
<td>253,000</td>
</tr>
</tbody>
</table>

Beams are divided into two main groups. The first group is the normal reinforced concrete beams without external reinforcement. The second group consists of the strengthened beams with two layers of CFS. CFS was attached all over the tension surface of the beams as shown in Fig. 1(b).

Concrete beams were left for curing for 30 days before CFS installation started. The concrete surface was cleaned by sand blasting before primer and putty were applied as a substratum to the CFS. At last, CFS was applied to the concrete beam by using epoxy resin as a bond agent. Both layers were applied at one time. Again, concrete beams after strengthening were left for epoxy curing for at least 21 days before test.

Four beams, two unstrengthened beams and two CFS strengthened beams, were tested under monotonic bending load in order to obtain the load-deflection characteristics of both unstrengthened and CFS strengthened beams. The average maximum moments from monotonic bending test are 18.1 KN·m and 30.5 KN·m for unstrengthened and CFS strengthened beam respectively. From these values, the maximum applied moments to the beams under fatigue test were determined.

Ten other beams were applied to the fatigue loading until failure with a sinusoidal load history at a frequency of 1 Hz with minimum load of 1.8 KN·m and the various maximum loads. Beams in fatigue test were monitored closely by measuring their natural frequency and deflection at the center of the beams, strain in reinforced steel, surface of the CFS and top concrete compression surface. Loading was stopped during each measurement.

The accelerometer used has a contact face area of approximately 10 cm² and nominal voltage acceleration sensitivity of 0.2040 V/(m/s²). Data signal acquired was transferred to the personal computer by Data Acquisition (DAQ) card. Measuring signal was 4,096 points per time and the measuring frequency was 1,000 Hz. Fourier transform of each signal was computed using and FFT algorithm.

Accelerometer was placed in the middle of the beam and the impact loads were given to the beam by hammer. Impacts were given three times for each position at the 35, 70 and 105 centimeters from one side of the support. The vibration waves measured from different points had different magnitudes; however, the frequencies measured were comparable and varied in ±1.0 Hz range. The measured results are the average of all nine measured data.
3. EXPERIMENTAL RESULTS AND DISCUSSION

3.1 OUTLINE OF TEST RESULTS

Test results are shown in Table 3. FU and FC series are beam in fatigue tests in the unstrengthened and strengthened conditions respectively. The FST beams were previously loaded by fatigue loading with the maximum load of 14.1 KN-m before being strengthened with CFS and continued loading until failure with the same applied load. FST-1 and FST-2 were previously loaded up to 150,000 cycles and FST-3 was previously loaded up to 80,000 cycles before strengthened with CFS.

Unstrengthened beams failed by first the rupture of steel reinforcements followed by crushing of concrete in compression. CFS strengthened beam’s failure started from a delamination of concrete at the level of internal steel followed by yielding of steel reinforcements. Then, steel reinforcements were ruptured and followed by crushing of concrete in compression.

Table 3. Test Results for CFS Strengthened Beams under cyclic loading

<table>
<thead>
<tr>
<th>Beam Name</th>
<th>External Reinforced</th>
<th>Maximum applied load (KN-m)</th>
<th>Applied Load Ultimate Load</th>
<th>Applied Strain range</th>
<th>Failure cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>FU-1</td>
<td>None</td>
<td>10.5</td>
<td>0.58</td>
<td>0.59</td>
<td>1,522,724</td>
</tr>
<tr>
<td>FU-2</td>
<td></td>
<td>14.1</td>
<td>0.78</td>
<td>0.78</td>
<td>373,162</td>
</tr>
<tr>
<td>FU-3</td>
<td></td>
<td>15.0</td>
<td>0.83</td>
<td>0.83</td>
<td>129,966</td>
</tr>
<tr>
<td>FC-1</td>
<td>CFS two layers</td>
<td>14.1</td>
<td>0.46</td>
<td>0.56</td>
<td>1,538,888</td>
</tr>
<tr>
<td>FC-2</td>
<td></td>
<td>15.0</td>
<td>0.49</td>
<td>0.65</td>
<td>918,588</td>
</tr>
<tr>
<td>FC-3</td>
<td></td>
<td>17.8</td>
<td>0.58</td>
<td>0.75</td>
<td>209,835</td>
</tr>
<tr>
<td>FC-4</td>
<td></td>
<td>23.8</td>
<td>0.78</td>
<td>1.14</td>
<td>2,986</td>
</tr>
<tr>
<td>FCT-1</td>
<td></td>
<td>14.1</td>
<td>0.78 (0.46) *1</td>
<td>0.78 (0.65)</td>
<td>150,000 (284,206) *2</td>
</tr>
<tr>
<td>FCT-2</td>
<td></td>
<td>14.1</td>
<td>0.78 (0.46)</td>
<td>0.78 (0.65)</td>
<td>150,000 (325,538)</td>
</tr>
<tr>
<td>FCT-3</td>
<td></td>
<td>14.1</td>
<td>0.78 (0.46)</td>
<td>0.78 (0.65)</td>
<td>80,000 (526,150)</td>
</tr>
</tbody>
</table>

*1 Ratio after strengthened  *2 Additional load cycles until failure after strengthened

Table 3 shows that for beams under the same applied load to ultimate load ratio, the fatigue life of the unstrengthened beam is greater than the fatigue life of the CFS strengthened beam. The increase in the ultimate load of the beam is not proportional to the increase in fatigue life of the beam due to the difference between a monotonic failure and fatigue mode of the beam.

