

# SHEAR CARRYING CAPACITY OF PRESTRESSED CONCRETE I-BEAMS REINFORCED WITH STEEL FIBERS

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## ABSTRACT

The test results of four I-section prestressed concrete beams with and without steel fibers inside concrete are presented in this paper. Steel fibers provided significant improvements in the shear strength as well as deformation ability and these improvements were greater at a specified fiber fraction of 1.0%. At a higher fraction of 1.5% and 2.0%, the compressive strength of concrete decreased and the shear strength dropped approximately 11% compared with a fraction of 1.0% and no significant increase in the shear strength occurred at a volume fraction of 2.0%.

**Keywords:** prestressed concrete, steel fibers, deflection, crack patterns, shear carrying capacity.

## 1. INTRODUCTION

The contribution of steel fibers to the shear-resisting mechanisms has been investigated during the past three decades [1, 2]. Investigations on the effect of fibers [3, 4] indicated that the bridging of steel fibers across crack interfaces results in narrower cracks and it allows in loading of beams to a higher value of strain in the longitudinal steel and improving the shear carrying capacity of structural concrete. Several previous researchers [5, 6] tested fiber reinforced concrete (FRC) beams with crimped steel fibers up to 3% by volume and found that the first crack tensile strength and ultimate tensile strength were increased by the contribution of steel fibers. They concluded that little improvement in shear strength was noted more than 1% of fiber volume, and also found that fibers were more effective in high strength concrete than the normal strength concrete.

Prestressed concrete (PC) structures have been used in recent years, primarily for today's bridges. Due to its development, the use of high compressive strength concrete (more than 80 N/mm<sup>2</sup>), which is relatively more brittle compared with the normal strength concrete (around 30 N/mm<sup>2</sup>), may require the use of fiber reinforcement to mitigate the brittleness and catastrophic shear failures in beams.

In usual structural design, shear is accounted for the contribution of concrete and shear reinforcements such as stirrups in beams. Most of PC structures are designed to fail in flexural failure which is not severe and predictable compared with the shear failure mechanisms. In order to resist the shear force, the cross-section of the members were designed with thick width or reinforced with excessive steel bars. Therefore, the appearances of structures become lack in visual and in economic. Thus, it is realized that the study on the shear carrying capacity of PC beams with steel fibers

becomes a significant research topics in order to solve the above shortcoming.

The main objective of this study is to determine the shear carrying capacity of prestressed concrete beams with and without steel fibers and the contribution of steel fibers to shear resistance at the various loading stage. Therefore, the beams were designed to fail in a shear mode and the fibers were introduced to the beams at a different fraction up to 2.0% for the determination of the possible enhancement of the shear carrying capacity provided by steel fibers.

## 2. SHEAR CARRYING CAPACITY

### 2.1 Shear Carrying Capacity of Concrete, $V_{PC}$

In order to get the high accuracy for predicting the shear carrying capacity of PC beams, Simplified Truss Model [7] was adopted in this study. Figure 1 illustrates the simplified truss model for evaluating the shear carrying capacity of PC beams without transverse reinforcement. The model consists of seven nodes and eleven elements for flexural compression, transverse tension, diagonal compression and flexural tension members. The model is fixed in X-direction at both nodes along the center line and in Y-direction at the support. The parameter  $m$  is used in the model to represent the inverse of the concentrated stress flow slope, where  $m = \cot \theta$ ,  $\theta$  is the angle of concentrated stress flow. The equation for estimating the value of  $m$  can be expressed as the following equation:

$$m = 2.55 \left( \left( 1 + 0.2 \frac{\sigma_u}{\sigma_u + \sigma_l} \right) \sigma_l \right)^{\frac{3}{5} \left( \frac{b_f}{b_w} \right)^{-1}} \left( \frac{a}{d} \right)^{\frac{1}{5} \left( \frac{b_f}{b_w} \right)} \left( \frac{b_f}{b_w} \right)^{\frac{3}{5}} \left( \frac{f'_c}{100} \right)^{\frac{3}{5}} \quad (1)$$

where  $\sigma_u$  and  $\sigma_l$  are the upper and lower extreme fiber stress (N/mm<sup>2</sup>),  $b_f$  and  $b_w$  are the width of flange and web (mm),  $a$  is the shear span,  $d$  is the effective depth,

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and  $f'_c$  is the compressive strength of concrete ( $\text{N/mm}^2$ ).

