- Technical Paper -

LIFE CYCLE REPAIRING COST CONSIDERING UNCERTAINTIES OF DETERIORATION PREDICTION MODEL

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ABSTRACT
To ensure the safety and serviceability of the structure, a lifetime maintenance program has to be planned based on result of the future prediction of structural conditions. Reliability of decided maintenance program depends on degree of uncertainty of the data used in the future prediction model. The method to determine the expected probability and cost of lifetime maintenance by considering parameters uncertainties of the prediction model is proposed. Example of material selection by using the proposed method to minimize the life cycle repairing cost is given.
Keywords: life cycle cost, deterioration, maintenance, rehabilitation, probability, uncertainty, corrosion

1. INTRODUCTION

In order to maintain the safety and serviceability, most structures need the appropriate repair and maintenance (R&M) program to be applied during their service life. In 2002, Japan allocated approximately 13.5 trillion yen, which is 21.5% of the total construction budget, to the repair and maintenance projects. The ratio of maintenance budget to the overall construction budget is expected to continuously increase in the future because of increasing number of aging structures. In the near future, R&M will become the major task which has to be significantly concerned instead of the new construction project.

In general, decision makers have to decide when and how to repair, rehabilitate, replace, or terminate the structures. Because there are limited resources and budget available for R&M, but there is a large number of aging structures. R&M program has to be effectively planned. Life cycle cost evaluation is one of the methods that can help decision makers to decide the most suitable plan. Usually, initial construction cost, inspection cost, maintenance cost, user cost, failure cost, and salvage cost are necessary to evaluate the life cycle cost of deteriorated structures.

Instead of using deterioration diagnosis software (Yokosawa et al. [1]) developed to assist a decision maker to understand about deterioration mechanism and current condition of the structure without necessity of the expert's decision, prediction of the future condition of structure by deterioration prediction model is another method to understand the level of structural performance. Prediction of deteriorated structural conditions in the future is one of the most important processes in order to decide the conducting time and the suitable method of the required maintenance actions. However, results of future prediction depend on various parameters, and their variables. Uncertainties associated with those variables are due to many reasons relating since in the beginning of the construction such as material properties, workmanship, structural dimensions, and environmental conditions. As a result, predicted results of structural conditions are not exactly the same as the actual conditions and normally show the variation. Therefore, the prediction of application time and methods of required actions have to be represented by the probability distribution instead of the deterministic value.

In this paper, the method to evaluate the expected probability of required R&M actions at any specific time during service life is proposed. Event tree analysis is used to facilitate the determination of all possible sequences of R&M

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actions and their expected probability during the service life. The final objective of this study is to optimize the life cycle cost of R&M program from many alternative strategies. Therefore, method to calculate the expected life cycle cost of R&M is also proposed. Effect of time-value of money is considered in term of discount rate. Total and annually required resources can be allocated to the specific structure or group of structures based on the expected cost. Finally, the optimal repairing method is selected based on the minimum expected life cycle cost. Example of material selection by using the proposed method to minimize the life cycle repairing cost is given.

2. DETERIORATION PREDICTION MODEL

In order to decide the maintenance plan for the structure, predictions of its future conditions are important. Preferably, values of parameters used in the prediction model have to be deterministically and accurately specified in order to get the reliable prediction result. However, in reality, there is variation of those values due to many uncertainties. Therefore, it is more suitable to use probabilistic model to present the prediction result. In this study, mainly model to predict corrosion crack width due to chloride attack is discussed.

2.1 Corrosion initiation time

Deterioration of structure can be categorized to many stages. The first stage is the duration before corrosion is initiated by increasing of the concentration of chloride ion diffused from the environment. Fick's diffusion law is used to predict the time when chloride concentration will reach the threshold value and corrosion will be initiated as shown in Eq. 1.

$$t_i = \frac{(0.1x)^2}{4D_{cl}} \left[erfc^{-1}\left(\frac{C_{\lim}}{C_s} \right) \right]^{-2} \qquad (1)$$

where, t_i is corrosion initiation time (year), x is steel covering depth (mm), D_{cl} is chloride diffusion coefficient (cm^2/year), C_{lim} is threshold chloride content that corrosion will be initiated (% by weight of concrete), C_s is the chloride content at concrete surface (% by weight of concrete). Values of variables of parameters are shown in Table 1.

