Numerical Modeling of Self-Compacting Mortar Flow Using Discrete Element Method

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ABSTRACT
A numerical modeling of Self-Compacting Mortar (SCM) flow, based on the 3D Discrete Element Method (DEM), was presented in this paper. SCM was modeled as a set of spherical elements and the shear strength and damping coefficient between mortar elements were proposed as DEM parameters, which modeled the yield stress and the plastic viscosity of SCM, respectively. After setting up appropriate set of parameters, SCM with different properties can be successfully simulated, and the feasibility of the simulation was validated by the observed different types of SCM flows.

Keywords: self-compacting mortar, simulation, three-dimensional discrete element method, parameter

1. INTRODUCTION
Since Self-Compacting Concrete (SCC) was first developed in 1988[1], it has been used all over the world and has been recognized as potentially the greatest development in concrete technology. However, as formworks become more and more complex and reinforcements become denser and denser, one of its main problems during application is the risk of improper form filling[2]. Nowadays, such problems can only be predicted by experimental simulations, which are always time-expensive and money-consuming. Therefore, it is important to develop some effective numerical tool to predict such behavior of SCC before actual construction.

Moreover, as an integral part of SCC, Self-Compacting Mortar (SCM) serves as a basis for the design of the concrete, and the workability properties of SCC could be assessed by SCM[3]. With these regards, this paper, as the first step, mainly focused on the numerical modeling of the flow behaviors of Self-Compacting Mortar. Though the SCM is usually considered as a kind of continual material, it was simulated by the employment of three-dimensional Discrete Element Method (DEM) in this study, in order to take the aggregate into account in the following SCC simulation.

DEM was first proposed by Cundall and Strack[4] for simulating rock and granular flow, analyzing micromechanical behavior of granular media. Then it grown rapidly to become an effective tool to simulate the dynamics in many other scopes of scientific and industrial, such as, silo discharge and process of mixing. Recently, DEM has been adopted in the area of concrete simulation, and the concrete is assumed to be represented by a collection of discrete elements. However, so far, the research has been limited to qualitative simulation because of the difficulty of defining appropriate DEM parameters of mortar[5]-[9]. In this study, the shear strength $f_s$ and damping coefficient $h$ between mortar elements were proposed as the DEM parameters to simulate the flow behaviors of SCM. And the feasibility of the proposed DEM simulation was validated by the observed different types of SCM flows.

In this paper, the DEM simulation principle and the numerical modeling of SCM are introduced at first. Then, the effects of the proposed parameters on prediction of flow behaviors of SCM and the method of parameters selection are described. Finally, the comparison between experiment results and numerical simulations are presented and some conclusions of this research were drawn.

2. NUMERICAL MODELING
2.1 Basic simulation principle of DEM
DEM is a tool with great capacity of modeling the movement including the separation and automatic contact detection of granular media. In DEM simulations, prototypes are assumed as discrete rigid balls. The contact mechanism between SCM elements is using sets of basic rheological elements, such as spring, dashpot and slider, for both the normal and shear direction. And as shown in Fig.1, in order to simulate the adhesive and cohesive behaviors, a set of joints for both the normal and shear direction were also added as failure criterions in this study, which would be explained later.

Based on the simulation principle, the code programming work has been finished and the...
developed program was used in this research as a basic program. In this program, each calculation cycle of DEM includes two steps. Firstly, the differential equations for the motion of particles are applied to all particles to calculate the movements and update the particles’ positions. Then, the contacts between elements are automatically detected and the constitutive law is applied to calculate the forces between elements.

2.2 Rheological parameters of SCM in DEM

SCM, as well as SCC, in its fresh stage, can be considered as a kind of Bingham fluid\[10\]. The rheology at a mesoscale can be written as a relation of the shear stress and the shear strain rate and there are two rheological constants namely the yield stress and the plastic viscosity in the relationship. In numerical modeling, it is also regarded that the SCM behaves as Bingham model. The constitutive law [Obara\[11\]], which uses rheological parameters in DEM simulation, has been adopted in this study. The spring rates is calculated with the consideration of spring combination in series between elements, and the damping coefficient are calculated by the differential equation of damping vibration, which are given in the following equations:

\[
k_n = \frac{3}{4} \sqrt{3} \frac{E_m}{r_m}
\]  
(1)

\[
k_s = \frac{3}{4} \sqrt{3} \frac{G_m}{r_m}
\]  
(2)

\[c_n = 2h\sqrt{m_m k_n}
\]  
(3)

\[c_s = 2h\sqrt{m_m k_s}
\]  
(4)

\[G_m = \frac{1}{2(1 + \nu_m)} E_m
\]  
(5)

where,

- \(E_m\): elastic modulus of mortar element
- \(G_m\): shear modulus of mortar element
- \(h\): damping coefficient
- \(m_m\): mass of mortar element
- \(\nu_m\): poisson’s ratio of element

Since the mortar is treated as a perfectly incompressible material in this study, the poisson’s ratio of mortar element \(\nu_m\) is set to be 0.5 and the elastic modulus of mortar element \(E_m\) should be enormous. However, generally speaking, the higher the \(E_m\), the smaller time step is required, the larger CPU time is consumed. With this consideration, the \(E_m\) is set to be 10,000Pa based on the trial calculation.

