- Technical paper -

CRACKS WIDTHS EVALUATION AND DAMAGE STATES DRIFTS OF REINFORCED CONCRETE COLUMNS

Hossein MOSTAFAEI *1 and Toshimi KABEYASAWA *2

ABSTRACT: An approach is studied to evaluate the damage states drifts of reinforced concrete columns based on crack widths. Shear cracks and flexural cracks are the main crack types, considered for the lateral drifts estimation of reinforced concrete columns. Load-Lateral drifts relationships at the damage states were estimated based on shear crack angles and truss mechanism. A constitutive law was modified for compression diagonal strut concrete of the analogues truss. The analytical outputs were compared with the experimental results of sixteen-tested reinforced concrete columns subjected to static cyclic unidirectional reverse lateral load and applied axial load with different shear capacities.
KEYWORDS: Reinforced concrete columns, shear failure, crack width, extreme damage state

1. INTRODUCTION

Columns play a very important role in behavior of reinforced concrete structures. Previous post-earthquake inspections and evaluation of damage in buildings have demonstrated that most of the old reinforced concrete buildings, constructed based on the old design codes, are vulnerable to seismic loads due to lack of shear capacity in columns. To implement an efficient means of rehabilitation, damage evaluation need to be carried out to determine the proper retrofit measures. In order to evaluate post earthquake damages of structures, in most cases, visual inspection is the only practical method to estimate the damage states. Experimental studies have shown that characteristics of crack, appeared in reinforced concrete members, are related to the damage states of the structures. In this study, a relationship between crack's parameters and lateral drifts of reinforced concrete columns is expressed. The measured widths and angles of the cracks are the main parameters involved in the drifts evaluation method. A lateral load-drifts relationship at damages states was computed for reinforced concrete columns in order to obtain the lateral load level corresponding to the lateral drifts derived from cracks parameters. The calculated results, obtained by applying the analytical approach, were compared with sixteen reinforced concrete columns tests results[1),2)]. The specimens were designed by considering main parameters contributing in shear failure of conventional reinforced concrete columns.

2. CRACK WIDTH AND LATERAL DRIFTS RELATIONSHIP

Lateral drifts of the reinforced concrete columns can be obtained based on the crack widths and the crack angles, appeared in the face of the columns, by dividing deformations in two categories: flexural deformation and shear deformation. Flexural cracks usually appeared before the shear cracks on the face of the columns at the two ends, tension sides, horizontally. As the drift increases, the flexural cracks develop until shear failure occurs which is followed by complete failure. Therefore it is likely to see the decreasing of the flexural crack widths and increasing the shear cracks at this time. The widths of shear cracks could be directly related to lateral drifts, which are signs of capacity loss states of the members.

2.1 DRIFTS DUE TO FLEXURAL CRACKS

Flexural cracks can be evaluated based on curvature moment analysis. The general equation for computing the lateral drifts due to flexural mechanism for beams is expressed as:

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*1 Earthquake Research Institute, University of Tokyo, PhD Student,
*2 Earthquake Research Institute, University of Tokyo, Dr. Eng., Professor

$$\delta = \int \phi x dx \tag{1}$$

where, δ= lateral drifts, ϕ= curvatures and x= a variable changes from 0 to L which is the length of the column. In which curvature ϕ is expressed by equation (2):

$$\phi = \frac{\varepsilon_s}{d-c} \tag{2}$$

where, ε_s= tension steel strain, d= effective depth of tension steel and c= distance from extreme compression fiber to neutral axis of reinforce concrete column sections.

Since the tension strains of concrete can be neglected it can be assumed that flexural crack widths ratio is related to ε_s by,

$$\frac{D_f}{L} \approx f(\varepsilon_s) \quad or \quad \frac{D_f}{L} \approx \frac{B-c}{d-c}\varepsilon_s \Rightarrow D_f \approx L\frac{B-c}{d-c}\varepsilon_s \tag{3}$$

where, B= section width, and D_f= crack width. Since, usually maximum flexural cracks widths at the bottom of the sections are measured; ε_s in equation (3) is multiplied by (*B-c/d-c*). Therefore, it could be acceptable to relate cracks widths to the curvatures (after substituting (2) in equation (3)) by:

$$D_f \approx \frac{B-c}{d-c}L\varepsilon_s \Rightarrow D_f \approx \frac{B-c}{d-c}L(d-c)\phi \Rightarrow D_f = (B-c)L\phi \quad or\,\phi = \frac{D_f}{(B-c)L} \tag{4}$$

