- Technical paper -

CONTROL OF DIAGONAL CRACKING IN PARTIALLY PRESTRESSED CONCRETE BEAMS

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ABSTRACT: An experimental program was conducted to investigate diagonal cracking behavior in reinforced and partially prestressed concrete beams under static loading by focusing on the influences of compressive stress in concrete due to prestress and stirrup spacing on maximum diagonal crack widths. Relationship between diagonal crack widths and crack displacements in stirrup direction was also examined. Test results have shown that crack displacements along stirrup direction had a linear relationship with stirrup strains. In partially prestressed beams, prestressing bars were found to have an important effect on controlling shear crack widths apart from the amount of stirrups.
KEYWORDS: diagonal cracking, stirrup, partially prestressed concrete, crack width

1. INTRODUCTION

Cracks of excessive width in concrete structures may be safe from a viewpoint of strength design but are often unacceptable for serviceability considerations as they are unsightly and may lead to a public concern. Further, wide cracks allow water to penetrate into the structures and may cause corrosion of embedded steel reinforcements, which adversely affect the long-term durability performance. Extensive research studies on the cracking behavior of reinforced concrete (RC) and partially prestressed concrete (PPC or PRC) beams have been conducted over the last 50 years. Most of them, however, are concerned with the flexural cracking behavior; little work has been done on the diagonal cracking [1].

Mechanism of diagonal cracking is more complex than cracking due to axial tension or bending because a diagonal crack is generally not perpendicular to the web reinforcement. Previous studies [2-4] have shown that strain in web reinforcement is the most important factor affecting the diagonal cracking in RC beams. Such parameters include the characteristic of web reinforcement (area, spacing, size, angle with member axis and bond property), concrete strength, web width and shear span-to-effective depth ratio. In the study by Hassan et al. [2], the crack displacement in stirrup direction ($w_{c,w}$) was found to have a close relationship with the stirrup strain. To estimate the diagonal crack width (w_c), the ratio of $w_c/w_{c,w}$ was proposed and can be determined from the experimental results.

Another concept for the diagonal cracking is that the diagonal crack width can be estimated by multiplying the normal strain perpendicular to the diagonal crack by the diagonal crack spacing [1]. The diagonal crack spacing is mainly dependent on the longitudinal and transverse crack spacing, which can be calculated from the method to determine crack spacing in a member subjected to uniaxial tension using the characteristics of longitudinal and transverse reinforcement, respectively. The recent study by Zararis [5] also supports this concept by indicating that the amount of web reinforcement may not be the only factor for adequate control of diagonal cracking. It was pointed out that the amount of longitudinal reinforcement can have a significant effect on the opening of critical diagonal cracks.

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Table 1 Test variables

Beam	Type	Nonprestressed reinforcement		Prestressing bar (d_p = 150 mm)	$\sigma_{c,ps}$ (MPa)	Stirrup spacing, s (mm)	f_c' (MPa)
		Top	Bottom				
S-1	RC	4D16 (d_s' = 40 mm)	4D29 d_s = 250 mm (ρ_s = 3.43 %)	-	-	100 (p_w = 0.21%)	35.3
S-2						200 (p_w = 0.11 %)	37.6
S-3	PRC			1 ϕ19 mm	2.0	100	41.1
S-4						200	41.1
S-5				1 ϕ26 mm	4.0	100	44.0
S-6						200	40.9