In addition, in order to simulate the tensile properties of SCM, the tensile strength \(f_n\) and shear strength of elements \(f_s\) were also under consideration. And the strength properties of element could be formulated as follows:

\[
k_n \delta_n \geq -A_m \times f_n
\]  
(6)

\[
|k_s \delta_s| \leq A_m \times f_s
\]  
(7)

\[f_n = 2f_s
\]  
(8)

where,

- \(\delta_n\): overlap in normal direction
- \(\delta_s\): displacement in shear direction
- \(A_m\): contact area between elements

Here, the tensile strength \(f_n\) was proposed as twice as the shear strength \(f_s\) according to the Mohr-Coulomb Theory. With these considerations, two rheological parameters of SCM elements, shear strength \(f_s\) and damping coefficient \(h\), should be input to simulate the flow behaviors of SCM, and the two parameters modeled the two rheological constants of SCM, the yield stress and the plastic viscosity, respectively.

It should be noted that, as shown in Fig.2, if the normal tensile stress carried by contact forces equals or exceeds the tensile strength, all the contact forces become zero and the particles detach from each other independently. While, if shear stress reaches up to the shear strength, the slider works to allow slip between
elements, and the elements still contact with each other until the failure happens in tensile direction.

2.3 DEM model of SCM

In DEM simulation, the smaller the particle sizes, the larger CPU time is consumed. It is important to choose a appreciate model of SCM to finish the simulation in an acceptable computation time. In this study, the SCM was modeled as single sized mortar elements with a diameter of 5mm, since the mortar element simulates the behavior of the combined water, powder and sand particles, and the maximum diameter of these materials is 5mm in the real SCM.

Though SCM is discrete at some levels, it shows continuity as a part of SCC. While after packing the SCM elements into a container, the elements cannot fill all the space of the whole volume. In order to model the SCM elements have the same weight with the SCM prototype, the elements should have a greater density than the SCM. According to Obara[11], as shown in Fig.3, orthorhombic body center structure conforms well of the arrangement of the discrete elements with the stiffness given above after random packing. The void ratio was 0.3424 and the density of SCM was chosen as 3.67 g/cm$^3$ in this study. The contact area $A_{m}$ between elements with the radius of $r_{m}$ could be calculated with the following equation:

$$A_{m} = \frac{3}{2} \sqrt{3} r_{m}^2$$

(9)

3. EVALUATION OF THE PARAMETERS

3.1 Basic description of experimental devices

Since the mortar flow test and mortar funnel test have been being proposed for testing the workability properties of SCM for many years, these two types of tests have also been used in this research to study the effects of the proposed parameters on prediction of flow behaviors of SCM.

As show in Fig.4 and Fig.5, the mortar flow test is used to evaluate the deformability of mortar and the mortar funnel test is used to evaluate the viscosity of mortar. The indices for deformability and viscosity are defined as relative flow area $\Gamma_{m}$ and relative funnel speed $R_{m}$.\[1\]

$$\Gamma_{m} = \frac{(d_{1} \cdot d_{2} - d_{0}^2)}{d_{0}^2}$$

(10)

Fig.3 SCM model in DEM simulation

Fig.4 Mortar flow test

Fig.5 Mortar funnel test

Fig.6 DEM simulation of Mortar flow test

Fig.7 DEM simulation of Mortar funnel test
\[ R_m = \frac{10}{t} \]  

where,
- \( d_1 \) & \( d_2 \): measured flow diameter
- \( d_0 \): bottom diameter of mortar flow cone
- \( t \): measured time (sec) for mortar to flow through the funnel

3.2 DEM simulations

As shown in Fig.6 and Fig.7, the DEM models of mortar flow test and mortar funnel test are set up, respectively. Both models are simulated as one quarter of the real apparatus to reduce the enormous number of particles. Then, in order to evaluate the effects of parameters, various cases have been performed on mortar flow test and mortar funnel test. The parameters of these cases are shown in Table1. The Relationship between \( \Gamma_m \) and \( R_m \) of SCM in DEM simulation are shown in Fig.8. According to Okamura and Ouchi [1], the range of \( \Gamma_m \) and \( R_m \) of SCM for achieving self-compactability of fresh concrete is \( \Gamma_m \) of 2.5 to 6 and of 0.75 to 1.25, which is also illustrated as the box in Fig.8, and it is shown that by given appropriate set of parameters, SCM with different workability properties can be successfully simulated with the developed program.

The effects of parameters of simulation of mortar flow test and mortar funnel test are shown in Fig.9 and Fig.10, respectively. It shows that \( \Gamma_m \) is mainly dominated by \( f_s \) than \( h \), and the \( R_m \) is determined by both \( f_s \) and \( h \). It is similar to say, the shear strength \( f_s \) was shown to possess significant influence on the deformability and velocity of SCM flow, while, the damping coefficient \( h \) mainly influence the velocity of SCM flow.

