# DRIFT AT AXIAL LOAD FAILURE UNDER CRACKED－REGION－SWAY MECHANISM FOR REINFORCED CONCRETE COLUMNS 

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#### Abstract

A simple method to evaluate the lateral drift at axial failure for reinforced concrete columns is suggested．The analysis is based on a modification of an existing analytical procedure combined to test results．The theory of fracture mechanics is applied to evaluate the critical damaged zone of the reinforced concrete columns which are supposed to be subjected to axial and lateral loadings．Comparison of the analytical results to the results obtained from tests showed a fair correlation．Another particularity in collapse modes，which may appear similar to the one supposed in the analysis was also pointed out for convenience．


Keywords：axial collapse，column，fracture band，lateral drift，mechanism identification

## 1．INTRODUCTION

In earthquake engineering，collapse refers to the stage at which structural elements or systems are unable to sustain the applied loads during or after a seismic event．Collapse mechanisms have been found complex and are still not well understood．Experimental research and post－earthquake reconnaissance have identified that reinforced concrete columns with light or widely spaced transverse reinforcement are susceptible to shear failure during earthquakes and consequently to a reduction in the axial load capacity．However，reconnaissance of recent earthquakes $[1,2]$ confirmed that structural elements can experience significant damage， including shear failures without collapse of the elements or the entire system．Investigations on the process of collapse or the collapse of isolated elements have recently increased and few methods have been developed［3，4，5］．The method proposed by Uchida and Uezono for the failure－mechanism （presented later as Type－II）［4］is improved herein by giving a procedure to define the height $l_{c}$ of the damaged zone through test results and by introducing the participation of the concrete， which was omitted by Uchida and Uezono，in the evaluation of the internal work．

## 2．MINIMIZATION ENERGY BASED MODEL

The model proposed by Uchida and Uezono
is based on the minimization of the developed energy and the equilibrium of the internal and external works developed under an axial load $N_{c r}$ at the ultimate stage of axial load failure，which corresponds to the time when the column has no lateral strength．This model assumes the failure to occur when a plastic hinge mechanism develops on the longitudinal reinforcement inducing a considerable elongation of the transverse reinforcement．Such mechanism can be formed along a clear diagonal crack and called Type I，or through an inclined damaged region，called Type II，which is the subject of this study and illustrated in Fig．1．The virtual works，respectively，external $\delta W_{e x}$ and internal $\delta W_{i n}$ ，developed by the different loads and components of the columns are expressed by


Fig． 1 Failure mechanism Type－II

[^0]\[

$$
\begin{align*}
& \delta W_{e x}=N_{c r} \delta v  \tag{1}\\
& \delta W_{i n}=\delta W_{c}+\delta W_{w}+\delta W_{s} \tag{2}
\end{align*}
$$
\]

where $\delta v$ is the vertical displacement, $\delta W_{c}$ is the virtual work developed by the concrete through friction and interlocking, $\delta W_{w}$ is the virtual work developed by the transverse reinforcement due to the elongation, and $\delta W_{s}$ is the virtual work developed by the longitudinal reinforcement through the plastic hinges at the extreme parts of the damaged region. Uchida and Uezono neglected the effect of interlocking in the concrete for reinforced concrete columns and considered the part $\delta W_{c}=0$, however, herein, its effect is considered.

The part of the internal work developed by the longitudinal steel bars, of number $n_{s}$, diameter $d_{b}$ and yield strength $f_{s y}$, due to the rotation $\theta^{\prime}$ of the plastic hinges at both ends of each bar in the damaged region when the axial failure is initiated, is expressed by

$$
\begin{align*}
& \delta W_{s}=2 n_{s} M_{p} \delta \theta^{\prime}  \tag{3}\\
& M_{p}=\frac{1}{6} d_{b}^{3} f_{s y} \tag{4}
\end{align*}
$$

The work developed by the hoops, due to their elongation $\delta l_{i}$, depends on the position $i$ of each hoop in the damaged region (Fig.2). The number of the participating hoops $n_{w}$ depends on the plane-inclination $\theta$ and the dimension of the damaged region $l_{c}$.