$\sigma_{c,ps}$: compressive stress in concrete section due to prestress

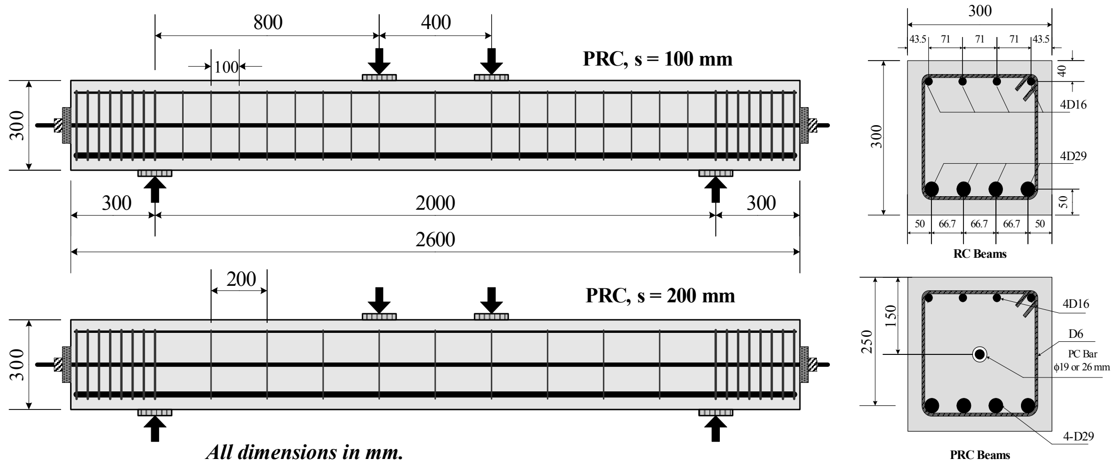


Fig. 1 Layout and cross-sectional details of specimens

Based on the literature review, it is clear that there is room for doubt on the mechanism of diagonal cracking, particularly the influential parameters affecting diagonal crack width. In addition, available experimental data on diagonal cracking in PRC beams are very limited compared to those on flexural cracking. The main objective of this study is to experimentally investigate diagonal cracking behavior in RC and PRC beams, with emphasis on the effects of compressive stress in concrete due to prestress and stirrup spacing on maximum diagonal crack width.

2. EXPERIMENTAL PROGRAM

Test specimens consisted of two RC and four PRC beams, having a rectangular cross-section of 300x300 mm and a total length of 2.6 m. The typical layout and cross-section details of specimens are shown in **Fig. 1**. All test specimens had the same tension and compression nonprestressed reinforcements, 4-D29 and 4-D16, respectively. Closed-shaped and vertical stirrups made of 6-mm deformed bars were provided as shear reinforcement. Ready mixed concrete with a maximum size of aggregate of 20 mm was used. The test variables are summarized as shown in **Table 1**. In RC specimens (beams S-1 and S-2), all parameters were identical except for the stirrup spacing: 100 and 200 mm for S-1 and S-2, respectively. These correspond to the web reinforcement ratios (p_w) of 0.21 and 0.11%. In PRC specimens, prestressing bars were provided at the mid-depth of

Table 2 Properties of reinforcements

Type of bar	Diameter	Type	Area (mm^2)	f_y (MPa)	f_u (MPa)	E_s x 10^3 (MPa)
Deformed bar	D6	SD345	31.7	359.0	-	206.7
	D16		198.6	382.0	-	207.3
	D29		642.4	403.5	-	198.7
PC bar	ϕ19	SBPR 1080/1230	283.5	1199	1276	201.0
	ϕ26		530.9	1214	1265	201.0

E_s: Young's modulus, f_y: yield strength, f_u: tensile strength

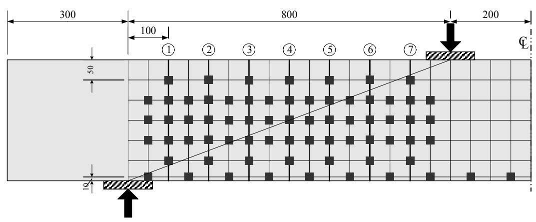


Fig. 2 Location of contact points (s = 100 mm)

concrete section and were straight-lined along the beam length. The level of prestress was varied to produce different compressive stress in concrete section ($\sigma_{c,ps}$) of 2.0 and 4.0 MPa. The mechanical properties of reinforcements used are given in **Table 2**.

