*- Technical Paper -*

# THE EQUATION CONSIDERING CONCRETE STRENGTH AND STIRRUPS FOR DIAGONAL COMPRESSIVE CAPACITY OF RC BEAM

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ABSTRACT

Objectives of this study are to investigate the effect of compressive strength, stirrup ratio and stirrup spacing on the diagonal compressive capacity and propose the predicting equation for the diagonal compressive capacity of RC beams considering the effect of compressive strength and stirrup spacing. Five I-beams were tested by three-point bending. As a result, the effect of compressive strength and stirrup spacing on the diagonal compressive capacity is interrelated. The proposed equation provides a better estimation in case of using high-strength concrete than the existing equations.

Keywords: diagonal compressive capacity, high-strength concrete, web crushing, prediction equation

## 1. INTRODUCTION

Recently, high-strength concrete with the compressive strength ($f'_c$) more than 100 N/mm$^2$ and high-strength reinforcing bars with yield strength ($f_y$) more than 685 N/mm$^2$ have been developed. Such advanced materials have been applied into the construction practice since designers can take their advantages and propose economical infrastructure by reducing material volumes, allowing the increase in the span length of concrete bridges and the reduction of cross-sectional area of members. However, in thin web T- or I-shaped reinforced concrete (RC) beams with excessive shear reinforcements, an uncommon type of shear failure may occur. This type of failure is known as diagonal compression failure. It is caused by the crushing of web concrete prior to the yielding of stirrups.

Research on the mechanism of diagonal compression failure was insufficient since it was usually avoided. The predicting equation for the diagonal compressive capacity of RC beams in the current JSCE Standard Specifications for Concrete Structures [1] has underestimated the diagonal compressive capacity and not been appropriate for the application to RC beams using high-strength concrete as reported by Kobayashi et al. [2].

Besides, there are limited studies on diagonal compression failure in RC beams using high-strength concrete except for Kobayashi et al. [2]. They reported that the factors affecting the diagonal compressive capacity were $f'_c$, the shear reinforcement ratio ($r_w$), the shear-span to effective depth ratio ($a/d$) and the spacing of stirrups ($s$). However, in regard to some factors, e.g. $f'_c$, $r_w$ and $s$, there is a lack of experimental data especially in $f'_c$ > 100 N/mm$^2$ and $r_w$ < 2%.

Objectives of this research are to investigate the effect of $f'_c$, shear reinforcement ratio and stirrup spacing on the diagonal compressive capacity of RC beams and to propose the predicting equation for the diagonal compressive capacity of RC beams considering the effect of $f'_c$ and stirrup spacing. Furthermore, the accuracy of existing equations for the diagonal compressive capacity is verified and compared with the proposed equation.

## 2. REVIEW OF EXISTING EQUATIONS FOR DIAGONAL COMPRESSIVE CAPACITY OF RC BEAMS

### 2.1 The equation by Placas and Regan

Placas and Regan proposed an empirical equation for evaluating the diagonal compressive capacity as the following [3]:

$$V_{Placas} = (1.04 + 0.21 r_w)\sqrt{f'_c}\, b_w d \tag{1}$$

The factors involving the diagonal compressive capacity in this equation are $f'^{1/2}_c$ and the ratio of stirrup $r_w$ (%).

### 2.2 JSCE Standard

In JSCE standard specifications [1], only $f'^{1/2}_c$ is the influencing parameter of the diagonal compressive capacity. This equation is a conservative estimation and only valid for concrete with $f'_c$ lower than 50 N/mm$^2$.

$$V_{JSCE} = 1.25\sqrt{f'_c}\, b_w d \tag{2}$$

### 2.3 Eurocode 2 1992-1-1:2004

In Eurocode 2 [4], the variable strut inclination method was used to determine the maximum shear

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force which can be sustained by the member, limited by crushing of the compression struts. Assuming the angle of concrete strut, $\theta$, to be 45 degree, the following expression can be obtained.

$$V_{EC2} = \frac{1}{2} v f'_c b_w 0.9d \quad (3)$$

where; $v$ is a coefficient that takes into account the increase of fragility and the reduction of shear transfer by aggregate interlock with the increase of $f'_c$. It may be taken to be 0.6 for $f'_c \leq 60$ N/mm$^2$ and the minimum between 0.9-($f'_c$/200) and 0.5 for $f'_c > 60$ N/mm$^2$.