However, when considering the beams of the same measured applied strain to yield strain of the steel reinforcement, the fatigue life of both types of beam becomes equivalent. It can be concluded that fatigue life of a steel reinforcement has much influence on the fatigue life of both unstrengthened and strengthened beams.

Moreover, the condition of steel reinforcement before the beam is strengthened is critical for determining the effectiveness of a strengthening system. The previously damaged beams, FST-1, FST-2 which were previously loaded with 150,000 cycles had their steel reinforcement yield before strengthening. As a result FST-1 and FST-2 had an additional life by strengthening system only 284,206 cycles and 325,538 cycles respectively. It must be emphasized that the steel reinforcements of the FST series did not yield because of the applied load to the ultimate load ratio was 0.78, but the yielding was due to the load cycles. The FST-3 specimen was only loaded with 80,000 cycles, and the steel reinforcements did not yield. Hence, FST-3 was able to endure more cycles after being strengthened than FST-1 and FST-2.
3.2 CHANGE IN NATURAL FREQUENCY AND BEAM STIFFNESS

Degradation of beams in this experiment is defined as the change in beam’s stiffness i.e. Beam’s Elastic Modulus (E) multiplies by Beam’s moment of inertia (I). In this experiment, beam stiffness along the load history was measured by beam’s natural frequency.

By theory, natural frequency is proportional to the square root of stiffness. The degradation level of beam at any cycle is given as a ratio of the cycle residual beam stiffness to the undamaged beam stiffness. Thus, the degradation index (DI) can be written as:

\[
DI = \left( \frac{f_0}{f_x} \right)^2
\]

Where,
\( E_{I_0} \) and \( E_{I_x} \) = Initial beam stiffness and beam stiffness measured after x cycles respectively
\( f_0 \) and \( f_x \) = Initial natural frequency of beam and natural frequency measured after x cycles respectively

The measurement shows that deflection of beam, strain in steel reinforcement, and the strain on the compression face of a concrete beam did not change much until close to failure. On the other hand, change of natural frequency of a beam is a good indicator of the deterioration under fatigue loading. Thus, hereafter, the ratio of residual stiffness and initial stiffness, \( EI_x/EI_0 \), will be defined as a square of a ratio of beam natural frequency at after x cycles and initial frequency, \( (f_x/f_0)^2 \).

3.3 DEGRADATION MECHANISM

The degradation curves of unstrengthened beams and undamaged two layers CFS strengthened beams are presented in Fig. 2 and Fig.3 respectively.

All beams show a great drop in stiffness during the first cycles, and then stiffness of beams gradually dropped until failure. Also, the higher the maximum load, the higher the damage to the beam during the first cycles. However, despite the different maximum loads, the unstrengthened beams failed when stiffness dropped to almost 60% that of undamaged beams. The actual last stiffness before failure of all beams can not be measured as the measurements were done in interval basis.

In the case of strengthened beams, the degradation curve shows the same trend with that of unstrengthened beams. The stiffness dropped to near 60% that of the undamaged condition. Yet, again at near failure after delamination of CFS occurred, the stiffness of beams sharply dropped again to less than 60% that of the undamaged condition. The reductions in stiffness of the strengthened beams are greater than in the case of the unstrengthened beams due to the higher initial stiffness in undamaged condition. Beam FC-3 and FC-4 could stand only few cycles after delamination of concrete since load applying on these beams were more than the ultimate strength of an unstrengthened beam. In the case of FC-1 and FC-2, after delamination, the beams were still able to carry load to some extent before failure.

The test results suggest that there exists a “failure stiffness” where all beams with similar configurations will fail when their stiffness is reduced to a particular value.
4. FATIGUE MODEL

The degradation model is developed under the following three observations from the test.

1. Cumulative damage (D) occurs in discrete increments during each load cycle (N) and can be written in differential form as $dD / dN$. $D$ is defined as the following.

\[
D = \frac{E_{I_0} - E_I}{E_{I_0} - E_{I_f}}
\]  

Where, 
$E_I$ and $E_{I_0}$ = Residual stiffness and initial stiffness respectively
$E_{I_f}$ = Stiffness of member just before failure and $E_I \geq E_{I_f}$
Structures are considered failed when cumulative damage, $D$, equals to one.