Four-point symmetrical loading with a distance between loading points of 400 mm, giving a shear span-to-effective depth ratio (a/d) of 3.2, was statically applied to all specimens. Before conducting the loading test, contact points (or Demec points) were mounted on concrete surface in shear-span regions (see **Fig. 2**) Two sets of contact points with an interval of 100 mm were used to measure the crack displacements in horizontal and vertical directions with an accuracy of 0.001 mm. By measuring the inclinations of diagonal cracks to the member axis, shear crack widths (w_c) can be determined as shown in **Fig. 3**. A pocket-type microscope with a precision of 0.01 mm was also used to measure diagonal crack widths, especially in case that two or more shear cracks penetrate into one set of contact points. Strains in each stirrup were taken from electrical resistance strain gages, which were attached at three different levels with an interval of 50 mm as shown in **Fig. 4**.

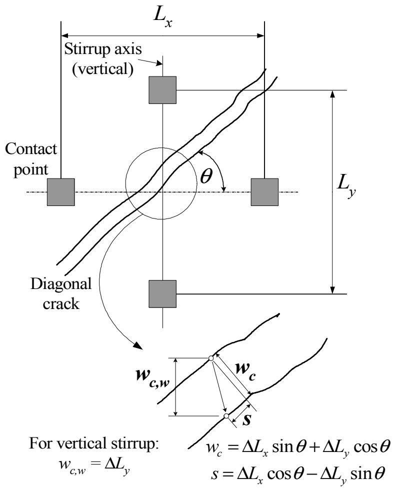


Fig. 3 Evaluation of diagonal crack width (w_c)

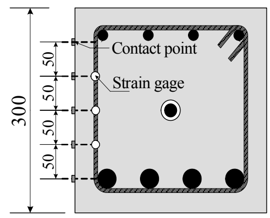


Fig. 4 Location of strain gages for stirrups

3. TEST RESULTS AND DISCUSSION

3.1 CRACK PATTERNS

Crack patterns at failure of all specimens are shown in **Fig. 5**. It can be seen that flexural cracks developed in the mid-span region between loading points and in the shear-span regions where diagonal cracks occurred subsequently. With increasing load, diagonal cracks rapidly widened and propagated towards the concrete compression zone under the loading points, whereas the widths of flexural cracks in mid-span region were almost constant. The inclinations of diagonal cracks were taken from the average value of crack angles measured from several locations where diagonal cracks intersect the grid lines in shear-span regions. It can be seen from **Fig. 5** that the inclinations of diagonal cracks slightly decrease in beams with higher compressive stress in concrete due to prestress and the effect of stirrup spacing was found to be insignificant. The failure mode of beams S-1 and S-2 (RC) was shear- diagonal-tension type, while it was shear- compression failure in beam S-3. All the other beams failed in flexural-compression mode with yielding of tension reinforcements and occurrence of wide diagonal cracks in shear-span regions.