Since the existing equations did not consider some influential factors and are not applicable to high-strength range, further investigation is required.

## 3. EXPERIMENTAL PROGRAMS

### 3.1 Specimen details

The experimental program prepared five RC beams with I-shaped cross section. Three-point bending tests were conducted by a 2000kN capacity hydraulic testing machine. The summary of test variables and details of specimens are provided in Table 1 and Fig. 1. The main parameters were compressive strength of concrete $f'_c$, stirrup ratio $r_w$ with different diameter and spacing $s$. The experimental cases can be classified into two series: one is the cases for the effect of $r_w$ and $s$ in high-strength concrete beam with various diameters of stirrups (7.1, 10, 13 mm). The other is the cases for the effect of the $s$ with different $f'_c$. The other parameters and the specimen's dimension were the same as in the study by Kobayashi et al. [2] in order to compare and discuss with their results. The constant variables were the following: diameter of tensile bars of 22 mm, shear span ($a$) of 660 mm, the effective depth ($d$) of 220 mm, $a/d$ ratio of 3.0 and the total length of 1800 mm.

All specimens were designed to be symmetric and be able to resist against the flexure failure and the diagonal tension failure by using high-strength reinforcing bars ($f_y > 930$ N/mm$^2$) as tensile and shear reinforcements. In addition, the combination of thin web cross section with dense reinforcement will cause specimens to exhibit the diagonal compression failure. In order to avoid the local failure, the web width outside support was increased to that of the bottom flange. Anchor plates and bolts were used to ensure the sufficient anchorage of the tensile bars and prevent anchorage failure.

### 3.2 Instrumentation and test procedures

For all specimens, applied load, mid-span deflections and strains of concrete, longitudinal bars and stirrups were measured. Strain gauges were attached at the mid span to measure the strain of longitudinal bars whereas at the distance of $d$/2 from compression fiber for all stirrups in the shear spans. Besides, both surfaces of all specimens were painted by white color to ease the drawing and observation of crack during the experiments.

## 4. EXPERIMENTAL RESULTS

The experimental results including Kobayashi et al. [2] are summarized in Table 2. Data of the strains of

Table 1 List of experimental cases

| Specimen | $f'_c$ [N/mm$^2$] | $b_w$ [mm] | $d$ [mm] | $a/d$ | $b_f^{*1}/b_w$ | $p_w^{*2}$ [%] | $D^{*3}$ [mm] | $f_y$ [N/mm$^2$] | $r_w^{*4}$ [%] | $\phi_d^{*5}$ [mm] | $s^{*6}$ [mm] | $f_{wy}^{*7}$ [N/mm$^2$] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| UH1.2 | 100 | 40 | 220 | 3.0 | 6.25 | 8.8 | 22.2 | 1204 | 1.2 | 10 | 150 | 953 |
| UH1.5 | | | | | | | | | 1.5 | | 120 | |
| UH1.8 | | | | | | | | | 1.8 | | 100 | |
| SUHs90 | 130 | | | | | | | 930 | 3.5 | 13 | 90 | 955 |
| SUHs160 | | | | | | | | | 2.0 | | 160 | |

*1 upper flange width, *2 longitudinal reinforcement ratio, *3 nominal diameter of longitudinal bars, *4 stirrup ratio, *5 diameter of stirrups, *6 spacing of stirrups, *7 yield strength of stirrups.