$$
\begin{equation*}
\delta W_{w}=\rho_{w} b S_{t} f_{w y} \sum_{i=1}^{n_{i}} \delta l_{i} \tag{5}
\end{equation*}
$$

where

$$
\begin{align*}
& \delta l_{i}= \begin{cases}\left(i S_{t}-R_{w}\right) \sin \theta^{\prime} & \text { if iS } S_{t}<L_{w}, i S_{t}<R_{w}+l_{c} \\
l_{c} \sin \theta^{\prime} & \text { if iS } S_{t}<L_{w}, i S_{t} \geq R_{w}+l_{c} \\
\left(L_{w}-R_{w}\right) \sin \theta^{\prime} & \text { if iS } \geq L_{w}, i S_{t}<R_{w}+l_{c} \\
\left(l_{c}-i S_{t}+L_{w}\right) \sin \theta^{\prime} & \text { if iS } S_{t} \geq L_{w}, i S_{t} \geq R_{w}+l_{c}\end{cases}  \tag{7}\\
& L_{w}-R_{w}=(D-2 c) / \tan \theta \tag{6}
\end{align*}
$$

$b$ is the section width, $c$ is the concrete cover including half-diameter of steel bars and $\rho_{w}, S_{t}$ and $f_{w y}$ are, respectively, the content ratio, spacing and yield strength of transverse reinforcement,

The internal work developed by the compression of the concrete in the damaged region is given by

$$
\begin{equation*}
\delta W_{c}=\sigma_{c} V_{o} \delta \varepsilon_{c} \tag{8}
\end{equation*}
$$

where $\sigma_{c}$ is the concrete compression stress, $\delta \varepsilon_{c}$ is the average deformation of a part of the damaged concrete block shown in Fig. 3 and $V_{o}$ the volume of the concrete strut participating in the mechanism. Due to the concrete damage level and large cracks opening, it is assumed herein that not all the damaged region participate in sustaining the axial load. The part included is just the shaded block that is acting as a strut with a width equal to the hoops spacing. The deformation $\delta \varepsilon_{c}$ is calculated from the diagonal lengths $l_{0}$ before deformation and $l_{l}$ after deformation as shown in Fig. 3.

$$
\begin{gather*}
\varepsilon_{c}=\left(l_{1}-l_{0}\right) / l_{0}  \tag{8}\\
l_{0}=\sqrt{\left(l_{c}+\frac{D-2 c}{\tan \theta}\right)^{2}+(D-2 c)^{2}}  \tag{9}\\
l_{1}=\sqrt{\left(l_{c} \cos \theta^{\prime}+\frac{D-2 c}{\tan \theta}\right)^{2}+\left(D-2 c-l_{c} \sin \theta^{\prime}\right)^{2}} \tag{10}
\end{gather*}
$$

As to the concrete compression stress and its reduction, it was assumed following the AIJ guidelines [6] and given by Eq.11, which is formulated in term of the lateral drift $R=\delta h / L=l_{c} \sin \theta^{\prime} / L$ and becomes limited beyond a lateral drift ratio of $5 \%$ of the columns' length $L$.

$$
\sigma_{c}= \begin{cases}(1-15 R) v_{0} f_{c} & R \leq 0.05  \tag{11}\\ v_{0} f_{c} & R>0.05\end{cases}
$$

Fig. 2 Hinges position and hoops elongation


Fig. 3 Damaged block before and after deformation
where $f_{c}$ is the concrete compression strength and

$$
\begin{equation*}
v_{0}=0.7-f_{c} / 200 \quad\left[f_{c} \text { unit: MPa }\right] \tag{12}
\end{equation*}
$$

By expressing the vertical displacement $\delta v$, the elongation $\delta l_{i}$ and the deformation $\delta \varepsilon_{c}$ in term of the hinge rotation $\delta \theta^{\prime}$ through differentials, the critical axial load can be formulated as

$$
\begin{equation*}
N_{c r}=\frac{2 n_{s} M_{p}+\rho_{w} b S_{t} \sum_{i=1}^{n_{c}}\left(\partial l_{i} / \partial \theta^{\prime}\right)+\sigma_{c} V \partial \varepsilon_{c} / \partial \theta^{\prime}}{\partial v / \partial \theta^{\prime}} \tag{13}
\end{equation*}
$$

while its corresponding lateral drift is

$$
\begin{equation*}
\delta h=l_{c} \sin \theta^{\prime} \tag{14}
\end{equation*}
$$

## 3. DAMAGED ZONE

The axial failure mechanism is a result of the progression of cracking patterns till the stage of shear failure, which involves progressive evolution of distributed damage and its localization into fracture or crack bands that develops in the concrete struts. The size of these bands, shown in Fig.4, appears to be a parameter that conditions the axial load failure mechanism.