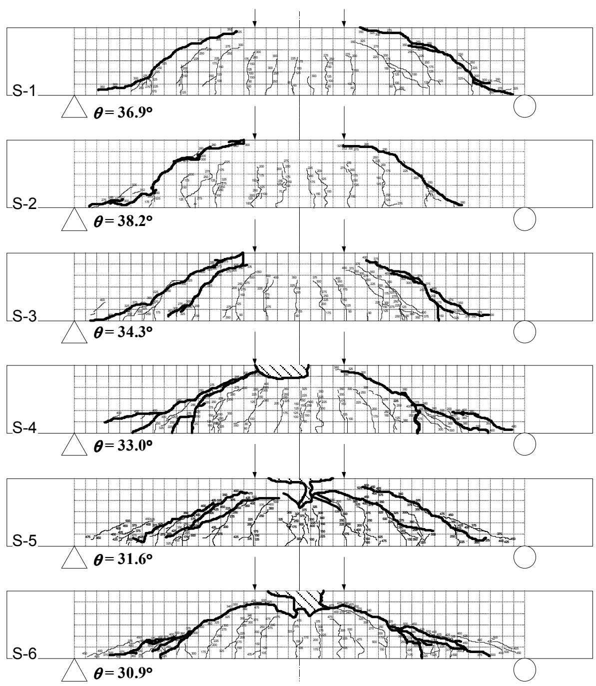
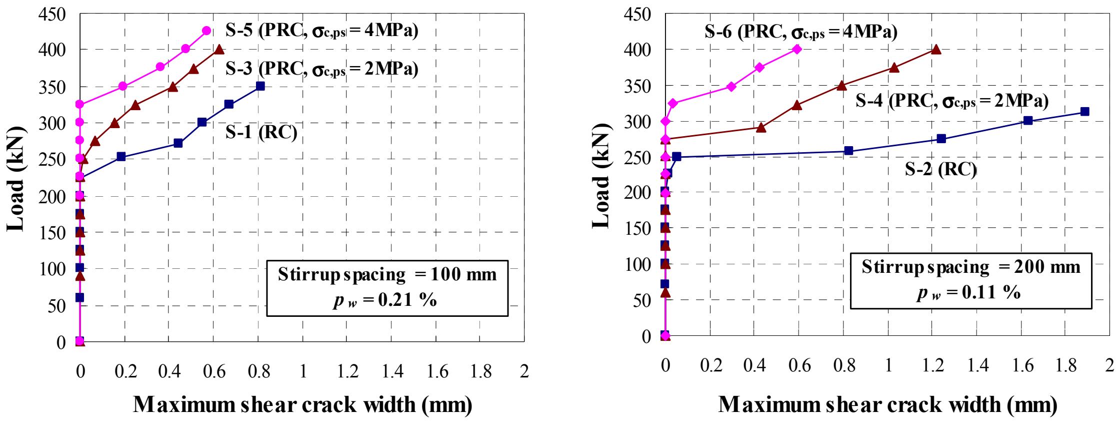


Fig. 5 Crack patterns

3.2 SHEAR CRACK WIDTH (w_c)

Fig. 6(a) shows the relationship between load and maximum shear crack width in beams with stirrup spacing of 100mm (p_w= 0.21%). The load when the crack width starts to increase can be considered