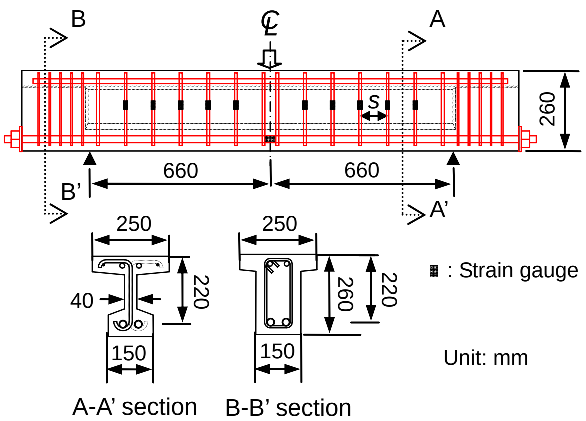


Fig. 1 Dimensions and steel layout of specimens

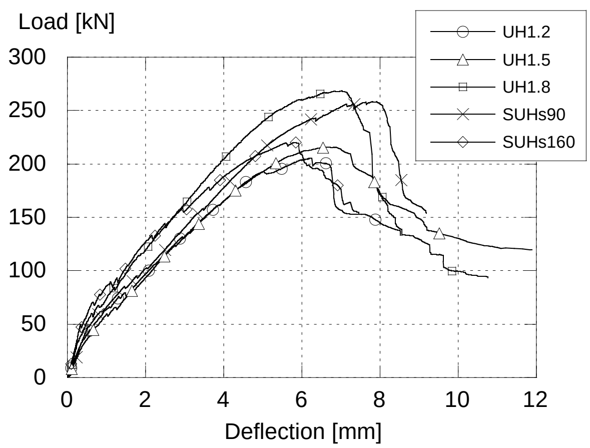


Fig. 2 Load-deflection relationships

Table 2 Summary of experimental results

| Specimen | $f'_c$ [N/mm$^2$] | $r_w$ [%] | $s$ [mm] | $a/d$ | $b_f/b_w$ | $V_{exp}{}^{*1}$ [kN] | $V_{exp}/V_{Placas}$ | $V_{exp}/V_{JSCE}$ | $V_{exp}/V_{EC2}$ | $v_{exp}{}^{*2}$ [N/mm$^2$] | $v_{exp}/f'_c{}^{1/2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| UH1.2 | 105 | 1.2 | 150 | | | 102.7 | 0.84 | 0.91 | 0.49 | 11.7 | 1.14 |
| UH1.5 | 101 | 1.5 | 120 | | | 108.0 | 0.86 | 0.98 | 0.54 | 12.3 | 1.22 |
| UH1.8 | 107 | 1.8 | 100 | | | 134.3 | 0.99 | 1.18 | 0.63 | 15.3 | 1.47 |
| SUHs90 | 131 | 3.5 | 90 | | | 129.3 | 0.69 | 1.03 | 0.50 | 14.7 | 1.28 |
| SUHs160 | 128 | 2.0 | 160 | | | 110.1 | 0.72 | 0.88 | 0.43 | 12.5 | 1.11 |
| UH2[2] | 102 | 2.0 | 90 | | | 137.0 | 1.00 | 1.23 | 0.68 | 15.6 | 1.54 |
| UH3[2] | 98.2 | 3.0 | 60 | | | 144.9 | 0.95 | 1.33 | 0.75 | 16.5 | 1.66 |
| UH4[2] | 99.1 | 4.0 | 45 | | | 167.6 | 0.97 | 1.53 | 0.85 | 19.0 | 1.91 |
| UH2s50[2] | 110 | 2.0 | 50 | 3.0 | 6.25 | 168.5 | 1.19 | 1.46 | 0.78 | 19.1 | 1.83 |
| UH2s160[2] | 115 | 2.0 | 160 | | | 96.6 | 0.67 | 0.88 | 0.43 | 11.0 | 1.03 |
| SUH3[2] | 133 | 3.0 | 60 | | | 162.0 | 0.96 | 1.28 | 0.62 | 18.4 | 1.6 |
| N06[2] | 32.0 | 0.63 | 160 | | | 59.4 | 0.97 | 0.95 | 0.78 | 6.7 | 1.19 |
| N1[2] | 33.4 | 1.0 | 160 | | | 58.6 | 0.88 | 0.92 | 0.74 | 6.7 | 1.15 |
| N2[2] | 35.8 | 2.0 | 80 | | | 61.6 | 0.76 | 0.94 | 0.72 | 7.0 | 1.17 |
| N3[2] | 35.0 | 2.9 | 55 | | | 61.7 | 0.68 | 0.95 | 0.74 | 7.0 | 1.18 |
| H2[2] | 73.8 | 2.0 | 80 | | | 114.5 | 0.99 | 1.21 | 0.74 | 13.0 | 1.52 |
| H3[2] | 61.5 | 2.9 | 55 | | | 107.8 | 0.89 | 1.25 | 0.75 | 12.3 | 1.56 |
| SSUH3[2] | 165 | 3.0 | 60 | | | 192.1 | 0.97 | 1.36 | 0.50 | 21.8 | 1.7 |

*1 the shear capacity from experimental result, *2 shear stress from experiment (=$V_{exp}/b_w d$).

longitudinal bars and stirrups revealed no yielding and the values were much less than their yield strength. It implies that the failure mode is neither flexure failure nor diagonal tension failure.