2. Cumulative damage (D) for both unstrengthened and CFS strengthened beams follows the same degradation curve as represented by Fig.4. Unstrengthened beams fail by rupture of the steel reinforcement and CFS strengthened beams are considered to fail when CFS delamination occurs. The cumulative curve in Fig. 4 can be best describe by

\[
\frac{dD}{dN} = \frac{C}{D^n}
\]  

Where, $N$= number of cycles of the load, $C$ and $n$ = constant and $n>1$

3. Cumulative damage is sensitive to the change in applied strain per cycle, $\Delta \delta$. Then, the constant $C$ in Eq.3 is rewritten as following.

\[
C = C'' (S_{max})^m
\]

Where, $C''$ = Constant, and strain ratio $S_{max} = \frac{\Delta \delta_{max}}{\delta_{yiled}}$ in which $\Delta \delta_{max}$ is the maximum applied strain range to the steel reinforcement and $\delta_{yiled}$ is the yield strain of the steel reinforcement.

Substitute Eq.4 to Eq.3 and integrate Eq.3 by substitute the initial condition, $D=0$ at $N=0$, we finally obtained:

\[
D = \left[ nC'' (S_{max})^m N \right]^{1/n}
\]  

Then constants are determined from the test. Constant $m$ can be found by the slope of the S-N curve for each type of specimens as describe in the following. S-N curve here is defined as plot of applied strain range to yield strain ratio $(S_{max})$ versus failure cycles of a beam $(N_f)$.

When $N$ approaches failure cycle $(N_f)$, $D$ approaches 1, therefore Eq.5 becomes

\[
\frac{1}{n} = C'' S_{max}^m N_f
\]

Apply logarithm and then Eq.6 can be rewritten as

\[
\ln(S_{max}) = -\frac{1}{m} \ln (N_f) + \text{Constant}
\]

Therefore, constant $m$ is a negative inverse of a slope of $\ln(S_{max})$-$\ln(N_f)$ curve.

Constant $n$ can be found from the measured data of the same applied strain $S_{max}$ as the following.
From the degradation curve, $D_1$, $N_1$ and $D_2$, $N_2$ are obtained from two measured points. Substitute $D_1$, $N_1$ and $D_2$, $N_2$ in to Eq.5. For the value in between, i.e. $N_1, D_1 \neq 0$, $n$ can be calculated as following:
\[ n = \frac{\ln \left( \frac{N_1}{N_2} \right)}{\ln \left( \frac{D_1}{D_2} \right)} \]  

(8)

Finally, constant \( C'' \) is obtained by plotting Eq.5 to fit the data after obtained \( m \) and \( n \).

Constant \( m \) obviously depends on condition of steel reinforcement alone. Hence, \( m \) is equal to 9.0 for both unstrengthened and CFS strengthened beam. Constant \( n \) depends on beam and load setup. Since there is only one setup in this test, \( n \) is equal in both types of beam and is equal to 13.0. Lastly, \( C'' \) is depend on type of beam. \( C'' \) found in this study is equal to 0.000003 for unstrengthened beams and 0.000007 for CFS strengthened beams.

Then, the cumulative damage equation for an RC beam is given by the following equations:

For unstrengthened beam

\[ D = \left[ 0.000039 S_{\text{max}}^{9.0} N \right]^{1/13} \]  

(9)

For CFS strengthened beam

\[ D = \left[ 0.000091 S_{\text{max}}^{9.0} N \right]^{1/13} \]  

(10)

It should be noted that the experimental data in this study agrees very well with the proposed model as shown in Fig.5. The correlation coefficient, \( R^2 \), from three unstrengthened beams and four CFS strengthened beams equals 0.976.

The effect of beam dimension and loading configuration need to be further investigated. The validation of the model for bridge slabs and beams is also being investigated.

The model could give a rough estimate of the number of cycles to failure of beams under repeated load. The crucial factor in the precision of this method is correctness of the input strain ratio, \( S_{\text{max}} \).

5. CONCLUSIONS

Based on the limited number of tests conducted in this research, the following conclusions can be presented.

1. By CFS, stress in steel reinforcement is reduced, thus life of a strengthened beam is longer than the life of an unstrengthened beam under the fatigue test. However, for the beams in the same a/d like in our test, shear-moment crack could lead to the delamination of the concrete at the level of tension reinforcement. As a consequence, CFS strengthened beams cannot help extended life of beam as much as expected by looking at the reduction of stress in steel reinforcement alone.
2. It is better to strengthen beams before they receive too much damage. The steel reinforcement condition before strengthening is very important in the effectiveness of a strengthening system. Earlier strengthening can extend life of a beam to a much longer extent than late strengthening.
3. Cumulative damage concept can be used to predict fatigue life of unstrengthened beams, strengthened beams, and pre-damaged strengthened beams.

REFERENCES