(a) s = 100 mm (b) s = 200 mm

Fig. 6 Load vs. maximum shear crack width

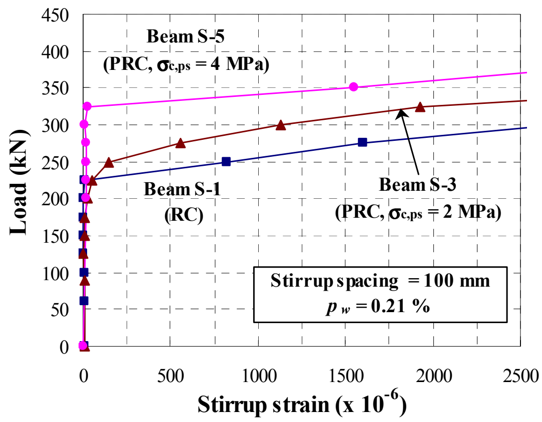


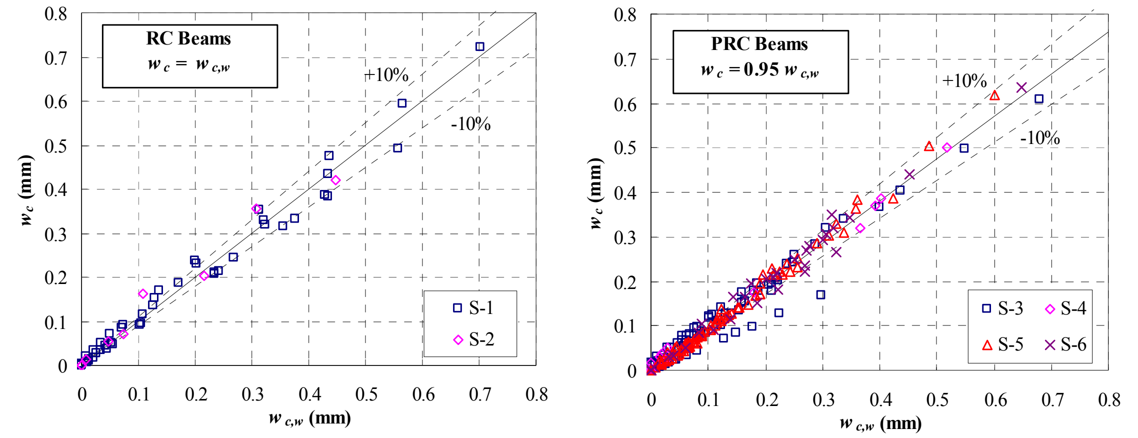
Fig. 7 Load vs. stirrup strain (*s*=100 mm)

as the diagonal-cracking load. It can be seen that the higher the compressive stress in concrete due to prestress ($\sigma_{c,ps}$), the greater the diagonal-cracking load. In addition, a higher $\sigma_{c,ps}$ causes a reduction in the maximum shear crack width at the same load level. This clearly indicates the effectiveness of prestress on controlling diagonal crack widths in PRC beams. Similarly, the impact of $\sigma_{c,ps}$ on shear crack widths was also observed in specimens with larger stirrup spacing (*s*=200mm), as shown in **Fig. 6(b)**. It is interesting to note that such influence of $\sigma_{c,ps}$ on reducing shear crack widths becomes more pronounced. This may be due to the larger stirrup spacing (or, smaller stirrup ratio) used in these beams, hence, the $\sigma_{c,ps}$ plays an important role in resisting shear forces induced by external load and effectively controls the shear crack widths..

By comparing **Fig. 6(a)** with **6(b)**, the effect of stirrup spacing on maximum shear crack widths can be investigated. In RC specimens, beam S-1 with closer stirrup spacing (s = 100mm) showed a lower rate of increase in shear crack widths than that of beam S-2 (s = 200mm). In PRC specimens, however, the influence of smaller stirrup spacing on reducing shear crack widths was found to be less pronounced, particularly in specimens having $\sigma_{c,ps}$ = 4MPa (S-5 and S-6). Relationship between load and stirrup strain (ε_w) in beams with stirrup spacing of 100 mm is shown in **Fig. 7**. It can be seen that although beam with a higher $\sigma_{c,ps}$ yields a smaller ε_w at a particular load, the increasing rates of ε_w are almost identical in both RC (S-1) and PRC (S-3, S-5) specimens. This implies that the $\sigma_{c,ps}$ has an influence only on increasing the diagonal cracking loads; after the occurrence of diagonal cracks, the increase in stirrup strains seems not to be affected by the compressive stress in concrete due to prestress.

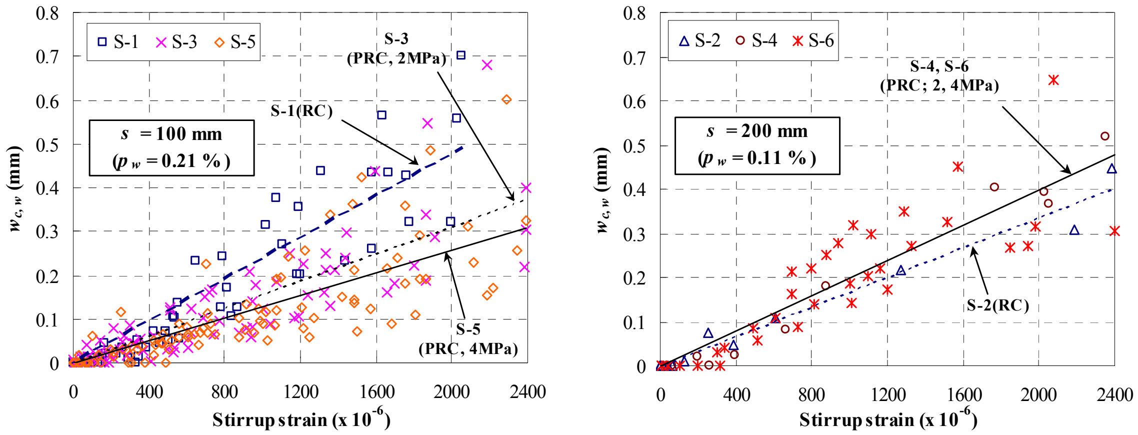
3.3 CRACK DISPLACEMENT IN STIRRUP DIRECTION ($w_{c,w}$)