Due its simplicity the classical truss model for reinforced concrete columns is used through fracture phenomena during failure in order to assess the size of the fracture bands. To do so, the approach proposed by Bazant for beams [7] is considered herein for columns. The approach is based on stress redistribution outside the fractured zone and the use of complementary energy concepts. At maximum shear strength, the propagation of fractures, considered of tension mode (expressed as Mode I in Fracture mechanics), across the compression concrete strut during the loading history is governed by the buckling of the concrete portion between fractures in the bands.

### 3.1 Assumptions

For the convenience in the analysis, some simplified assumptions are needed:
1 . The compression strut (thickness: $t d$, where $t$ is a factor and $d$ the effective depth) between the major inclined cracks develops when the maximum lateral strength is reached.
2. The inclination angle $\theta$ of the compression strut is evaluated through Mohr's circle.

$$
\begin{gather*}
\tan \theta=\sqrt{\left(f_{0}+f_{t}\right) /\left(\sigma_{0}+f_{t}\right)}  \tag{15}\\
f_{0}=\rho_{w} f_{w y}, \sigma_{0}=N / b D, f_{t}=0.623 \sqrt{f_{c}} \tag{16}
\end{gather*}
$$



Fig. 4 Fracture zone propagating across a strut
where $f_{0}$ is the confinement stress, $N$ the axial load and $f_{t}$ the concrete tension strength.
3. The cracked band develops soon after reaching the maximum strength and cracks run in the direction of the maximum principal compressive stress, parallel to the axis of the strut.
4. The transmitting crack-bridging normal and tangent stresses are negligible compared to the compressive stresses in the strut.
5. The fracture band is assumed to relieve the compression stress from the entire length of the strut causing a release of strain energy from the strut and the fracture band widens to its full length $h$ and then propagates sideways till reaching its full thickness $a$.
6. The fracture band cracks are assumed equally spaced. The spacing $s_{c}$ is considered equal to the dimension of the larger aggregate size ( 20 mm ).
7. The fracture band length $h$ is reached when the elements between the band cracks buckle and become constant only after a certain initial growth.

### 3.2 Size of fracture band

The truss model allows an easy calculation of the energy release on the basis of complementary energy $\Pi c$. The cell limited by the shaded zone in Fig. 4 must alone be capable to resist the applied shear force $V$ and the fracture band relieves the stress completely from the inclined strip. Considering the shear force $V$ being constant, the stress in the cell must redistribute such that the remaining strips on both sides of the cracked band carry all of the compression force in the strut. After that, all of the complementary energy in concrete in the cell is contained in the two shaded strips and may be expressed as

$$
\begin{align*}
& \Pi_{c}=\sigma_{c}^{2} Z / 2 E_{c}  \tag{17}\\
& \sigma_{c}=V /\left[b(t d-a) \sin ^{2} \theta\right]  \tag{18}\\
& Z=b d(t d-a) \tag{19}
\end{align*}
$$

where $b$ is the column's section width and $E_{c}$ the concrete Young Modulus.

From fracture mechanics, the energy release rate is obtained by differentiation of the complementary energy $\Pi_{c}$ at constant shear force

$$
\begin{equation*}
\zeta=\frac{\partial \Pi_{c}}{\partial a}=\frac{V^{2}}{2 E_{c}} \frac{d}{b(t d-a)^{2} \sin ^{4} \theta} \tag{20}
\end{equation*}
$$

The value $\zeta$ must be equal to the energy dissipation rate $\psi$ of the fracture band, which is expressed by the cracks spacing $s_{c}$ and the plane stress crack toughness factor of reinforced concrete $K_{c}$ that is considered of Mode I $\left(K_{I}\right)$ in the fracture theory for a band of length $2 w$ subjected to tension.

