*- Technical paper -*

# CRACKING AND TENSION-STIFFENING OF CHEMICALLY PRESTRESSED CONCRETE UNDER UNIAXIAL TENSION

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**ABSTRACT:** The behaviors of CPC under uniaxial tension were compared with those of RC. A special specimen profile was designed to avoid the effect from end parts. The tension stiffening of both RC and CPC compared with the current tension stiffening model. The crack pattern was observed after loading. The results show that the CPC has superior tension stiffening and the conventional model for RC underestimates the value. Moreover, cracking can be retarded in CPC so that the number of cracks in CPC is less than in RC at the same load. These properties of CPC should be related to bond of CPC.
**KEYWORDS:** chemical prestress (CPS), chemical prestrain (CPN), cracking, tension stiffening, bond, crack spacing

## 1. INTRODUCTION

Because of its inherent weakness in tension, concrete is commonly reinforced by steel bars which can carry tensile forces across the cracks after tensile failure of the concrete. The sudden structural collapse of reinforced concrete (hereinafter, RC) under tension can thus be prevented. In addition, in RC, the concrete can carry tensile stresses between cracks as the result of bond action between concrete and reinforcement. This phenomenon is called tension stiffening and it plays important role in assessing serviceability requirement after cracking, including member stiffness, deformation, and crack widths.

Tension stiffening effect is important part of an analysis that use averaged stresses and strains to predict member behavior, such as smeared finite elements or a layered beam section analysis. This type of approaches require a suitable materials model for cracked concrete, and tension stiffening results can be used to obtain the post-cracking stress-strain response of concrete.

Chemically prestressed concrete (CPC), which is made from expansive concrete, has higher structural performance than RC, for example, higher cracking load and structural properties after first cracking. The better performance of CPC under bending loads or shear loads have been reported [1]. CPC has also higher resistance to cracking [2]. It was reported that CPC could be used to reduce number of cracks and crack width in structure. These merits of CPC are likely to relate with properties of CPC under tension. However, the information about the CPC loaded under uniaxial tension and its tension stiffening is still insufficient.

This study is therefore an attempt to investigate the tension stiffening of CPC and compare it with the tension stiffening of RC and the available tension stiffening models. It is also an aim of this study to compare the cracking properties of CPC and RC under tension. These investigations would provide an insight about bonding of CPC.

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## 2. TENSION STIFFENING MODEL

The typical tensile response of an RC member is shown in Fig. 1. The difference between the bare bar response and the RC response is the so-called tension stiffening. Before the first cracking, stresses and strains are theoretically uniform along the length of the member. Equilibrium and the strains compatibility can be linked together by assuming linear elastic material properties for both concrete and steel. The external load is thus shared between the concrete and the steel in relation to their rigidities.

Once the tensile stress in concrete reaches the concrete's tensile capacity, cracking takes place. At cracking, the steel experiences a jump in stress at the crack locations, and the distribution of stresses and strains is no longer uniform. The average stress and strain are therefore used to represent the member response on account of this variation in force and deformation. The steel reinforcement is generally assumed to carry all of the tension at crack locations, while the concrete portion between cracks still carry a part of tensile force (see Fig.2), and tension stiffening results from the presence of this tensile force in concrete. The amount of tension transferred to the concrete portions between cracks mainly depends on the bond between steel and concrete and the crack spacing.

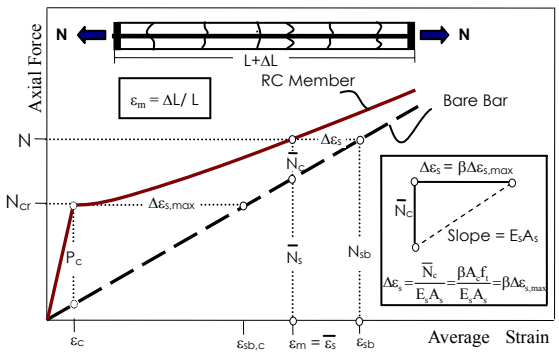



**Fig.1 Typical response of RC under direct tension**

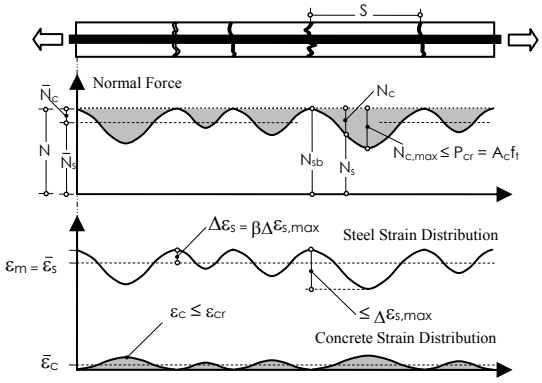



**Fig.2 Load distribution of concrete and steel bar in cracked RC under tension**

The load is transferred from the steel to concrete by bonds and the load carried by the concrete reaches its highest value, which cannot exceed the cracking force $P_{cr}$, at the middle at each cracked RC portion. The average force ($\overline{N}_c$) carried by the concrete and average tensile stress ($\overline{\sigma}_c$) in concrete can then expressed by