### 4.1 Load-deflection relationship

Load-deflection relationship is illustrated in Fig. 2. Firstly, specimens behaved in elastic manner until the first flexural crack occurred, which is reflected in the graph as a rate of inclination decreased. After the first flexure crack, the load-deflection curve remains to advance linearly with the initiation of diagonal crack at the web concrete. In the pre-peak region, the deflection increased with a relatively small increase in applied load until the web concrete began to crush. Data from the strain gage attached in stirrups and tensile bars reveals no yielding strain in all stirrups and tensile bars. After the peak load, applied load rapidly decreased.

### 4.2 Effect of $r_w$ and $s$

(1) Effect of $r_w$ and $s$ in specimens with different diameter of stirrups

Kobayashi et al. [2] adapted a method to eliminate the effect of $f'_c$ by normalized the obtained shear capacities by $f'_c{}^{1/2}$ which is used in the predicting equations of JSCE [1] and Placas et al. [3]. This method is also applied in this study. The relationships between $r_w$ and $v_{exp}/f'_c{}^{1/2}$ for the specimens with $f'_c$ = 100 N/mm$^2$ are demonstrated in Fig. 3. With the increase in $r_w$, the diagonal compressive capacity increases when using stirrups of $\phi_d$ = 10mm. A linear relationship agrees well with the test results. However, in the case of constant value of $r_w$ = 2% with different $\phi_d$ and $s$, $v_{exp}/f'_c{}^{1/2}$ differs significantly. Furthermore, the experimental results of the same set of data were plotted against $s$ despite of $r_w$ in Fig 4. Notwithstanding the diameter of stirrups and $r_w$ are different, the diagonal compressive capacities are distributed along the same linear line. It means that the diagonal compressive capacity can be represented by not $r_w$ but $s$. Therefore, the diagonal compressive capacity will be discussed by $s$ in the following section.

The difference in the diagonal compressive capacity with the change in $s$ can be explained by two mechanisms [2]. One is the increase of confinement effect on cracks by providing closer shear reinforcements. It results in smaller diagonal crack width ($w$); therefore, the critical average stress in web concrete ($f_{2max}$) would be greater because $w$ affected the diagonal compressive capacity as reported by Schäfer et al. [6] and Reinneck [7]. Second is the localization of compressive strut. Fig. 5 explains the model of compressive strut formation under different stirrup spacing proposed by Kobayashi et al. [2]. Fig. 5(a) demonstrates that the diagonal stress generates uniformly along the beam with close-spacing stirrups. In contrary, as shown in Fig. 5(b), the diagonal stress was concentrated in a local portion of the beam with wide-spacing stirrups. This stress concentration caused early crush in the web concrete; hence the diagonal compressive capacity decreased.

This mechanism can also be observed in this study. Fig. 6 illustrates crack patterns after loading. The thicker lines and the grey areas represent the wider crack width and crushing areas, respectively. In specimens with close spacing (UH1.8 and SUHs90), cracks distribute more finely and the crushing area at web distributes more widely than that of the wide-spacing specimens (UH1.2 and SUHs160). It implies that the failure localization occurred in UH1.2 and SUHs160. Since the confinement of stirrups helps stress to transfer across a crack after the crack initiation, the stress can distribute along the beam. Therefore, the localization of compressive strut seemed to be induced by a lack of the confinement effect. However, further study by some analytical approach is required to clarify actual stress flow.