After computing the values of  $m$ , each of the member force ( $F_i$ ) caused by the external applied shear force  $V$ , can be determined by employing the Castigliano's second theorem. The stiffness of the concrete at the ultimate stage where strain at the maximum compressive strength,  $\epsilon'_{0s}$ , is 0.002, as the secant modulus,  $E_{cu}$  is applied for diagonal compression members. The initial elastic stiffness of concrete  $E_c$ , is employed to flexural compression members which are considered to behave in the elastic range.

By considering the effect of bearing plates and effective depth, the values of the horizontal thickness of the diagonal compression members at the loading point  $w_l$  and support  $w_s$ , are expressed in the following equations:

$$w_l = (r_l + 0.1d) \left( \frac{b_f}{b_w} \right)^{\frac{1}{5}} \quad (2)$$

$$w_s = (2r_s + 0.1d) \left( \frac{b_f}{b_w} \right)^{\frac{1}{5}} \quad (3)$$

where  $r_l$  is the loading plate width,  $r_s$  is the support plate width and  $d$  is the effective depth. The cross sectional area of each strut member ( $A_i$ ) can be computed as the values of  $w_i$  multiplied with  $b_w$  (width of the web) and it's inclination.

The resistance capacity of each diagonal compression member can be obtained from  $f'_c$  incorporating the compressive softening parameter  $\eta$ , the cross sectional area  $A_i$  and the inclination of each member are expressed in the following equation:

$$R_i = \eta f'_c A_i \sin \theta_i \quad (4)$$

By comparing the value of  $F_i$  and  $R_i$ , the shear carrying capacity of PC beams,  $V_{PC}$  can be determined when one of the diagonal compression members become critical. The critical member refers to the member among four diagonal compression members where it's ratio of  $F_i$  and  $R_i$  becomes greatest and equal to 1.0.

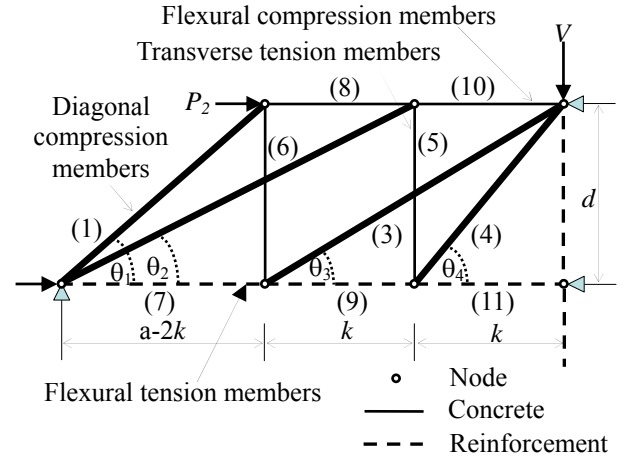
## 2.2 Contribution of Steel fibers to Shear Stress

The effect of steel fibers on the shear carrying capacity was taken into account which is based on the composite mixtures rule [8]. The contribution of steel fibers to shear stress after the formation of shear crack for the nonprestressed concrete beam was given by the following empirical equation [9]:

$$v = \eta_o \tau_{bF} V_f \frac{L_f}{d_f} \quad (5)$$

where  $v$  is the contribution of the fiber pullout mechanisms to the shear stress ( $\text{N/mm}^2$ ),  $\eta_o$  is the orientation factor of steel fibers,  $\tau_{bF}$  is the bond shear strength of steel fibers having crimped-ends ( $\text{N/mm}^2$ ),  $V_f$  is the volume of fibers expressed as (%),  $L_f$  is the fiber length (mm) and the  $d_f$  is the fiber diameter (mm).