$$
\begin{gather*}
\psi=b h K_{c}^{2} /\left(s_{c} E_{c}\right)  \tag{21}\\
K_{c}=K_{I}=f_{t} \sqrt{\pi h} \sqrt{(2 w / \pi h) \tan (\pi h / 2 w)} \tag{22}
\end{gather*}
$$

where $2 w$ is considered herein equal to $(D-2 c) / \sin \theta$. As to cracks length, Bazant expressed it simply by

$$
\begin{equation*}
h=h_{0} a /\left(a_{0}+a\right) \tag{22}
\end{equation*}
$$

where $a_{0}$ is a constant and the crack length $h_{0}$, limited geometrically, is evaluated from stability criteria at buckling stage ( $\alpha=0.5$ ) of the portion between two cracks, assuming the critical compressive stress equal to the concrete strength.
$h_{0}=\pi s_{c} /(2 \alpha) \sqrt{E_{c} / 3 f_{c}} \leq(D-2 c) / \sin \theta$
At limit of stability, the curve of energy release rate at constant load must be tangent to the resistance fracture curve

$$
\begin{equation*}
\partial \zeta / \partial a=\partial \psi / \partial a \tag{24}
\end{equation*}
$$

because $\zeta=\psi$, a more convenient condition equivalent to Eq. 24 is expressed by

$$
\begin{equation*}
1 / \zeta(\partial \zeta / \partial a)=1 / \psi(\partial \psi / \partial a) \tag{25}
\end{equation*}
$$

By substituting the expressions of $\zeta$ (Eq.20), $\psi$ (Eq.21), Eq. 22 and Eq. 23 into Eq.25, and by carrying out the differentiations, a quadratic equation for crack band width $a$ is reached, whose only real solution represents the theoretical expression for the length of the crushing band at stability loss.

$$
\begin{equation*}
a=\frac{3 a_{0}}{4}\left(-1+\sqrt{1+\frac{8 t d}{9 a_{0}}}\right) \tag{26}
\end{equation*}
$$

By considering $4 t d /\left(9 a_{0}\right)$ small compared to unity, the next approximation is introduced in Eq. 26

$$
\begin{equation*}
\sqrt{1+2 \frac{4 t d}{9 a_{0}}} \approx 1+\frac{4 t d}{9 a_{0}} \tag{27}
\end{equation*}
$$

and the length of the crashing band becomes

$$
\begin{equation*}
a=t d / 3 \tag{28}
\end{equation*}
$$

Substituting this expression of $a$ into the fracture propagation criterion $\zeta=\psi$ (from Eq. 20 and Eq.21), the equation whose solution furnishes the simple result is obtained, after neglecting $t d$ in front of $3 a_{0}$.

$$
\begin{equation*}
t d=\left[3 a_{0} \frac{9}{4}\left(\frac{V}{K_{c} b \sin ^{2} \theta}\right)^{2} \frac{d s_{c}}{h_{0}}\right]^{1 / 3} \tag{29}
\end{equation*}
$$

The width $a$ or $t d$ are computed assuming the constant $a_{0}$ equal to hoops spacing and by using the maximum shear strength value $V$ obtained by an appropriate method, for instance from Arakawa equation [8].

### 3.3 Size of damaged zone

Actually, for elements without axial loading, like the case of beams, the cracked-region-sway mechanism may only be engaged when the damaged region length $l_{c}$ reaches at least the value $D / \tan \theta$, from which the extreme struts at the both edges of the damaged region would not be able to convey the forces through the existing transverse reinforcement [8]. While this is the case observed in absence of axial loading, the effect of axial loads reduces the height of the damaged region and the length of $D / \tan \theta$ may be a maximum value, as observed during the experimental work. Also, while the influence of the amount of the transverse reinforcement on the height of the damaged region $l_{c}$ is easily understood, the influence of the slenderness of elements seems not so important.
(1) Selected tested columns

Some columns, designated in Table 1, from two test programs carried out by the author $[9,10]$ and gathered from literature [11], were selected in order to complete the analysis. The columns, all of square sections ( $300 \times 300 \mathrm{~mm}^{2}$ ) but with different heights and reinforcement steel contents, are shear-critical ones and subjected to different loading patterns where tests were carried out till full collapse under the axial loads. All selected columns experienced a failure mechanism judged reflecting the cracked-region-sway mechanism.