(2) Effect of $s$ with different $f'_c$

Fig. 7 shows $v_{exp}/f'_c{}^{1/2}$ as a function of $s$ for specimens with different $f'_c$. It is confirmed that the effect of $s$ depends on $f'_c$. There were slightly variation in $v_{exp}/f'_c{}^{1/2}$ by the change in $s$ when using normal strength concrete while the greater effect of $s$ can be observed in high-strength concrete beams. As for beams with $f'_c > 60$ N/mm$^2$, the diagonal compressive capacity reduces as $s$ increases in linear relationship. With higher $f'_c$, the inclination of trend line increases with a limitation when $f'_c$ approximates 100 N/mm$^2$. It can be explained by considering that the capacity depends on not only $f'_c$ but also the width of compressive strut. If the width of compressive strut becomes smaller with higher $f'_c$, there is a possibility that the diagonal compressive capacity is limited.

### 4.3 Effect of $f'_c$

Fig. 8 plots the shear strength $v_{exp}$ ($=V_{exp}/b_wd$) as a function of $f'_c$. Since there was slight effect of $s$ when using $f'_c$ = 30 N/mm$^2$, the specimen N3 [2] is used as a representative of specimen with $f'_c$ = 30 N/mm$^2$ and $s$ = 60mm in this figure. It can be observed that $v_{exp}$ increases with the increase in $f'_c$. As discussed before, the $f'_c$ and $s$ are interrelated. It can be observed again in Fig. 8 as the incremental rate of $v_{exp}$ is different when using different $s$ and this increment rate decreased as $f'_c$ increased. From Fig. 6, there is not much difference in crack patterns after loading between specimens with different $f'_c$ but approximately the same $s$.

## 5. DEVELOPMENT OF EQUATION

### 5.1 Accuracy of the existing equations

The shear capacity from experimental results of

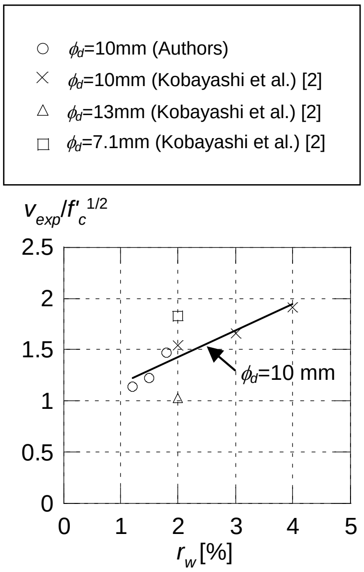

Fig. 3 Effect of $r_w$ ($f'_c$=100 N/mm$^2$)

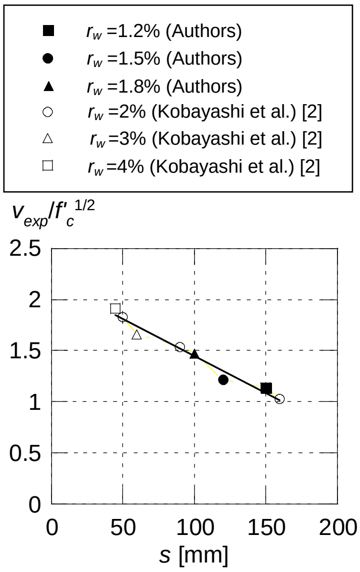

Fig. 4 Effect of $s$ ($f'_c$=100 N/mm$^2$)