Because diagonal crack is not perpendicular to stirrups, shear crack width may not be equal to crack displacement in stirrup direction. Previous study by Hassan et al. [2] has shown that, in RC beams subjected to static and fatigue loading, the ratio of $w_c/w_{c,w}$ was mainly influenced by the angle of stirrup and was proposed to be 1.2 for beams with vertical stirrup. To verify this concept, particularly for PRC beams, the relationships between w_c and $w_{c,w}$ of RC and PRC beams are shown in **Figs. 8(a)** and **(b)**, respectively. It can be seen that while the ratio of $w_c/w_{c,w}$ is approximately 1.0 for RC beams, it is slightly lower in case of PRC beams (= 0.95). As such, it can be concluded that the ratio of $w_c/w_{c,w}$ is not significantly affected by the $\sigma_{c,ps}$ and can be assumed to be 1.0 for the specimens used in this study. This means that shear crack widths can be directly estimated from crack displacements in stirrup direction, which have a close relationship with stirrup strains. This will be discussed in the following section.



(a) RC Beams (b) PRC Beams

Fig. 8 Shear crack width (w_c) vs. crack displacement in stirrup direction ($w_{c,w}$)



(a) s = 100 mm (b) s = 200 mm

Fig. 9 Crack displacement in stirrup direction vs. stirrup strain

Crack displacement in stirrup direction ($w_{c,w}$) is basically equal to the elongation of stirrup minus the elongation of concrete in stirrup direction. Since the concrete elongation is relatively small and can be neglected, $w_{c,w}$ is largely dependent on stirrup strains [2]. Cracking behavior of concrete is intrinsically random and highly uncertain, hence, it is by no means possible to obtain stirrup strains at locations where diagonal cracks cross stirrups. In this study, all available stirrup strains at the locations near diagonal cracks were used to plot the relationship between $w_{c,w}$ and ε_w as shown in **Figs. 9(a)** and **(b)** for specimens with stirrup spacing of 100 and 200mm, respectively.

It can be observed from **Fig. 9(a)** that, at the same stirrup strain, $w_{c,w}$ shows a significant variation and greatly differs in each beam. Fitting curves obtained from a linear regression analysis represent the relationship between the average values of $w_{c,w}$ and ε_w. Clearly, $w_{c,w}$ increases with ε_w in a linear manner but with a different rate of increase. Beam S-1 (RC) shows the highest rate of increase, implying that crack widths are larger compared to those of PRC beams (S-3, S-5), both of which have nearly the same relationship. The reason may be such that prestressing bars, which are provided at the mid-depth of concrete sections, may actively restrain the opening of diagonal cracks in longitudinal direction, thereby reducing diagonal crack widths. Surprisingly, it can be observed from **Fig. 9(b)** that beam S-2 (RC) registers the least $w_{c,w}$ compared to that of PRC beams (S-4, S-6) at the same stirrup strain. This may be caused by errors in obtaining the stirrup strains in beam S-2, in which stirrups yielded rapidly after the occurrence of diagonal cracks. The effect of $\sigma_{c,ps}$ on the $w_{c,w}$-ε_w relationship in beams with s=200 mm (S-4 and S-6) was also found to be insignificant, similarly to the case of beams with s=100 mm (S-3 and S-5).