The orientation factor for randomly oriented



where  $k$  is  $0.5md$  if  $m > 1.0$  and  $1.0md$  if  $m \leq 1.0$

The number in parenthesis ( ) shows member number

Fig. 1 Details of simplified truss model (half model)

steel fibers is taken as 0.37. From the experimental results of frictional slip between fiber and corresponding concrete [10], the bond shear strength  $\tau_{bF}$  of the crimped-ends steel fiber (with aspect ratio of 60) was adopted in the following equation:

$$\tau_{bF} = 0.6(f'_c)^{\frac{2}{3}} \quad (6)$$

According to the previous study [11], bond shear strength of steel fibers mainly depends on the compressive strength and fibers aspect ratio has little effect on the compressive strength than the split tensile strength. Therefore, Eq. (6) can be applied in present investigation even if the aspect ratio of steel fiber is 50. Finally, the shear carrying capacity is calculated as shown in the following equation:

$$V_{cat} = V_{PC} + V_F \quad (7)$$

where  $V_{PC}$  is the shear carrying capacity of PC beams and  $V_F$  is the contribution of steel fibers to shear carrying capacity.  $V_F$  can be computed as the values of  $v$  multiplied with width of the beam ( $b_w$ ) and effective depth ( $d$ ) of the beam.

## 3. TEST PROGRAMS

### 3.1 Materials

#### (1) Concrete

Details of the mix proportion are summarized in Table 1. The materials used in concrete were high early strength cement, fine aggregates, coarse aggregates, and superplasticizer. To achieve adequate fiber dispersion and workability, superplasticizer (Type; SP8N), 1.25% by weight of cement was used as the admixture for each concrete mix. A water-to-cement ratio was set to 0.35 in order to achieve 14 days concrete cylinder strength of  $60 \text{ N/mm}^2$ .

#### (2) Reinforcements

Reinforcements and cross-sectional details of the test specimens have been given in Fig. 2. Two PC bars

with the nominal diameter of 19 mm were used as tensile reinforcements, and two deformed steel bars with the nominal diameter of 6 mm were used as build-up steel bar. Their yield strengths were 1080 N/mm<sup>2</sup> and 355 N/mm<sup>2</sup>. Shear reinforcements consisting of deformed steel stirrups with the nominal diameter of 6 mm with the yield strength of 355 N/mm<sup>2</sup> were provided in one shear span. Stirrups were not used in the test shear span.

### (3) Steel fibers

Material properties and layout for steel fibers are shown in Table 2 and Fig. 3, respectively. The parameter that affects the shear strength of prestressed concrete beams identified in this study was the volume of steel fibers in the concrete mix up to 2.0%. Only one type of steel fiber was investigated: 30 mm long crimped end steel fiber with a circular cross section of 0.6 mm diameter.

### (4) Casting procedure

First, cement and sand were mixed in dry for about 30 seconds, and then water was added and mixed for about 1 minute. After that steel fibers were added and mixed again for 30 seconds. Finally, coarse aggregates were added and mixed again for 2 minutes. Concrete cylinders with size of 100x200 mm, and 150x300 mm were cast and cured with the same condition as the beam specimen to determine the compressive strength and splitting tensile strength.

After seven days from casting, prestressing force was applied to each PC bar of the beam and recorded in a data logger to determine the loss of prestress in PC bars at the loading day. And then cement grout was introduced between the steel sheath and PC bars.

### (5) Experimental setup

The test program consisted of four shear critical I-section beams having the same cross-section of width,  $b = 200$  mm, height,  $h = 300$  mm, shear span to effective depth ratio,  $a/d = 2.8$ , total length of 1800 mm and a span length of 1600 mm as shown in Fig. 2.

One control beam specimen (with no fiber reinforcement) was cast to study the effect of parameter used in this study. The rest of the beams had different fiber volume content of 1.0%, 1.5% and 2.0%. The beams were designated to indicate the type of the beams and the presence of steel fibers in the concrete mixture. The following terminology is used: the former letter represents the prestressed concrete beam (PC) and the latter shows fiber percentage contained in each specimen, 0.0%, 1.0%, 1.5% and 2.0%, respectively.