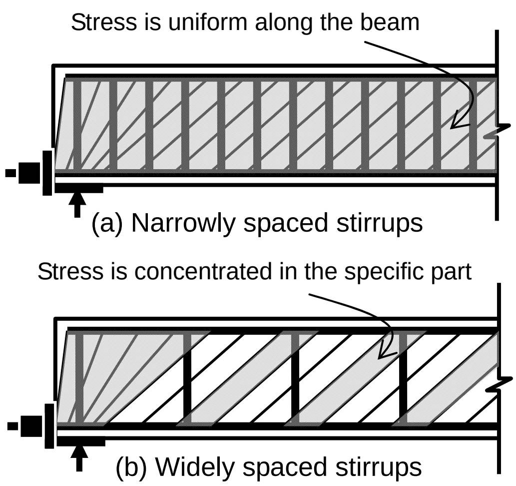

Fig. 5 Effect of spacing of stirrups

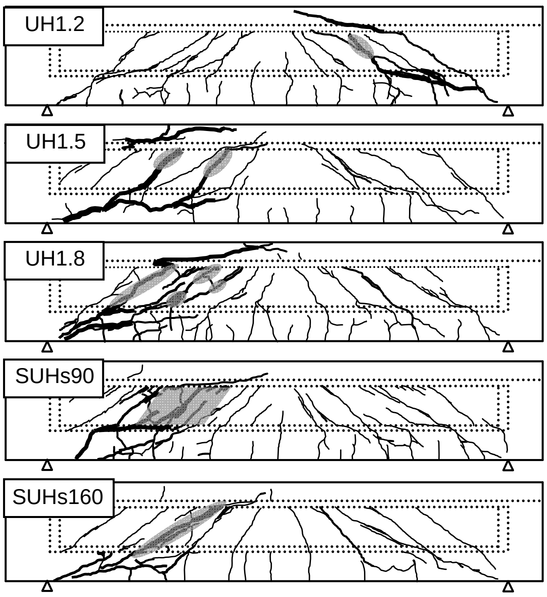

Fig. 6 Crack patterns after loading

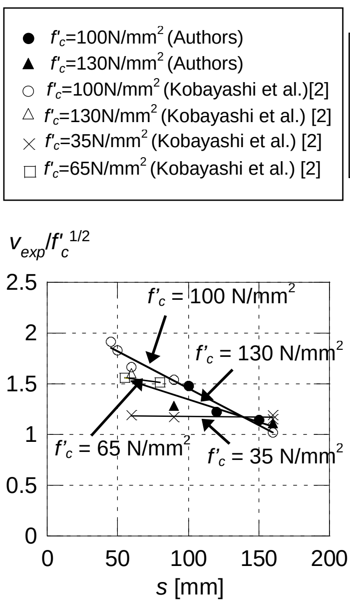

Fig. 7 Effect of $s$ with different $f'_c$

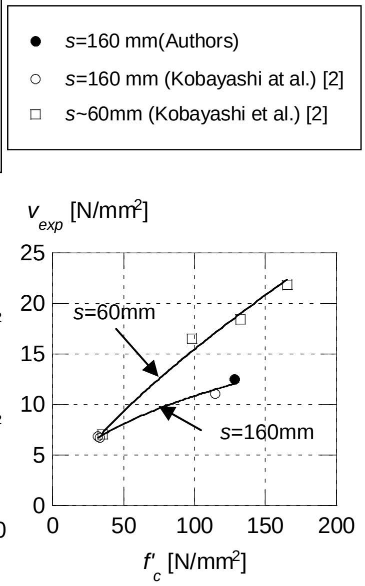

Fig. 8 Effect of $f'_c$ ($v_{exp}$- $f'_c$)