$$\overline{N}_c = A_c\overline{\sigma}_c = A_c\beta f_t = \beta P_{cr} \qquad \text{or} \qquad \overline{\sigma}_c = \beta f_t \tag{1}$$

Where; $f_t$ is the tensile strength of concrete and $A_c$ is the absolute area of concrete. β is a bond factor that accounts for the variation of concrete tensile stresses between the cracks. β represents the average tensile stress in concrete after cracking, but is generally expressed as a ratio of the cracking stress. Previous research [3,4] has shown that the bond factor, β, decreases as the applied load or the member strain increases. Okamura et al [4] suggest that bond factor of RC made with normal steel reinforcement can be expressed as a function of smeared tensile concrete strain ($\varepsilon_t$) as shown in Fig.4.

In CPC, which is initially prestressed by the expansion of expansive concrete, the origin of the tension stiffening should start from the point where the prestress completely diminishes (see Fig. 3 and Fig. 4). This point can be determined as the point where CPC member response and response of tensioned bars cross each other (see Fig. 4). The relationship between member's strains and CPC material's strain can be obtained as shown in Fig. 4.

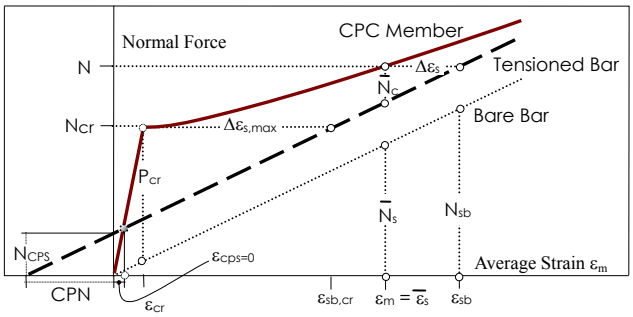



**Fig.3 Response of CPC member under tension**

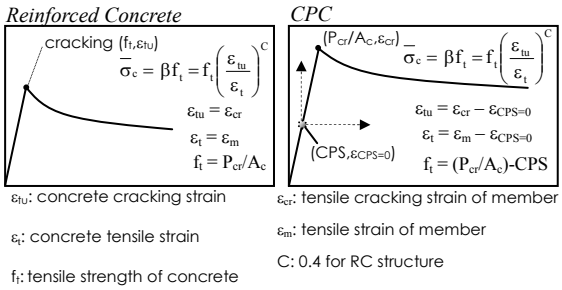


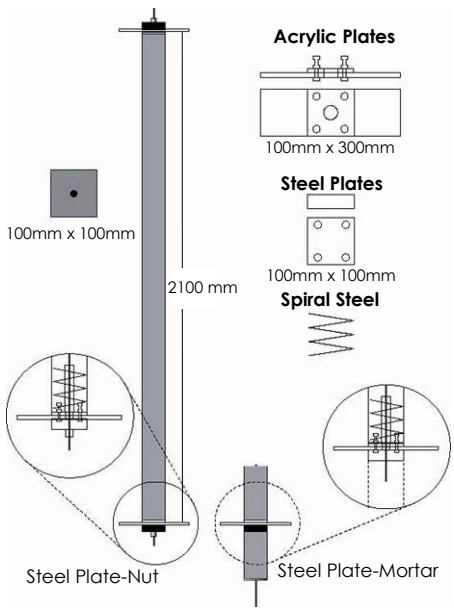

**Fig.4 Okamura's Tension stiffening model and adjustment for CPC**

## 3. EXPERIMENTS

### 3.1 SPECIMEN DETAILS

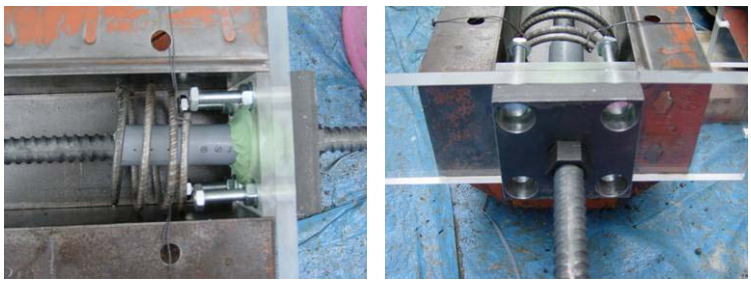
Four RC and six CPC columns reinforced with D-19 steel bar were tested under direct tension in this program. The total length of specimen was 2100 mm with 100x100 mm$^2$ cross-section. Because the end parts of specimen, similar to pre-crack, might cause early splitting cracks and can affect the accuracy of measurement. The 100-mm unbonded length was provided by inserting PVC pipes at both ends of each specimen to avoid these effects. The ϕ6 spiral steel was also installed at the ends to prevent longitudinal cracks in the unbonded zone at high load level. Acrylic plates with four bolts were provided at the end of all specimens in order to set the displacement transducers. The profile of specimens is shown in Fig. 5a.

Steel with two different shapes, i.e., ribbed bar and screw-shaped bar, were used as the tensile reinforcement in this experiment in order to check the effect of lug pattern on tension stiffening. In case of CPC columns, sufficient restraint is necessary; therefore, two different restraining methods; i.e., steel plate-nut method and steel plate-mortar methods were used in this study (see Fig. 5b and Fig. 5c). These two methods were applied at the same time in order to verify their appropriateness. The thickness of steel plates used in each method is 30 mm. The mortar used in steel plate-mortar method is a high-early strength expansive mortar cast two day before the casting of specimen. List of the specimens is given in Table 1.