Name of the specimens, all values of compressive strength ( $f'_c$ ), tensile strength ( $f_t$ ), concrete stress at top fibers ( $\sigma_t$ ) and bottom fibers ( $\sigma_b$ ) resulting from the tests corresponding to each specimen are listed in Table 3.

## 4. RESULTS AND DISCUSSION

### 4.1 Testing Procedure

All four I-beam specimens were tested under a four-point bending system. A load cell was used to measure the applied load. Steel plates, 100 mm width

Table 1 Mix proportion of concrete

W/C (%)	W (kg/m <sup>3</sup> )	C (kg/m <sup>3</sup> )	S (kg/m <sup>3</sup> )	G (kg/m <sup>3</sup> )	SP (g/m <sup>3</sup> )
35.0	165	471	914	790	3680

W = water

C = high early strength cement, density = 3.14 g/cm<sup>3</sup>

S = fine aggregate, density = 2.60 g/cm<sup>3</sup>, FM = 2.62

G = coarse aggregate, density = 2.64 g/cm<sup>3</sup>, FM = 6.89,

Maximum size of coarse aggregates,  $G_{max} = 20$ mm

SP = superplasticizer, density = 1.44 g/cm<sup>3</sup>

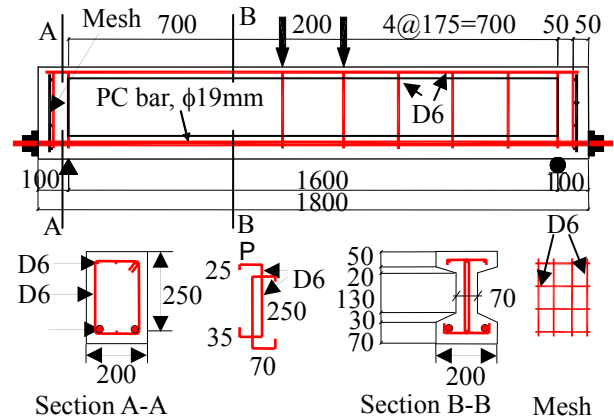


Fig. 2 Details of I-section PC beam, (Unit: mm)

Table 2 Properties of steel fibers

Weight (kg/m <sup>3</sup> )	Length (mm)	Diameter (mm)	Tensile strength (N/mm <sup>2</sup> )	Elastic modulus (kN/mm <sup>2</sup> )
7874	30	0.6	1000	210



Fig. 3 Layout of steel fibers

and 300 mm long were also placed at the loading points and supports to ensure adequate bearing. Vertical deflections were measured at mid-span and support using displacement transducers. Electrical-resistance strain gauges were used to record the strain in concrete as well as PC bars at the mid span of the beam. A set of measurements was taken and also recorded the development and propagation of cracks. The test was stopped when the crushing of the concrete in compression and considerable loss of load carrying capacity was observed.

### 4.2 General Behavior of Test Beams

Flexural cracks first appeared at the constant moment zone at the early stage of loading for all beams.

With further increase in load, existing cracks were extended and additional flexural cracks were formed in the mid-span region. As the load increased further, the diagonal crack was developed in the shear span of the beam leading from the loading point towards the support. In later stage of loading, the web cracks in the shear span were widened and the concrete near the crack tip in the compression zone was crushed, finally the beam failed in shear compression. The beams with fiber reinforcement showed improved resistance to crack growth as well as the shear strength at the ultimate stages. The load at first flexural crack,  $P_{cr}$ , diagonal cracking load,  $P_{diag}$ , peak load,  $P_u$  and the shear carrying capacity,  $V_u$  for all the specimens are summarized in Table 4.

### 4.3 Load-Deflection Responses

The overall behavior of a test beam is described by means of load-deflection responses. Key observations included ultimate loads in shear, strain distributions in the main tensile reinforcement and strain in concrete at the concrete compression zone. The effect of fiber content on the load carrying capacity for all the test beams are described in the followings.