Table 1 Designation of columns

| Reference | Column |
| :--- | :---: |
| $[9]$ | C4, C6, C10, C12 |
| $[10]$ | D1, D5, D13, D14, D16 |
| $[11]$ | N1, N4, N5, N18M, N18C |

(2) Height of damaged region

The height of the damaged region was obtained experimentally from observations made from the carried tests, though in limited number, where the observed size of the damaged regions on the selected columns related to the crack band length $a$ or $t d$, calculated by Eq. 28 and Eq. 29. Fig. 5 illustrates the empirical relationship of the observed size $l_{c}$ to the strut band length $t d$ by range of a parameter $k$, which contains the main intrinsic characteristics of the columns.

$$
\begin{equation*}
k=\rho_{w} f_{w y} /\left(\eta f_{c}\right) \tag{30}
\end{equation*}
$$

where $\eta=N /\left(b D f_{c}\right)$ is the axial load ratio.
The plotted data appear to be separated into two groups where each group appears gathered along a line evaluated from fitting the data by least squares method. The first group with $k$ values less or equal to 0.16 forms an upper line. The second group with $k$ values higher or equal to 0.27 forms a lower line. Due to the limited number of data, a limit $k=0.265$ separating the two groups is assumed subjectively. Consequently, the length $l_{c}$, measured in millimeter, may be expressed by

$$
l_{c}=\left\{\begin{array}{cc}
1580.8-9.82 t d & k \geq 0.265  \tag{31}\\
4168.2-24.15 t d & k<0.265
\end{array}\right.
$$



Fig. 5 Damaged zone size related to strut size


Fig. 6 Column C6 under varying axial load


Fig. 7 Column D5 under constant axial load


Fig. 8 Column N18M under constant axial load

## 4. DRIFT AT AXIAL LOAD FAILURE

Using the length of the damaged zone given in Eq. 31 and the inclination angle $\theta$ from Mohr's circle as given by Eq.15, the lateral drift ratio at ultimate limit, evaluated as to the total clear length of the column as given by Eq.14, is reached when the level of steel bars hinge rotation angle $\theta$, corresponds to the critical axial load $N_{c r}$ given by Eq.13. The relationship between the axial load and its corresponding lateral drift ratio of some columns, as examples, obtained through this analysis are shown below in Fig.6, Fig. 7 and Fig. 8 including the test results at collapse stage. Fig. 6 includes also all values at loading peaks.

According to the results, the method could reflect fairly the measured lateral drift ratios for the majority of the columns, while for few other columns it underestimates the lateral drift at axial failure. This underestimation is probably due to a possible occurrence of two simultaneous damaged regions at the opposite sides symmetrically away from the counter-flexure point in the concerned columns, as shown in Fig.9. Between the two cracked regions, a less cracked region happens to behave like a rigid body and the column as a whole behaves like a double-hinged element, which obviously allows larger lateral deformation than in the case of a single cracked
region element. This fact results into considering not the cracked region $l_{c}$ in the analysis but the region which is considered as a rigid part $l_{c}$, including the length of the two damaged zones, as given in Eq.32, which is evaluated empirically, too. Actually, identification of such mechanism is still not grasped and more studies and experiments should be carried out to investigate such case.
$l_{c}^{\prime}= \begin{cases}L-2\left(D / \tan \theta-2 t d+2 S_{t}\right) & k<0.265 \\ L-2\left(D / \tan \theta-t d+2 S_{t}\right) & k \geq 0.265\end{cases}$
Fig. 10 compares the test and analytical results after introduction of the modified length of the damaged region. The mean value of the calculated lateral drift divided by the measured lateral drift is 1.24 and the coefficient of variance is 0.23 . Compared to the drift calculated before modifying the damaged length $l_{c}$, the coefficient of variation ( 0.44 ) is largely improved while the mean value (1.07) increased.

## 5. CONCLUSIONS

This work suggests a simple evaluation method of ultimate stage of critical columns. The performance at ultimate limit is complex to assess and improving this method necessitates more tests. Solving the lack in the identification of the two-damaged-region mechanism is a key for better results.


Fig. 9 Simultaneous damaged regions


Fig. 10 Comparison of analysis and test results

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