(a) Specimen profile

(b) Steel plate-mortar method



(c) Steel plate-nut method

**Fig. 5 Specimen profile and restraining methods**

**Table 1 List of specimens**

| Code Name | Concrete | Steel | Restraints | Loading |
|---|---|---|---|---|
| NS－1 | Normal | S | × | Yielding |
| NS－2 | Normal | S | × | 100 kN |
| NR－1 | Normal | R | × | 100 kN |
| NR－2 | Normal | R | × | Yielding |
| ESN－1 | Expansive | S | N | 100 kN |
| ESN－2 | Expansive | S | N | Yielding |
| ESM－1 | Expansive | S | M | Yielding |
| ESM－2 | Expansive | S | M | 100 kN |
| ERM－1 | Expansive | R | M | Yielding |
| ERM－2 | Expansive | R | M | 100 kN |

Note: R: Ribbed bar; S: Screwed-shaped bar; N:Steel plate and nut restrain, M:Steel plate and mortar strain

**Table 2 Mix Proportion of concretes and mortar**

| | W/(C+E) | s/a(%) | Unit Content（kg/m3) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | W | C | E | S | G | 78S | 303A |
| Normal Concrete | 50 | 48 | 165 | 330 | 0 | 860 | 956 | 1.98 | 1.98 |
| Expansive Concrete | 50 | 48 | 164 | 268 | 60 | 860 | 956 | 1.98 | 1.98 |

Two specimens with exactly same materials were produced. Half of specimens were loaded until yielding took place while the loading of the other specimens was stopped at load of 100 kN (see Table 1) in order to observe the crack patterns at different loads.

### 3.2 MATERIALS

The mix proportions used in this experiment are given in Table 2. The superplasticizer (78S) and air entraining agent (303A) were added to control the workability and air content (see Table 2). The 28-day compressive strength of the normal concrete and the expansive concrete under free expansion condition are 40.45 MPa and 20.24 MPa, respectively. The restraining mortar was made from high-early strength cement with water cement ratio of 0.35 and expansive agent of 90 kg/m$^3$. This mortar had 2-day strength of 41MPa and 28-day strength of 48.96MPa. Both the ribbed bar and the screw-shaped bar have the same area of 286.5 mm$^2$ and same elastic modulus of 1.97x10$^5$ MPa. The yielding strain of the ribbed bar is around 2000 μ while that of the screw-shaped bar is around 4000 μ.

### 3.3 PROCEDURE

All specimens were cured under wet condition until the age of 28 days and then loaded uniaxially. The initial expansions (or chemical prestrain, CPN) were measured by strain gages at the middle of the rebars during this curing period. The measurement of initial strain was carried out until the start of loading. During loading, two invar lines with weights of 2.5 kg were hung to an acrylic plate at the top of the specimen and the weight of each side touched a displacement transducer. The elongation was measured by these transducers. The average strain of each specimen was then calculated from the measured elongation and initial length of specimen. The occurrence of cracking was carefully observed during loading and crack pattern of each specimen was recorded after loading.