Monte Carlo simulation (Fishman [2]) uses generated random numbers and probability statistics sampling of uncertainties variables to provide approximate solution to a variety of mathematical problems. Based on random sample size of 10,000, Monte Carlo simulation is performed to determine probability distribution of corrosion initiation time that depended on those four parameters. The obtained result is shown in Fig. 1.

Table 1 Values of variables

Variables	Mean	Coefficient of variation (Distribution Type)	Ref.
x	38.1 mm	0.05 (Log normal)	[3]
D_{cl}	1.29 cm^2/year	0.10 (Log normal)	[3]
C_{lim}	0.05 % by mass of concrete	0.10 (Log normal)	[3]
C_s	0.20 % by mass of concrete	0.10 (Log normal)	[3]
D	1.6 cm	0.015 (Normal)	[4]
f_c'	35.1 MPa	0.18 (Normal)	[4]
i_{corr}	2 μA/cm^2	-	[5]
d_0	12.5 μm	-	[5]
α_{rust}	0.57	-	[5]
ρ_{rust}	3600 kg/m^3	-	[5]
ρ_{st}	7850 kg/m^3	-	[5]
f_t'	$0.23\, f_c'^{2/3}$	-	[6]
E_c	30.1 GPa	-	[6]
φ_{cr}	1.1	-	[6]
υ_c	0.20	-	[6]

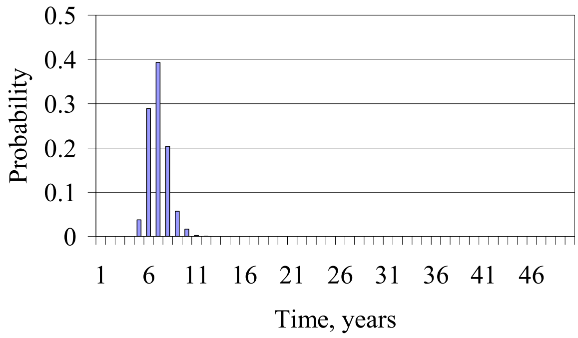

Fig.1 Probability distribution of corrosion initiation time

As shown in Fig.1, possible corrosion initiation time for the assigned values of variables in Table 1 is in the range of the 4th year to the 12th year after in service. The highest possibility of corrosion initiation time is the 6th year that is close to deterministic result determined by using the mean value of all parameters.

2.2 Corrosion crack width

After corrosion was initiated, corrosion product will create tensile stress inside the concrete and finally cause cracking. Bazant [7] proposed that concrete with embedded reinforcing bar can be modeled as a thick-wall cylinder of which the inner and outer radius are $a = (D+2d_0)/2$, and $b = x + (D+2d_0)/2$. Where, D is steel diameter (mm), and d_0 is thickness of pore band of steel and concrete interface (mm). After steel was corroded, corrosion products, mostly ferrous and ferric hydroxides, $Fe(OH)_2$, and $Fe(OH)_3$, firstly fill the pore band. Then, excess amount of corrosion products will create pressure and generate tensile stress to concrete and cause cracking. The thickness (mm) of excess corrosion product, $d_s(t)$, can be determined from Eq. 2 as proposed by Liu and Weyers [5].

$$d_s(t) = \frac{W_{rust}(t)}{\pi(D+2d_0)}\left(\frac{1}{\rho_{rust}} - \frac{\alpha_{rust}}{\rho_{st}}\right) \tag{2}$$

where, $W_{rust}(t)$ is a mass of rust product (mg/mm), ρ_{rust} is density of corrosion products, ρ_{st} is density of steel, α_{rust} is coefficient related to types of rust products. $W_{rust}(t)$ can be determined from Eq. 3 (Liu and Weyers [5]).

$$W_{rust}(t) = \sqrt{2\int_{t_i}^{t}\left(\frac{0.105\pi D i_{corr}(t)}{\alpha_{rust}}\right)dt} \tag{3}$$

where, t is the considering point of time (year), i_{corr} is annual mean corrosion rate ($\mu A/cm^2$).