3.3 Parameter selection method

With the goal of predicting the SCM flow behaviors during actual application, an appropriate set of parameters has to be defined at first.

According to the effect of parameters on prediction of flow behaviors of SCM obtained above, the mortar flow test and mortar funnel test were proposed as the standard tests for the quantitative selection of the DEM parameters of SCM in this research. And the procedure of parameters selection were proposed and shown in the flow chart in Fig.11.

At first, damping coefficient was set to be 0.5, and \( f_s \) was adjusted to satisfy the deformability condition in the simulation of mortar flow test. Then, using the selected \( f_s \), \( h \) was calibrated to satisfy the viscosity

![Fig.9 Simulation results of mortar flow tests](image)

![Fig.8 Relationship between flow area and funnel speed of SCM](image)

![Fig.10 Simulation results of mortar funnel tests](image)

![Fig.11 Procedure of parameters selection](image)
condition in the simulation of mortar funnel test. And according to the relationship shown in Fig.8, the DEM parameters of SCM could be easily fixed after 2~3 trial simulations. Finally, the appropriate set of parameters could be selected to proceed the SCM flow simulations. Some examples of SCM flow simulations will be presented in the following.

4. SIMULATIONS OF SCM FLOWS

After the DEM parameters of SCM could be selected by means of the simulation of mortar flow test and mortar funnel tests, a series of SCM flow tests were carried out both experimentally and numerically.

4.1 Description of experiment

In order to simulate the flow behaviors of SCM after pumping during actual application, four types of pipe cone with different size were used in this study. The size of the pipe cones are Φ107mm*300mm, Φ107mm*600mm, Φ56mm*300mm, Φ56mm*600mm, which are defined from 1# to 4#, respectively. An example of the pipe cone flow test is shown in Fig.12. The results of four types of pipe cone flow tests as well as mortar cone flow test are shown in Table2.

4.2 Simulation of experiment

The same SCM pipe cone flow tests have also been simulated, as shown in Fig.13. First, according to the parameter selection method proposed above, the shear strength $f_s$ and damping coefficient $h$ were selected as 10Pa and 0.5, respectively. The simulated mortar cone flow test was 247mm, which was close to the experiment value of 246mm. Then, the four types of pipe cone flow tests have been simulated with the same parameters obtained above. The results of simulations are also shown in Table2, and the comparison between the results of experiments and simulations are illustrated in Fig.14. It is found that four types of SCM pipe cone flows have been successfully simulated. The maximum relative difference between the results of simulations and experiments is only 5%, which means that the numerical modeling proposed in this study is quite capable of predicting the flow behaviors of SCM.

4.3 Typical procedures of SCM flow simulation

Finally, based on the simulation results of this study, the typical procedures of SCM flow prediction could be summarized in two steps:

(1) Parameter selection

The flow behaviors of SCM should be observed in the laboratory by means of mortar flow test and mortar funnel test. Then, an appropriate set of DEM parameters, shear strength and damping coefficient, of the SCM, can be selected with the procedures

<table>
<thead>
<tr>
<th>Table2</th>
<th>Experimental and numerical results of different types of mortar flow tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortar cone</td>
<td>1#Pipe_Φ107*300</td>
</tr>
<tr>
<td>Experiments</td>
<td>246 100%</td>
</tr>
<tr>
<td>Simulations</td>
<td>247 100%</td>
</tr>
</tbody>
</table>
illustrated in Fig.11.

(2) SCM flow simulation

Based on the strength and damping coefficient obtained above, the simulation of SCM flow is started. And the flow behaviors of SCM could be predicted from the final results.

5. CONCLUSIONS

DEM is a very potential tool for SCM flow simulation. A numerical modeling of SCM, based on the three-dimensional DEM, has been presented herein, and the following conclusions can be made from this part of research of simulation of SCM flow.
(1) The feasibility of the simulation with the proposed rheological parameters, the shear strength $f_s$ and the damping coefficient $h$, have been validated.
(2) The flow behaviors of SCM, with the relative flow area $\Gamma_m$ of 2.0-8.0 and relative funnel speed $R_m$ of 0.2-1.8, have been successfully simulated.
(3) In DEM simulation, the results of mortar flow test are mainly controlled by the shear strength, while, the results of mortar funnel flow test are controlled by both the shear strength and damping coefficient.
(4) The appropriate DEM parameters of SCM with certain workability could be easily defined by using the presented method.
(5) The typical procedures of SCM flow simulation have been proposed, and the behaviors of SCM pipe cone flow can be well predicted.

Ongoing work mainly focuses on finding out the relationship between the rheological parameters used in this study and the rheological constants of actual SCM, and the influence of the size of mortar elements. The study on numerical modeling of coarse aggregates using DEM is also currently in process with the goal of providing tools which are capable to predict possible segregation and blocking of aggregates during casting of SCC.

REFERENCES