## 4. EXPERIMENTAL RESULTS

The tension stiffening of RC and CPC (NR2 and ERM2) calculated without taking into account of CPN is shown in Fig. 6. High member cracking stress was observed in CPC because of CPS. Fig.7 illustrates the adjusted tension-stiffening of ERM2 which is calculated according to formula described in Fig.4. The adjusted parameters of all specimens are shown in Table 3. The CPS, CPN, member cracking strain, and member cracking stress were measured experimentally and the concrete tensile strength ($f_t$) and concrete cracking strain ($\varepsilon_{tu}$) were then calculated. Parameters of RC were also adjusted for its initial shrinkage by assuming linear elastic modulus of concrete. Both methods of restraining could sufficiently restrain expansion CPC. However, some difference in tensile modulus and cracking strain among members can be observed. This might result from different rate of drying shrinkage during the loading.

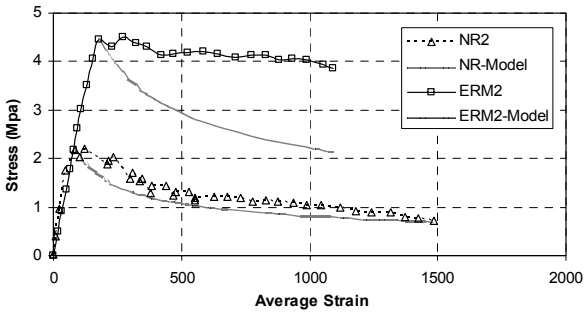



**Fig.6 Tension stiffening of NR2 and ERM2**

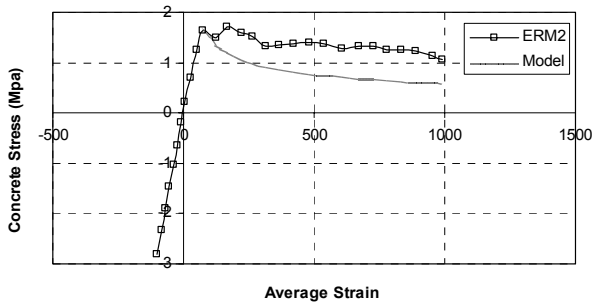



**Fig.7 Adjusted tension stiffening of ERM2**

**Table 3 Chemical prestrain, chemical prestress and tension-stiffening parameters**

| Specimen | Prestrain (μ) | Prestress (MPa) | $\varepsilon_{cr}$ (μ) | $\varepsilon_{CPS=0}$ (μ) | $\varepsilon_{tu}$ (μ) | $\sigma_{cr}$ (MPa) | $f_t$ (MPa) |
|---|---|---|---|---|---|---|---|
| NS1 | -7 | -0.038 | 60.2 | -1 | 61 | 2.24 | 2.28 |
| NS2 | -65 | -0.378 | 119 | -23 | 142 | 1.96 | 2.34 |
| NR1 | -58 | -0.334 | 138 | -26 | 164 | 1.77 | 2.10 |
| NR2 | -19 | -0.110 | 87 | -4 | 91 | 2.18 | 2.29 |
| ESN1 | 411 | 2.385 | 331 | 135.04 | 198 | 3.56 | 1.27 |
| ESN2 | 495 | 2.876 | 238 | 158.45 | 80 | 4.02 | 1.25 |
| ESM1 | 527 | 3.062 | 186 | 104.87 | 81 | 4.66 | 1.71 |
| ESM2 | 522 | 3.030 | 238 | 104.39 | 133 | 4.81 | 1.90 |
| ERM1 | 508 | 2.952 | 214 | 97.48 | 117 | 4.05 | 1.23 |
| ERM2 | 496 | 2.882 | 180 | 103.23 | 76 | 4.45 | 1.64 |

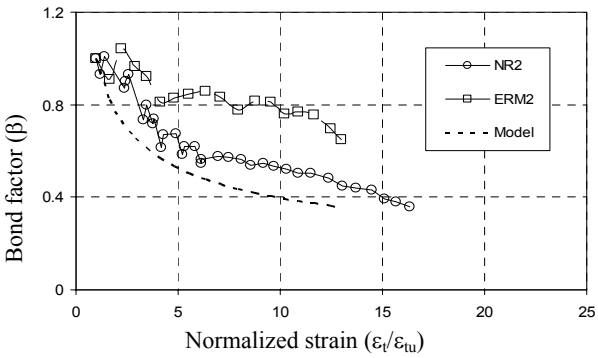



**Fig.8 Relationship between bond factor and normalized strain of NR2 and ERM2**

The adjusted tension-stiffening of each specimen was then normalized to obtain the relationship between bond factor (β) and the normalized strain ($\varepsilon_t/\varepsilon_{tu}$). The relationship between bond factor (β) and the normalized strain ($\varepsilon_t/\varepsilon_{tu}$) of NR2 and ERM2 is illustrated in Fig. 8. Although the current tension-stiffening model can properly estimate the average concrete tensile stress in RC, its application to CPC leads to more underestimation.