Expansion of corrosion product generates the pressure to surrounding concrete. Zheng et al. [8] discussed about the three different stages of cracking process including no cracking, partial cracked, and completely cracked. In this study, completely cracked concrete, which crack has already penetrated to the concrete surface, is focused in order to determine the crack width at the concrete surface. It can be determined from Eq. 4 as proposed by Zheng et al. [8]. This formula considers effects of amount of corrosion, concrete property, and dimension of structure.

Based on JSCE [6] to ensure the structural serviceability, permissible crack width is set based on type of reinforcement and environmental conditions as shown in Table 2. In this study, structure is assumed to be using deform bar in corrosive condition. Limit state equation can be simply formed as shown in Eq. 5. Similar to corrosion initiation time, Monte Carlo simulation is performed to determine probability distribution of the time that limits state equation will be violated. Result is shown in Fig. 2. Values of variables of parameters necessary to determine time that permissible crack width will be violated are shown in Table 1. Moreover, distribution of the corrosion initiation time obtained from Section 2.1 is also included in the simulation.

$$w_c = \frac{4\pi d_s(t)}{(1-\upsilon_c)\left(\frac{a}{b}\right)^{\sqrt{\alpha}} + (1+\upsilon_c)\left(\frac{b}{a}\right)^{\sqrt{\alpha}}} - \frac{2\pi b f_t}{E_{ef}} \tag{4}$$

where, w_c is crack width (mm), υ_c is concrete Poisson's ratio, α is stiffness reduction factor, f_t is concrete tensile strength (MPa), E_{ef} is effective modulus of concrete equals to $E_c/(1+\varphi_{cr})$ (MPa), E_c is elastic modulus of concrete (MPa), φ_{cr} is concrete creep coefficient.

$$w_c(t) \geq w_{\lim} \tag{5}$$

where, $w_c(t)$ is corrosion crack width as calculated from Eq. 4, w_{lim} is permissible crack width as shown in Table 2.

Table 2 Permissible crack width [6]

Type of reinforcement	Environmental condition		
	Normal	Corrosive	Severely corrosive
Deformed and plain bars	0.005x	0.004x	0.0035x
Prestressing steel	0.004x	-	-

3. PROBABILITY OF REPAIRING TIME

From probabilistic results obtained from the prediction models, probability of all possible repairing at each year during the service life of the structure will be determined. Repairing will be conducted based on criteria that corrosion crack width as calculated from Section 2.2 was exceed the permissible width as shown in Table 2. Because re-deterioration of repaired structure is also well known, repairing is considered to be repeatedly conducted various times during service life of the structure. In this study, performance of structure after being repaired is assumed to be recovered to the original level for simplicity. Event tree analysis is used to determine all possible events of repairing sequence conducted during the structural service life. Service life of structure is set to be 50 years. Example of event tree analysis to determine the probability of the third repairing will be annually conducted based on absolute time scale is shown in Table 3. Relative time scale of conducting the repair is interval between current repairing and the previous conducted repairing. While the absolute time scale is interval of time since the structure has in service.