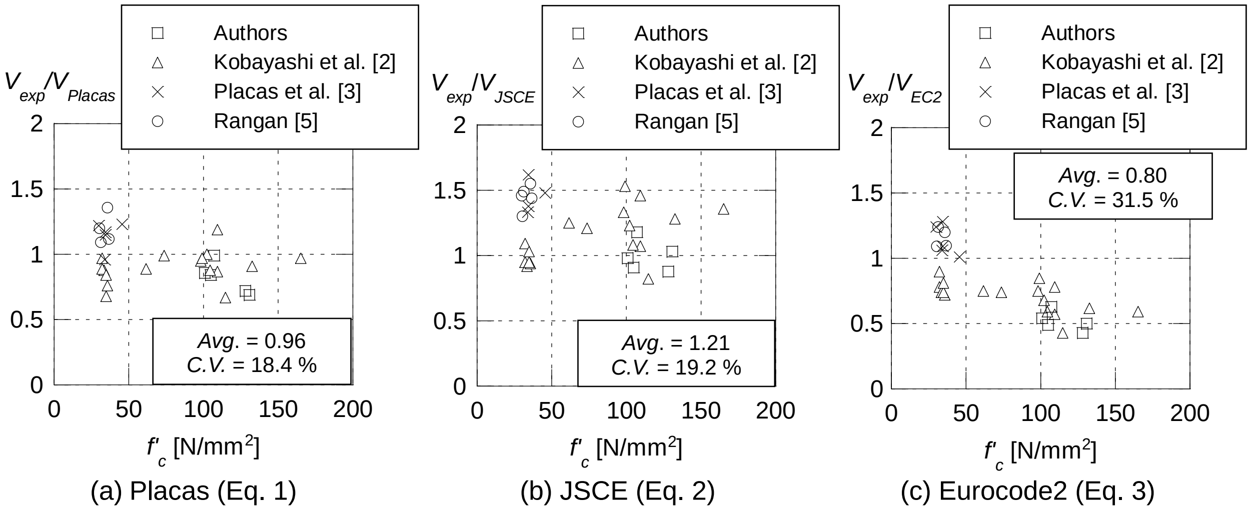


Fig. 9 Accuracy of the existing equations

the total of 31 beams including Kobayashi et al. [2], Placas et al. [3] and Rangan [5] was used to illustrate the accuracy of existing equations. The ratio between shear capacity and results calculated by the existing equation reviewed in chapter 2 are also listed in Table 2. The average of these ratios (*avg.*) with a coefficient of variation (*C.V.*) is presented in Fig. 9.

The average of $V_{exp}/V_{Placas}$ = 0.96 with a *C.V.* of 18.4%. It implies that Placas equation can evaluate an average value of the diagonal compressive capacity including the case of $f'_c$ > 100 N/mm$^2$. In contrary, *C.V.* of 18.4% implies that there are other factors affecting the diagonal compressive capacity except $f'_c$ and $r_w$. In addition, Fig. 9(a) presents that in the case of $f'_c$ > 50 N/mm$^2$, the equation of Placas et al. overestimates the diagonal compressive capacity.

JSCE standard demonstrated the average of $V_{exp}/V_{JSCE}$ = 1.21. Fig. 9(b) indicates that JSCE equation underestimates the capacity in almost all specimens. The specification may calculate the diagonal compressive capacity conservatively for safety reason. In the same way as Placas equation, the results calculated by the specification showed large variation as *C.V.* = 19.2%.

The accuracy of Eurocode2 for predicting the diagonal compressive capacity was shown in Fig. 9(c). The *avg.* of $V_{exp}/V_{EC2}$ is 0.80 with a *C.V.* of 31.5 %. Similar to Eq. 2, Eq. 3 overestimates the capacity in case of high-strength concrete and shows large scattering.

From the reasons mentioned above, the current design equations are not accurate enough to evaluate the diagonal compressive capacity of RC beams, especially in high-strength concrete. Besides, considering only $f'_c$ and $r_w$ in the existing equation are not appropriate.

### 5.2 Equation proposal

Assume a free body diagram cut in a vertical direction as shown in Fig. 10. The forces acting on cutting surface are 1) compressive force of concrete fiber, C', 2) diagonal compressive force of strut, D' and 3) tensile force of tension reinforcement, T. Considering the equilibrium in a vertical direction, the shear force, *V* must be resisted by the vertical component of *D'*. With the assumptions that the diagonal compressive force causes the failure of the member without yielding of stirrups, the following expression can be obtained.

$$V_{\max} = D'_{\max}\sin\theta = f_{2\max} b_w jd\cos\theta\sin\theta \qquad (4)$$

where; $V_{max}$ : shear force carried by web at web crushing stage (N).
$\theta$ : angle of inclination of concrete struts to the longitudinal reinforcement (degree)
$f_{2max}$ : critical average compressive stress in the web concrete (N/mm$^2$)

From influential factors discussed in chapter 4, $f_{2max}$ can be expressed by $\beta_c$ ($f_{2max}=\beta_c$). With the use of trigonometry, Eq. 5 is proposed based on Eq. 4

$$V_{\max} = \frac{1}{2}\beta_c b_w jd\sin 2\theta \qquad (5)$$

$\beta_c$ represents the effect of $f'_c$ and $s$. As mentioned in section 4.2, the relationship between the diagonal compressive capacity and $s$ demonstrated inverse variation. In addition, the effect of $s$ and $f'_c$ are interrelated. Section 4.3 indicated that the diagonal compressive capacity increases as $f'_c$ increases with different increment ratio depending on the value of $s$. This ratio decreases as $f'_c$ increases. The 3-D expression of $v_{exp}$-$f'_c$-$s$ was used to formulate $\beta_c$ having minimal variation and corresponding to the relationship observed from the experimental results. The $\beta_c$ was developed through a surface-fitting technique and was the following equation:

$$\beta_c = 3.93(1.25 - x)f'^{\,x}_c \qquad (6)$$

where; $x = 0.7 - \dfrac{s}{735}$ ($f'_c$: N/mm$^2$, $s$: mm) (7)