Considering equation (1), lateral drifts can be derived:

$$\delta_f = \int \phi x dx = \int_0^L \frac{D_f}{(B-c)L} x dx = \frac{L}{2(B-c)}\sum D_{f\,i} \tag{5}$$

where, δ_f= lateral drifts due to flexural cracks, and $\sum D_{fi}$ is the sum of the flexural cracks along the edges of the column. For sake of simplification, it could be acceptable assumption to presume B-c= B/2. Therefore the total lateral flexural drifts can be obtained by:

$$\delta_f = \frac{\sum D_{f\,i}}{B} L \tag{6}$$

2.2 DRIFTS DUE TO SHEAR CRACKS

Relationship between shear cracks and lateral drifts in reinforced concrete columns was derived based on shear cracks widths and cracks angles. Fig. 1 shows shear cracks and the horizontal component of the crack width, which is assumed to be equal to the resultants of shear diagonal concrete and stirrups deformation. In Fig.1 D_c is the crack width of an individual crack. Based on relationship illustrated in Fig. 1, δ_{shear}, lateral drifts due to shear deformation can be related to the summation of the horizontal components of shear cracks.

$$\delta_{shear} = \frac{\sum Dc_i}{\sin \beta_i} \tag{7}$$

where βi is individual shear crack angle to horizontal corresponding to the crack width D_{ci}

2.3 TOTAL DRIFTS DUE TO SHEAR AND FLEXURAL CRACKS

Total lateral drift can be estimated by combining equation (6) and (7) which gives,

$$\Delta = \frac{\sum D_{ci}}{\sin \beta_i} + L\frac{\sum D_{fi}}{B} \tag{8}$$

where, $\sum Dc_i$ = sum of the shear crack widths appeared on one face of the columns in the load direction, β_i = the individual shear cracks angles to horizontal direction, $\sum D_{fi}$ = sum of flexural cracks widths, L= length of the column, B= width of the column's section on the side that flexural cracks appeared and Δ= total lateral drift. Elastic deformation of the columns must be added to the lateral drift obtained by the above equation.

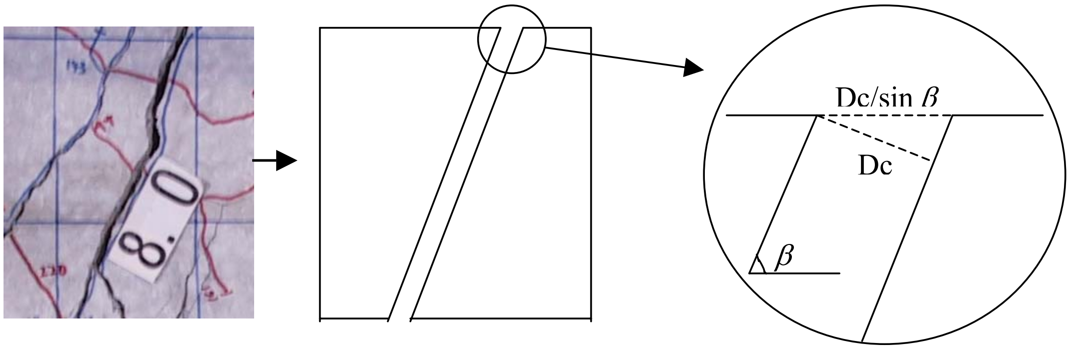

Fig.1 Shear crack widths in specimen[1)] No.1-2002 and shear cracks parameters in equation (7)

In order to evaluate the reliability of the expression, shear deformations of sixteen reinforced concrete columns specimens subjected to static cyclic unidirectional reverse lateral load and applied axial load with different shear capacities, were computed using equation (8) and the outputs were compared with the test results (see Fig.2).

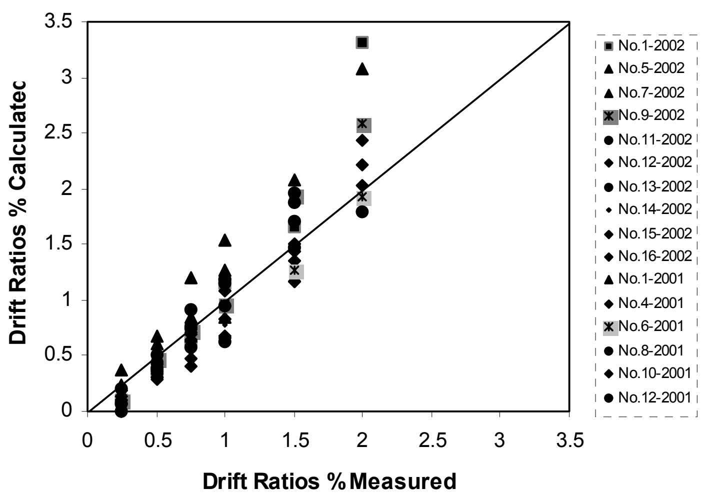

Fig.2 Calculated and measured shear drifts ratios of sixteen tested reinforced concrete columns