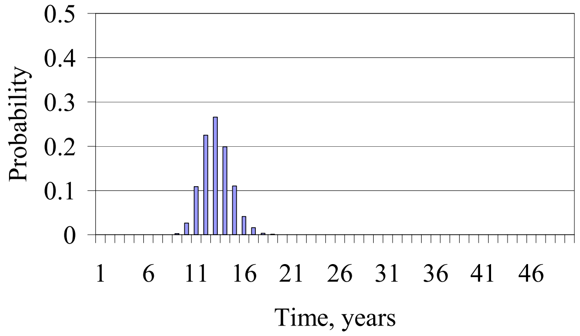

Fig.2 Probability distribution of required time to violate the permissible crack width

Probabilities of repairing ith at time t can be obtained by adding the probabilities of all events repaired maximum ith time and the absolute year of last repair is at the time t as shown in Eqs. 6 and 7. For example, there is only one event that the 3rd repairing will be conducted at year 24th. Therefore, probability of the third repairing will be conducted at year 24th is equal to the probability of that event. Results of probability of all repairing expected to be conducted at each year during service life based on absolute time scale are calculated by Eq. 7 and shown in Fig. 3.

$$P_{r_i,t} = \sum_{j=1}^{n} P_{r_j,t} \quad ; if \ t = t_1 + ... + t_i \qquad (6)$$

$$P_{r,t} = \sum_{i=1}^{n} P_{r_i,t} \qquad (7)$$

where $P_{ri,t}$ is probability of repairing ith at time t, $P_{rj,t}$ is probability of event j which the last repairing is the ith at time t, $P_{r,t}$ is probability of all repairing at time t.

As shown in Fig. 3, maximum 6 repairing are required during the service life of the structure. Mainly the first four repairing have high possibility to be conducted during the service life. While there is very low possibility that repairing 5th and 6th will be conducted.

Table 3 Example of event tree analysis of the third repairing

Time to conduct repairing (Relative (year), Absolute (year)) (Probability)			Event probability $P_{rj,t}$
1st	2nd	3rd	
(8,8) (0.0001)	(8,16) (0.0001)	(8,24) (0.0001)	1.00E-12
(8,8) (1E-4)	(8,16) (0.0001)	(9,25) (0.0027)	2.70E-11
(8,8) (0.0001)	(8,16) (0.0001)	(10,26) (0.0267)	2.67E-10
. .	. .	. .	. .
(8,8) (0.0001)	(9,17) (0.0027)	(8,25) (0.0001)	2.70E-11
(8,8) (0.0001)	(9,17) (0.0027)	(9,26) (0.0027)	7.29E-10
(8,8) (0.0001)	(9,17) (0.0027)	(10,27) (0.0267)	7.21E-09
. .	. .	. .	. .
(20,20) (0.0002)	(20,40) (0.0002)	(8,48) (0.0001)	4.00E-12
(20,20) (0.0002)	(20,40) (0.0002)	(9,49) (0.0027)	1.08E-10
(20,20) (0.0002)	(20,40) (0.0002)	(10,50) (0.0267)	1.068E-9

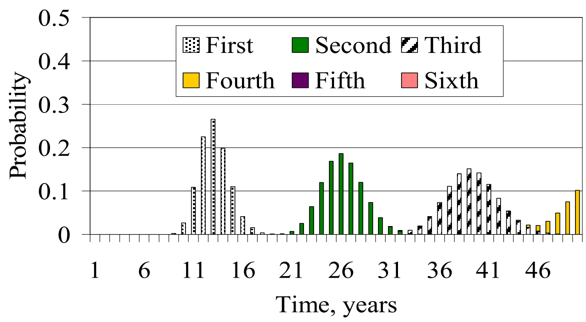

Fig.3 Probability of annual repairing

4. LIFE CYCLE REPAIRING COST

Firstly, the expected cost of repairing conducted at each year will be calculated as the present value at the base year as shown in Eq. 8. The time structure firstly serviced is assumed as the base year for discounting.

$$E\left[C_{r_i,t}\right] = \frac{C_{r_i}}{(1+\nu)^{t_i}} \cdot p_{i,t} \qquad (8)$$

where $E[C_{ri,t}]$ is expected present value of repair i^{th} at time t, C_{ri} is undiscounted cost of repair i^{th}, ν is discount rate, and $p_{i,t}$ is probability of repair i^{th} at time t. It is assumed that undiscounted repairing cost is same for all sequences repairing and equal to 100. Official primary credit discount rate, 5.25% (Federal Reserve [9]), is considered. Fig. 4 shows the present value of repairing cost at each year during the service life of structure. From this result, resources necessary for structural maintenance can be annually allocated to the structure or group of structures based on the obtained expected probability of repairing that will be conducted at each specific year.