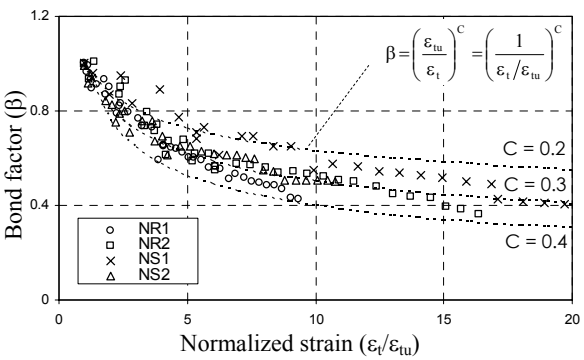



**Fig.9 Relationship between bond factor and normalized strain of RC**

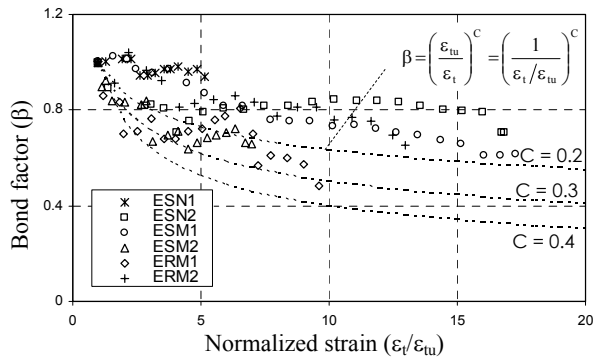
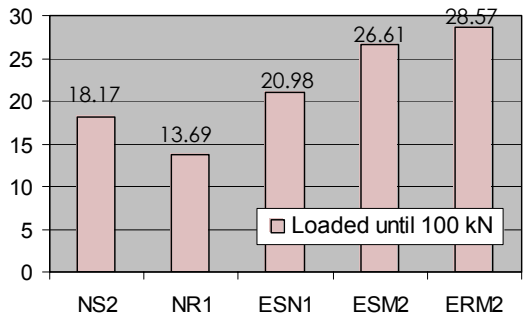
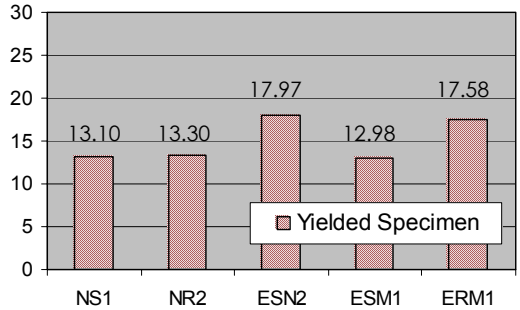



**Fig.10 Relationship between bond factor and normalized strain of CPC**




(a) Specimens loaded till 100 kN




(b) Specimens loaded till yielding

**Fig 11. Average Crack Spacing (cm)**

Fig. 9 and Fig. 10 show the relationship between bond factor and normalized strain of all specimens in this study. The results are also compared with various values of c (see Fig.4). It is obvious that CPC, which shows higher bond factor than model with c=0.2 (see Fig.10), can yield better bonding performance than RC with same reinforcement.

The average crack spacing of all specimens after loading is shown in Fig.11. The average crack spacing of CPC is much longer than that of RC at the load of 100 kN (see Fig.11a) and this difference in crack spacing become less when load increases (see Fig.11b). However, in general, it can be concluded that the crack spacing of CPC is longer in case of uniaxial tension. This result contradicts the report made by Ishimura et al [5] that the better tension-stiffening of CPC is caused by the better distribution of cracks.

Providing high tension-stiffening while reducing number of crack is a special property of CPC which cannot usually be observed in RC. In order to clearly explain this property, deeper investigation on local bond and deformability of CPC is necessary. Based on the results of this experiment, it is expected that local bond property of CPC should be better than that of RC while that the generation of cracks might be delayed by the deformability of CPC [2].

## 5. CONCLUSION

1. CPC shows good tension-stiffening even though the concrete tensile strength is lower.
2. The bond factor of CPC is remarkably higher than that of RC and a possible explanation for this should base on the local bond of CPC.


## ACKNOWLEDGEMENT

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