Fig. 2 demonstrates a close correlation between the test results and the analytical outputs. The different values between measured and calculated shear drift ratios were calculated and the averages and standard deviations were obtained, for each step of shear drift, based on a statistical analysis. The results of the analysis are listed in table 1. The results show less accuracy for large drifts. As the shear drifts increase, cracks spread through the whole column's face and cracks pattern becomes more complex therefore cracks parameters might be measured with less accuracy as well as lateral drifts. The standard deviation at shear drift equal to 0.25 might be improved by adding the elastic drifts of the columns.

Table 1. Statistical Analysis Results of Different Between Measured and Calculated Shear Drift Ratios

Drift Ratios	0.25	0.5	0.75	1	1.5	2
Average Values	0.055678	0.090411	0.046939	0.024075	0.07199	0.42299
Standard Deviations	0.162959	0.107387	0.186219	0.246991	0.291076	0.510038

3. TRUSS MECHANISM ANALYSIS BASED ON CRACK ANGLES

Truss mechanism is believed to be the major causes of shear drifts. The shear distortions, in the web of reinforced concrete columns, can be approximated by an analogous truss. Horizontal stirrups and α degree diagonal concrete struts are assumed to form the web members. The analogous truss shown in Fig. 3 depicts a conventional column with a web horizontal transverse reinforcement. The inclination of the diagonal compression struts, resisting a force C_d and running parallel to diagonal cracks, is assumed equal to the shear crack angle α to the vertical. The external shear force V_s, which is the same as the proportion of shear force resisting by truss mechanism, is equal to the resultant of all stirrup forces across the diagonal crack. $\Delta T'$ is the bond force or difference of tension forces between two truss sections. Horizontal displacement component due to truss mechanism can be calculated as:

$$\frac{\Delta_{ht}}{jd\cot\alpha} \approx \frac{\Delta_s}{\cot\alpha jd} + \frac{\Delta_c}{jd\cos\alpha} \tag{9}$$

where, Δ_s= stirrups deformation, Δ_c= strut deformation, Δ_{ht}= total lateral shear drift, and jd= distance from centroid of compressive forces in the steel and concrete to the centroid of the tension force (Fig. 3). After applying deformation compatibility relationships and equilibrium equations, equivalent initial shear stiffness of the truss mechanism can be obtained as equation (10).

$$K_t = b_w E_s jd \frac{\rho_{sw}\cos^2\alpha \ \ \sin^2\alpha}{\sin^4\alpha + \rho_{sw}(E_s / E_c)} \tag{10}$$

where, b_w = width of section, $\rho_{sw} = A_v / b_w s$, s= stirrups spacing, A_v = stirrups area. Consequently, the total shear force-shear drift relationship can be summarized as equation (11).

$$V_s = K_t \Delta_{ht} \tag{11}$$

V_s: External shear force (Fig. 3), Δ_{ht}: Total shear drift related by equation (9), and K_t: equivalent initial shear stiffness calculated by equation (10).

Inelastic shear behavior of truss mechanism can be modeled by introducing two nonlinear springs (see Fig. 3), Ks represents as a transverse reinforcement spring which can be modeled by a normal bilinear hysteretic model and Kc functions as a concrete strut spring with a constitutive law defined by equation 12 (see Fig. 4).

$$f_c = \nu_o f_c' \left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002}\right)^2 \right] \quad if \quad \varepsilon_c \le 0.002 \tag{12-a}$$

$$f_c = \nu_o f_c' (1 - Z(\varepsilon_c - 0.002)) \quad if \quad 0.002 \le \varepsilon_c \le \varepsilon_{20c} \tag{12-b}$$

in which

$$Z = \frac{0.5}{\varepsilon_{50u} + \varepsilon_{50h} - 0.002} \quad , \quad \varepsilon_{50u} = \frac{0.2109 + 0.002 f_c'}{f_c' - 70.31} \quad (f_c' : kgf/cm^2) \text{ and } \varepsilon_{50h} = a_w \bar{\rho}_w \left(\frac{\bar{b}}{s_w}\right)^{1/b_w}$$