Cumulative expected life cycle repairing cost of R&M program is shown in Fig. 5. Due to the effect of discount rate and the events that are not considered due to they cannot be implemented during the service life, total expected life cycle cost is less than the total of assumed total cost of six repairing, 600. Example is the low possibility to be conducted of repairing 5^{th} and 6^{th}. From this result, the total expected life cycle cost of R&M can be estimated.

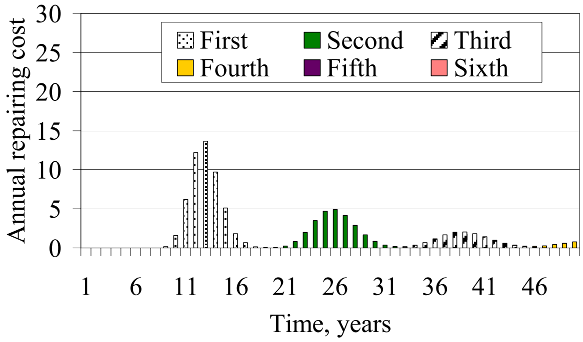

Fig.4 Present value of annual repairing cost

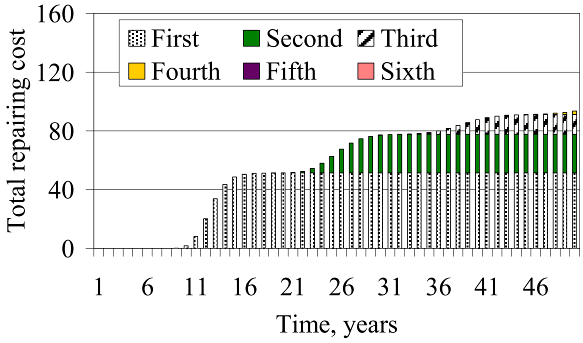

Fig.5 Present value of total repairing cost

The final objective of this study is to develop a tool to support decision makers relating to selection of material used in new construction or as repairing material to ensure the safety, serviceability of structure, while its life cycle cost is also minimized. Therefore, example to present the effect of changing material properties on the life cycle repairing cost is given.

In this study, durability of structure was modified by the changing of chloride diffusion coefficient (D_{cl}). Material with a higher performance is assumed to be used and its higher performance is represented by lower chloride diffusion coefficient. Assumed values of variable were 0.645 cm^2/year for mean, and 0.10 for coefficient of variation. Variables of other parameters are the same as shown in Table 1. Corrosion initiation time and required time for corrosion crack width to violate the permissible crack width are shown in Fig. 6 and 7. Probability of annual repairing is shown in Fig. 8.

Due to lower chloride diffusion coefficient, time required for the chloride content to exceed the threshold value is longer as shown in Fig.6. As a result, time required for corrosion crack to exceed the permissible crack width is also longer as shown when compared results of Fig. 7 to that of Fig. 3.

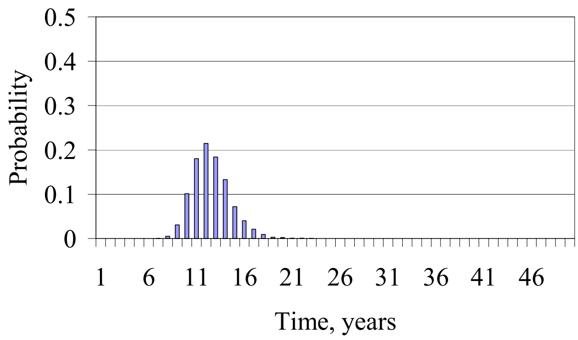

Fig.6 Probability distribution of corrosion initiation time (D_{cl} = 0.645 cm^2/year)