Figure 4 shows the applied load versus mid-span deflection responses for beams with and without steel fibers. The load slightly dropped after the formation of the first flexural crack, and it continued to increase in PC-0.0. After that, diagonal crack occurred in the shear span and the load sharply dropped, however soon after that, the load continued to increase and slightly dropped again with the formation of another diagonal crack that was parallel to the already existing diagonal crack in the shear span. Even though the diagonal crack took place, the beam was still able to resist the applied load by the dowel action of the longitudinal reinforcement and the aggregate interlocking effect on the two faces of the crack. Finally the beam failed in shear compression where the diagonal cracks in the test shear span were widened and the concrete near the crack tip in the compression zone crushed. A drop of load at the ultimate stage indicated due to the initiation of concrete crushing.

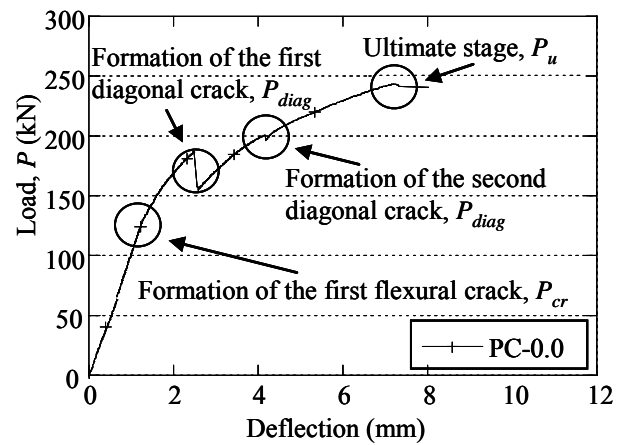
For beams with steel fibers, the load-deflection response indicates that the load slightly dropped after the initiation of the diagonal crack; however, it was not significant in the beams with fiber content of 2.0%. To a large extent, the addition of steel fibers enhanced the initial crack resistance, cracking and ultimate load as well as the shear carrying capacities compared with the control beam as shown in Table 4. The enhancement of the shear carrying load was most prominent at a fiber volume fraction of 1.0% and increased by approximately 21% of the control beam. Further increased in fiber volume, 1.5% and 2.0%, the shear carrying capacity was increased by 8% and 13% of the control beam. On the other hand, a significant increase in shear carrying capacity did not occur with an increase in the fiber volume more than 1.0%. It was found that the compressive strength of concrete played a major role and increases the shear carrying capacity of beams at higher volume of fibers because a further

Table 3 Properties of each specimen

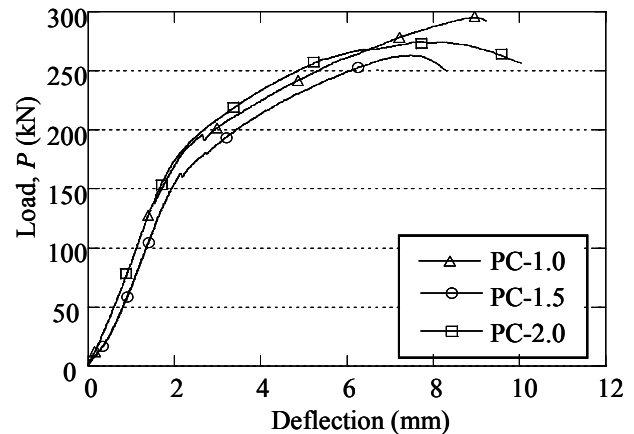
Name	Fibers (%)	$f'_c$ (N/mm <sup>2</sup> )	$f_t$ (N/mm <sup>2</sup> )	Concrete stress (MPa)	
				$\sigma_u$	$\sigma_t$
PC-0.0	0.0	63.6	3.6	-1.4	7.4
PC-1.0	1.0	62.9	4.4	-1.4	7.6
PC-1.5	1.5	54.9	4.3	-1.4	7.3
PC-2.0	2.0	55.8	4.8	-1.4	7.5

Table 4 Comparison of test results

Name	$P_{cr}$ (kN)	$P_{diag}$ (kN)	$P_u$ (kN)	$V_u=P_u/2$ (kN)
PC-0.0	127.0	186.6	243.1	121.5
PC-1.0	140.0	196.2	295.3	147.6
PC-1.5	142.9	163.0	263.1	131.5
PC-2.0	166.5	178.3	274.7	137.3



(a) Load-deflection curve of control beam



(b) Load-deflection curves of fiber reinforced beam

Fig. 4 Load-deflection curves of test beams

increase in fiber volume fraction did not result in a corresponding increase in compressive strength as well as the shear strength of the beams.