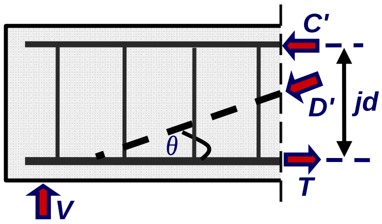


Fig. 10 Free-body diagram

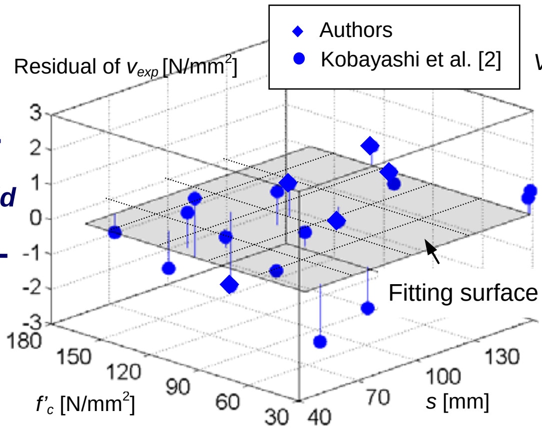


Fig. 11 Residual plot of $v_{exp}$-$f'_c$-$s$

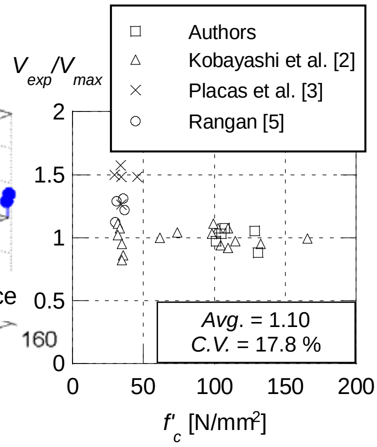


Fig. 12 Accuracy of proposed equation (Eq. 7)

The residual plot of $v_{exp}$-$f'_c$-$s$ demonstrated in Fig.11 shows a good accuracy of the fitting surface.

Kobayashi et al. measured the angle of principal compressive strain of struts of specimens N2AD45 and UH2AD35 [8]. It is revealed that this angle is constant and approximately 30 degrees at the peak load. This study assumes the angel of compressive strut, $\theta$ to be same as that of principal compressive strain. By substituting Eqs. 6 and 7 into Eq. 5, $jd$ = (7/8)$d$ as recommended by JSCE [1] and $\theta$ = 30 degrees, the diagonal compressive capacity can be calculated.

In Fig.12, the accuracy of the proposed equation was evaluated with the same database as in section 5.1. Although the average of 1.10 and *C.V.* of 17.8% are not satisfactory, Eq. 6 can predict the diagonal compressive capacity well with slight variation in case of $f'_c > 50$ N/mm$^2$. It is because this equation takes into account only the effect of $f'_c$ and $s$. However, $a/d$ and $b_f/b_w$ of Placas et al.'s specimens were designed to be 3.6 and 9.61, consequently. Kobayashi et al. observed a slight effect of $a/d$ in case of $a/d > 3.0$ [2]. It implies that the influence of $b_f/b_w$ should be considered. Although $b_f/b_w$ of 5.41-6.35 in Rangan's study approximates to the authors specimens, $a/d$ of Rangan's specimens were 2.5. It implies that the effect of $a/d$ is significant in case of $a/d$ = 2.5 to 3.0. It is hoped that the development of the equation considering all influential factors can be performed.

## 6. CONCLUSIONS

(1) The effect of compressive strength and stirrup spacing on the diagonal compressive capacity of RC beams is interrelated. The greater effect of stirrup ratio was observed in high-strength concrete. Shear capacity increases as compressive strength increases with different increment ratio depending on stirrup spacing. The increment ratio decreased as compressive strength increased.

(2) The predicting equation for the diagonal compressive capacity of RC beams which takes into account the effect of compressive strength and stirrup spacing was developed based on the experimental results. The proposed equation provided a better estimation of the diagonal compressive capacity in case of beams using high-strength concrete than the existing equations.

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