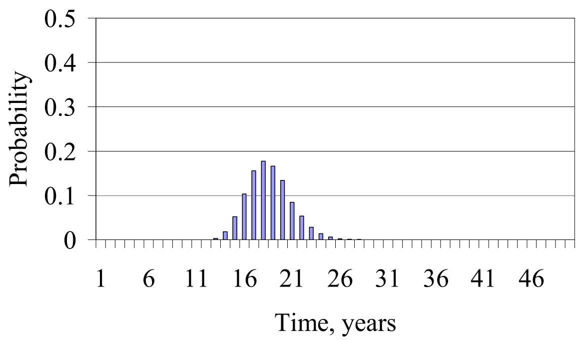

Fig.7 Probability distribution of required time to violate the permissible crack width (D_{cl} = 0.645 cm^2/year)

As shown in Fig. 8, the maximum number of required repairing during the service life reduces to 4 times comparing to 6 times as shown in Fig. 4 due to the higher durability of material.

Life cycle repairing cost is also determined; however, the undiscounted repairing cost is assumed to be 150 due to the higher durability of material compared to 100 of the previous one. Life cycle repairing cost is shown in Fig. 9.

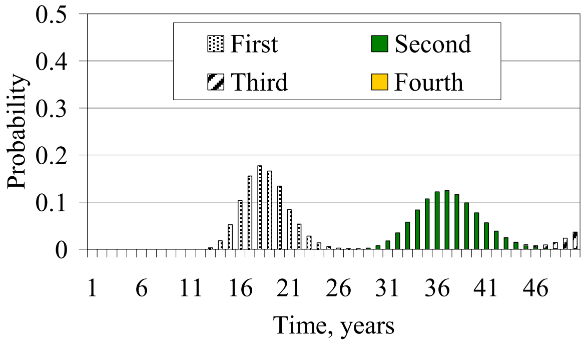

Fig. 8 Probability of annual repairing (D_{cl} = 0.645 cm^2/year)

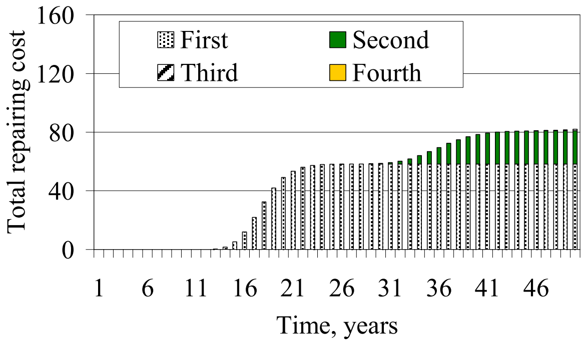

Fig. 9 Present value of total repairing cost

Compared with the result shown in Fig. 5, it is clearly shown that the present value of life cycle cost of case repaired by higher durability material as shown in Fig.9 is lower even though the repairing cost at each time is higher. This is due to the less number of repairing have to be conducted during the service life of structure as well as repairing is conducted later in the future. Based on assumed condition, the higher durability material is a better option due to its cheaper life cycle repairing cost.

5. CONCLUSIONS

Methods to evaluate the expected probability of required R&M at each year during the service life and the present value of expected annual R&M cost of deteriorating structure as well as its life cycle cost were presented.

Due to uncertainties of parameters used in the deterioration prediction model, predicted time to conduct R&M is represented by the probability distribution instead of deterministic value. Expected cost is calculated based on obtained expected probability and discount rate. Total and annual resources can be allocated to structure or group of structures based on expected total life cycle cost and expected annual cost, respectively.

This approach is a part of the effort to determine the most optimal life cycle R&M cost for deteriorated reinforced concrete structure. However, in the future, effort is needed to deal with currently assumed and insufficient data and its uncertainties associated with the life cycle analysis of reinforced concrete structure.

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