Table 3 shows that the addition of steel fiber does not change the compressive strength of concrete very much; however, the tensile strength of concrete is seen to increase with the increasing fiber content.

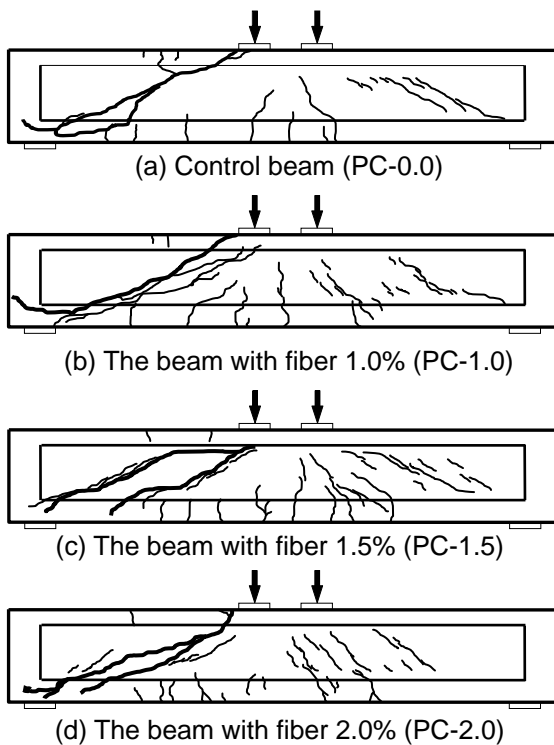


Fig. 5 Crack patterns of test beams



Fig. 6 Failure mode of control beam



Fig. 7 Failure mode of fiber reinforced beam (PC-2.0)

Owing to this property of the concrete, a rapid propagation of the crack was observed in comparison with those having steel fiber reinforcement. Fibers' role was to arrest the widening of cracks and to increase the stiffness of the member up to failure. The effectiveness of the bridging action of fibers across the cracks was indicated by relatively ductile failure because the fibers continue to transmit the tensile stresses across the cracks. This implies that steel fibers reasonably contribute to carry the shear in prestressed concrete beams.

#### 4.4 Cracking Behaviors

The first flexural and shear cracking loads were detected visually in all the beams. The significant effect of steel fibers in the concrete was reflected in the cracking behaviors and shear cracking loads of the beams as shown in Table 4 and Fig. 5. The increased number of both flexural and shear cracks with closer

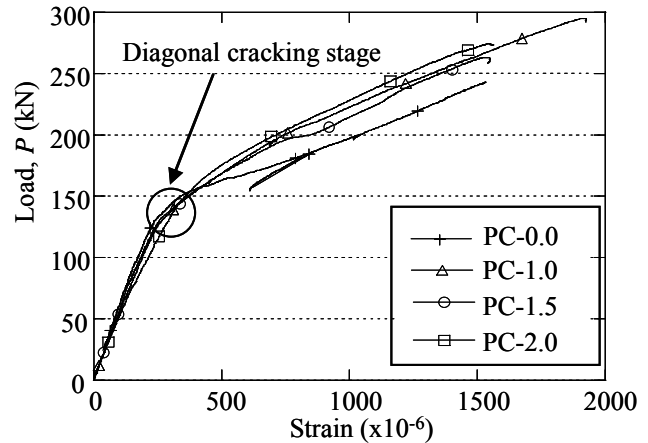


Fig. 8 Load-strain in prestressed steel

spacing were observed in the fiber reinforced PC beams and these were much narrower than the corresponding control beam without steel fibers. The bridging effect of steel fibers across the all cracks reduced spalling, bond cracking and resist the shear force to produce smaller deformations and then sustain the beams at higher loads compared to the control beam. For this reason, the control beam failed in the brittle manner at the ultimate stage as shown in Fig. 6; however, fiber reinforced beams provided a gradual softening in shear and finally showed the web crushing and the concrete near the crack tip in the compression zone crushed as shown in Fig. 7.