Fig. 10 shows the relationship between maximum $w_{c,w}$ and ε_w of all specimens, except for beam S-2. It is apparent that the maximum $w_{c,w}$ is linearly dependent on the ε_w in both RC and PRC beams and, at the same stirrup strain, a smaller maximum $w_{c,w}$ is obtained in case of PRC beams. Since the impact of $\sigma_{c,ps}$ on the maximum $w_{c,w}$ is found to be insignificant, the difference in $w_{c,w}$-ε_w relationships between RC and PRC beams is considered to be caused by the provision of prestressing bars at the mid-depth of concrete section. To incorporate this effect, a modification factor (K_{ps}) was proposed to be a function of diameter (ϕ_{ps}) and effective reinforcement ratio of prestressing bar ($\rho_{r,ps}$).

$$K_{ps} = \frac{\rho_{r,ps}}{3.67\times10^{-4}\phi_{ps}} \quad ;\rho_{r,ps} = A_{ps} / A_{e,ps} \qquad (1)$$

where $A_{e,ps}$ is the effective concrete area for prestressing bar as shown in **Fig. 11**. By multiplying the maximum $w_{c,w}$ with K_{ps}, the modified maximum $w_{c,w}$ for PRC beams can be calculated and plotted as shown in **Fig. 10**. It should be noted that the modification factor K_{ps} was determined to obtain a better fitting curve for the test specimens used in this study. Since the test variables and number of specimens are limited, it is recommended that further research be carried out to examine other parameters, such as type of prestressing steel, distribution of compressive stress due to prestress and arrangement of prestressing steel.

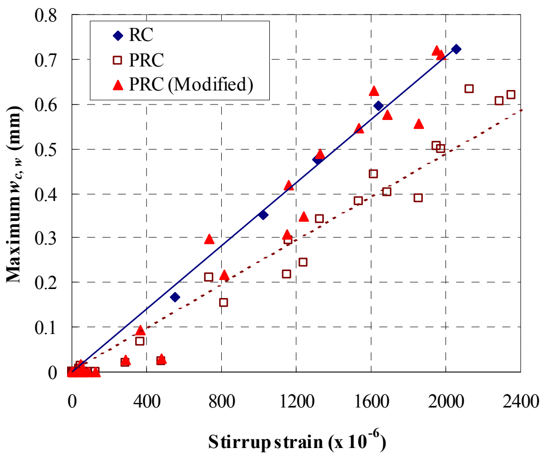


Fig. 10 Maximum $w_{c,w}$ vs. stirrup strain

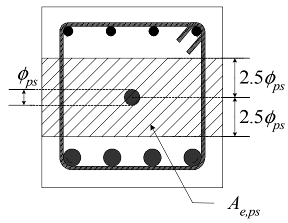


Fig. 11 Effective concrete area for prestressing bar ($A_{e,ps}$)

4. CONCLUDING REMARK

In order to study the effects of compressive stress in concrete due to prestress ($\sigma_{c,ps}$) and stirrup spacing on the diagonal crack width (w_c), a test program was conducted on RC and PRC beams under static loading. It was found that crack displacements in stirrup direction ($w_{c,w}$) increased with stirrup strains (ε_w) in a linear manner and the ratio of $w_{c,w}/w_c$ was observed to be approximately 1.0 for both RC and PRC beams. Use of closer stirrup spacing can greatly reduced shear crack widths in RC specimens, however, such effect was less pronounced in PRC specimens. By applying a higher $\sigma_{c,ps}$, an increase in diagonal cracking loads was obtained; nevertheless, it had little influence on the $w_{c,w}$-ε_w relationship. The presence of longitudinal prestressing bars at mid-depth of concrete section can significantly reduced the diagonal crack widths. To account for this impact of prestressing bars, a modification factor K_{ps} was proposed to be a function of diameter and effective reinforcement ratio of prestressing bar.

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