Where f_c'= concrete strength in direction of the compression stress kg/cm^2 [4], ν_o= is efficient factor for compression strength in direction of the strut obtained from AIJ code of Japan. ρ_w=ratio of volume of transverse reinforcements to volume of concrete core measured to outside of hoops, $\bar{b}$=width of confined core measured to outside of hoops, s_w = spacing of hoops, bw=2, aw=0.75 (constant factors for the mechanism), and Z= slope of the assumed linear falling branch in Fig. 4. A reduction factor is multiplied to ε_{50u} and ε_{50h} parameters in the equation. The reduction factor is depended on loading type (hysteretic characteristics), mainly effect of tension bars stress reduction (after shear failure) due to the axial load effects and compression failure mechanism, which can be obtained from a nonlinear curvature-moment analysis. In this study, it was assumed to be equal to reduction tension stresses ratio in steel bars due to applying axial load. Fig. 5 shows the correlation between experimental results and analytical outputs, based on the presented approach, of lateral drift ratios at the load level 50% of the maximum lateral load capacity of the columns after shear failure for the sixteen tested columns listed in Table 2. The average and standard deviation for the different values between experimental and analytical results of shear drift ratios are 1% and 0.195 respectively. The computed results show appropriate agreement with the test outputs. Comparison of output results of specimen[1)] No.11/2002 to No. 14/2002 implies that energy dissipation is another factor that has to be considered in the presented mechanism.

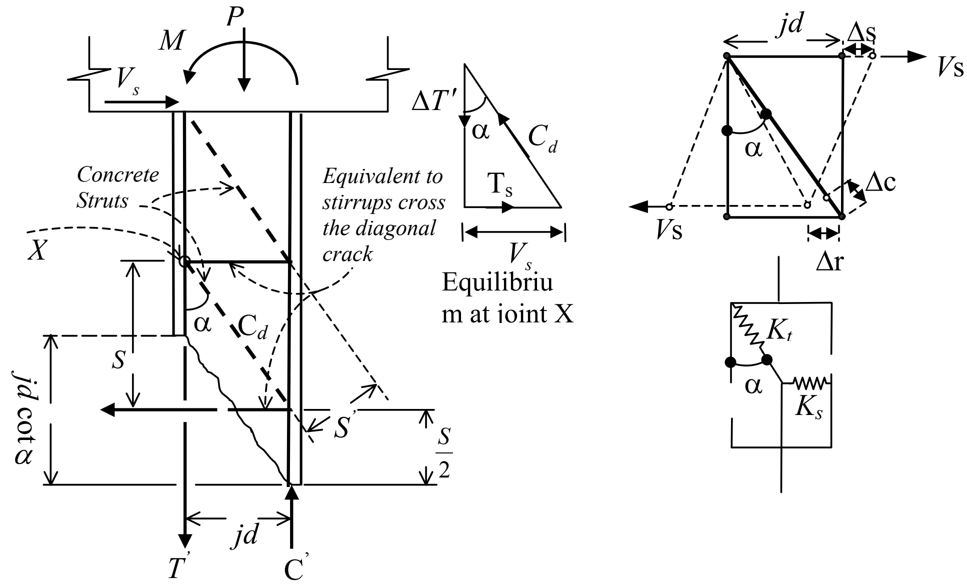

Fig. 3 Shear distortions in the web of a reinforced concrete column, and the analogous truss.

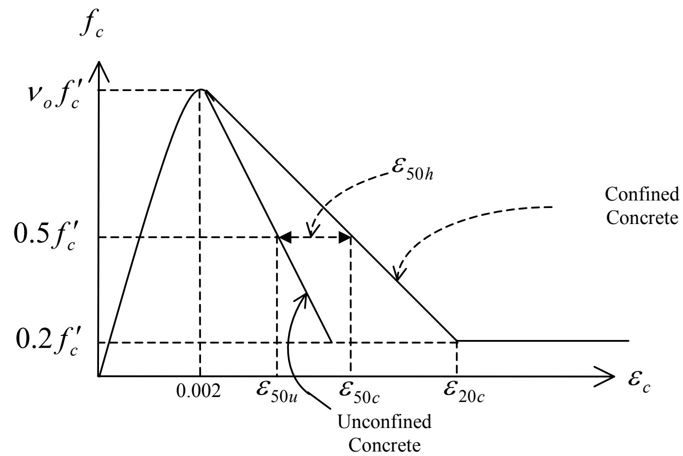

Fig. 4 Stress-strain curve for concrete confined by rectangular hoops, Kent and Park