#### 4.5 Load versus Strain in PC bars

Figure 8 shows the variation of strain in the prestressing steel with the applied load measured in the test. Slope changes points of the curves in the figure indicated due to the occurrence of diagonal crack in the shear span. Before cracking, the strain in PC bars showed a slight increase with the applied load in all beams. After cracking, the strain tended to increase significantly at a rate and these increments were more prominent in the control beam when compared with the beam with fiber 1.0%. This can be explained by the fact that a part of the applied load is sustained by the fibers prior to its stretching and de-bonding. The same response can be seen in the beam, PC-1.5 and PC-2.0. Hence, the greater the fiber content with the same properties, the greater the load carried by beam.

### 5. COMPARISON OF TEST RESULTS

The obtained results for the beams with and without steel fibers are summarized in Table 5. Figure 9 shows the variation of  $V_{exp}/V_{cal}$  ratio with respect to the percentage of fibers in the concrete.

The results mentioned in the table and figure indicates that the prediction method has reasonable accuracy between the predicted and experimental results only in the case of the beams with fiber 0.0% and 1.0%. As it can be seen from Fig. 9, even for the high volume of steel fibers, 1.5% and 2.0%, the predicted method for the shear carrying loads overestimates the experimental results. This is expected that at higher fiber volumes ( $>1.0\%$ ), fiber balling

Table 5 Experimental results versus calculated results

Name	Fiber volume (%)	Aspect ratio	Bond shear strength ( $\tau_{bF}$ )	$\nu$	$V_F = \nu b_w d$	$V_{PC}$	$V_{cal}$	$V_{exp}$	$V_{exp}/V_{cal}$
			(N/mm <sup>2</sup> )	(N/mm <sup>2</sup> )	(kN)	(kN)	(kN)	(kN)	
PC-0.0	0.0	50	9.57	0.00	0.00	112	112.00	121.59	1.09
PC-1.0	1.0	50	9.50	1.76	30.75	113	143.75	147.66	1.03
PC-1.5	1.5	50	8.67	2.41	42.10	108	150.10	131.58	0.88
PC-2.0	2.0	50	8.77	3.24	56.77	109	165.77	137.38	0.83

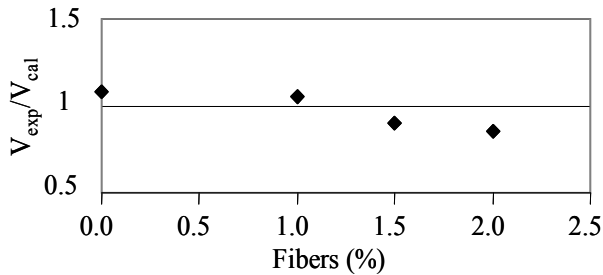


Fig. 9 Relationship of  $V_{exp}/V_{cal}$  on the fiber volume

invariably occurs although the mix proportions are selected carefully to provide proper fiber dispersions, resulting the decrease in compressive strength of concrete which leads to reduce the experimental shear carrying capacities of the beams. Thus, modification of the prediction method for evaluating the contribution of fiber pullout mechanism to shear stress and more testing are required in order to access this problem.

## 6. CONCLUSIONS

- (1) The addition of fiber reinforcement in prestressed concrete beams improved the shear carrying capacity.
- (2) Test results indicated that the enhancement on the shear carrying capacity was most prominent when adding 1.0% of steel fibers into the concrete mix.
- (3) The presence of steel fiber in concrete changed the basic characteristics of load-deflection, load-steel strain and cracking behaviors of the beams. After shear cracking stage, the strain in PC bar decreased with the inclusion of steel fibers resulting in higher load carrying capacity. The beam without steel fibers failed at lower load with a fewer cracks, higher crack widths and higher steel strain.
- (4) The theoretical method adopted in this paper has adequate accuracy in the case of the beams with fiber 0.0% and 1.0%. Beyond 1.0%, the predicted methods for shear carrying capacities are overestimate the experimental results.

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