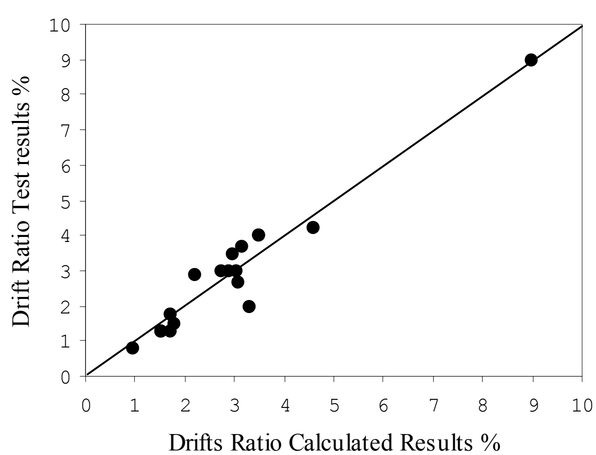

Fig. 5 Correlation between experimental and analytical results of lateral drift ratios at the load level 50% of maximum lateral load capacity of the columns after shear failure for the sixteen tested columns listed in table (2)

Table 2 Materials and characteristics of the specimens[1),2)].

<table>
<tr><th></th><th>Section Dimension</th><th>Heights mm</th><th>σ B kg/cm2</th><th>Main Bars</th><th>Hoops</th><th>Axial Load ratio</th></tr>
<tr><td>No.1/2002</td><td rowspan="16">300×300mm</td><td rowspan="2">600</td><td>280</td><td rowspan="2">12-D13</td><td rowspan="3">2-D6@50</td><td>0.21</td></tr>
<tr><td>No.16/2002</td><td>261</td><td>0.23</td></tr>
<tr><td>No.5/2002</td><td>750</td><td rowspan="2">296.5</td><td>12-D16</td><td>0.2</td></tr>
<tr><td>No.7/2002</td><td rowspan="13">900</td><td rowspan="2">12-13φ</td><td rowspan="2">2-4φ@100</td><td>0.2</td></tr>
<tr><td>No.9/2002</td><td rowspan="3">281.5</td><td>0.2</td></tr>
<tr><td>No.11/2002</td><td rowspan="5">16-D13</td><td rowspan="2">2-D6 @150</td><td>0.23</td></tr>
<tr><td>No.12/2002</td><td>0.23</td></tr>
<tr><td>No.13/2002</td><td rowspan="3">261</td><td rowspan="2">2-D6@50</td><td>0.23</td></tr>
<tr><td>No.14/2002</td><td>0.23</td></tr>
<tr><td>No.15/2002</td><td>4-D6@50</td><td>0.23</td></tr>
<tr><td>No.1/2001</td><td rowspan="3">135</td><td rowspan="6">12-D13</td><td>2-5φ@160</td><td>0.3</td></tr>
<tr><td>No.4/2001</td><td rowspan="5">2-D6@75</td><td>0.3</td></tr>
<tr><td>No.6/2001</td><td>-0.15~+0.85</td></tr>
<tr><td>No.8/2001</td><td rowspan="3">180</td><td>0.3</td></tr>
<tr><td>No.10/2001</td><td>-0.15~+0.85</td></tr>
<tr><td>No.12/2001</td><td>0.2</td></tr>
</table>

4. CONCLUSIONS

Two crack characteristics were applied to estimate the shear drifts of RC columns; crack width, which was applied in the first part of this study in order to obtain the shear drift, proportionately from summation of crack widths, and crack angle which was employed in a truss mechanism to estimate the shear drift and it's relationship with shear force. The main scope of the study was to employ the collaboration of two methods in a process of post-earthquake damage evaluation. Using the first approach the maximum approximate lateral drift that experienced by a column during an earthquake might be estimated and the shear force level of the RC columns would be obtained by the second approach applying the force-deformation relationship. The results of shear drifts estimation using the crack widths for the tested columns show that as the shear drifts increases, cracks spread through the whole column's face and cracks pattern becomes more complex therefore cracks parameters might be measured with less accuracy as well as lateral drifts. In order to obtain more accurate results in large lateral drifts and their relation with residual cracks widths, further study is needed. In the second part of study the results of lateral drifts at the 50% shear capacity after shear failure were obtained based on the analysis and compared with the experimental results. Study showed that a modified truss mechanism could be applied to estimate reliable lateral drifts results at damage states.

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