- Technical Paper -

THE EFFECT OF WIDE STIRRUP SPACING ON DIAGONAL COMPRESSIVE CAPACITY OF HIGH STRENGTH CONCRETE BEAMS

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ABSTRACT

To promote the rational design and wider use of high strength materials, this paper investigated the effect of wide stirrup spacing on the diagonal compressive capacity of RC beams and compared the results with the prediction by the existing equations. Two 500mm depth I-beams were tested by three-point bending. As a result, diagonal stress did not concentrate in local area of the specimens even wide stirrup spacing was used because of sufficient confinement effect provided by two-legged stirrups. The predictive equation by the authors cannot be implemented beyond its applicable range.

Keywords: diagonal compressive capacity, high strength concrete, web crushing, stirrup spacing

1. INTRODUCTION

Nowadays, designs of concrete infrastructures are required to be more economical and environmental friendly to establish sustainable society for future. The utilization of the recently developed high strength materials, e.g. high strength concrete with the compressive strength (f'_c) greater than 100 N/mm^2 and high strength reinforcing bars with the yield strength (f_y) greater than 685 N/mm^2 can satisfy this demand. Such advanced materials result in lower material consumption (smaller cross section, thin web concrete girders) and longer service life of infrastructures while maintaining remarkable structural performance. However, in case of reinforced concrete (RC) beams, the combination of thin web (T- or I-shaped cross section) and high strength shear reinforcing bars will lead to an uncommon type of shear failure known as diagonal compression failure. It is caused by the crushing of web concrete prior to the yielding of stirrups.

The research on the mechanism of the diagonal compression failure was insufficient since it was usually avoided because of its brittle phenomenon. The design equation for the diagonal compressive capacity of RC beams in the current JSCE Standard Specifications for Concrete Structures [1] only considers the effect of f'_c and limits the applicability to concrete with f'_c up to 50 N/mm^2.

Furthermore, limited studies on the diagonal compressive capacity of high strength RC beams have been performed, except for previous research by the authors (Tantipidok et al. [2]). They investigated the effect of various parameters on the diagonal compressive capacity of RC beams using high strength concrete and proposed an accurate and simple predictive equation for the diagonal compressive capacity of RC beams based on the experimental results. It was reported that the major factors affecting the diagonal compressive capacity were f'_c and stirrup spacing (s). The effect of shear-span to effective depth ratio (a/d), flange width to web width ratio (b_f/b_w) and effective depth (d) was found to have less influence on the diagonal compressive capacity in their study range. In the case of stirrup spacing, their proposed equation can be applied in the range from 45 mm to 160 mm. In real structures, which size of RC beams can be relatively larger, s can exceed 160 mm and the equation may not be applicable. Further validation is required.

The purposes of this research are to investigate the effect of stirrup spacing larger than 160 mm on the diagonal compressive capacity of relatively large RC beams and validate the predicting equation by the authors for the diagonal compressive capacity (Tantipidok et al. [2]). Finally, the accuracy of the existing equations is verified. This research is a step forward toward the development of new design equation for the diagonal compressive capacity of RC beams applicable to high strength concrete.

2. REVIEW OF THE EXISTING EQUATIONS FOR DIAGONAL COMPRESSIVE CAPACITY OF RC BEAMS

2.1 The equation by Tantipidok et al.

The authors investigated the effect of f'_c, r_w, s, a/d, b_f/b_w and d on the diagonal compressive capacity and proposed a simple predictive equation based on the experimental results as the following [2]:

$$V_{Tantipidok} = (1.9 - \frac{s}{190})\sqrt{f'_c}\,b_w d \qquad (1)$$

where; f'_c and s are in N/mm^2 and mm, respectively. The equation was proposed based on the results that the

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diagonal compressive capacity linearly decreased with larger stirrup spacing regardless of its diameter. This is because insufficient confinement effects provided by stirrups caused the localization of compressive stress in struts. Thus, s was prominent, rather than r_w, for evaluating the diagonal compressive capacity in range of s from 45 mm to 160 mm. The effect of spacing became more significant with higher concrete strength; hence, the effect of f'_c and s was interrelated. On the other hand, the effect of a/d from 3.0 to 4.5, b_f/b_w from 3.75 to 12.5 and d from 220 mm to 319 mm had almost no influence on the diagonal compressive capacity of RC beams. The equation can be implemented to beams with f'_c: 19-165 N/mm^2, r_w: 0.6-4%, s: 45-165 mm, b_f/b_w: 3.75-12.5, a/d: 2.5-4.5 and d: 220-563 mm.

2.2 The equation by Placas and Regan

Placas and Regan proposed an empirical equation for evaluating the diagonal compressive capacity as the following [3]:

$$V_{Placas} = (1.04 + 0.21 r_w)\sqrt{f'_c}\, b_w d \qquad (2)$$

Factors involving the diagonal compressive capacity in this equation are f'_c (N/mm^2) and the ratio of stirrup r_w (%). Although there is no upper limit of f'_c stated in this equation, the experimental evidences used to derive this equation approximates 35 N/mm^2.

2.3 JSCE Standard Specifications

In JSCE standard specifications [1], only f'_c (N/mm^2) is considered as the influencing parameter of the diagonal compressive capacity. Because this formula was originally proposed for application to normal strength concrete, the equation is only valid for concrete with f'_c not exceeding 50 N/mm^2.

$$V_{JSCE} = 1.25\sqrt{f'_c}\, b_w d \qquad (3)$$

3. EXPERIMENTAL PROGRAM

3.1 Specimen details

The experimental program prepared two RC beams with I-shaped cross section. Three-point bending tests were conducted by a 3000kN capacity testing machine. The summary of experimental cases and details of specimens are provided in Table 1 and Fig. 1, respectively. The main parameter was stirrup spacing (s) of 300 and 370 mm which are the maximum allowable stirrup spacing by the design standard ($s \le 0.75d$ or 300 mm) [1]. The constant variables were the following: the web width (b_w) of 80 mm, the effective depth (d) of 500 mm, shear span (a) of 1500 mm, a/d ratio of 3.0, longitudinal reinforcement ratio of 8.9% and the total length of 3600 mm. Assuming no effect of d based on the previous result [2], the size of the specimens in this research was greatly enlarged than in the previous one in order to provide wider s. Tensile reinforcements had two layers which D22 were used as top layer (T) while bottom layer (B) was D25. Compressive reinforcements were D32.

All specimens were designed to be symmetric and be able to resist against the flexure failure and the diagonal tension failure by using high strength

Table 1 List of experimental cases

Specimen	f'_c [N/mm^2]	b_w [mm]	d [mm]	a [mm]	a/d	p_w *1 [%]	D *2 [mm]	ϕ_d *3 [mm]	f_{wy} *4 [N/mm^2]	r_w *5 [%]	s *6 [mm]
s300	100	80	500	1500	3.0	8.9	22.2 (top) 25.4 (bottom)	13.1	1368	1.06	300
s370										0.88	370

*1 longitudinal reinforcement ratio (=100$A_s/(b_w d)$), *2 nominal diameter of tensile bars, *3 nominal diameter of stirrups, *4 yield strength of stirrups, *5 stirrup ratio (=100$A_w/(b_w s)$), *6 spacing of stirrups

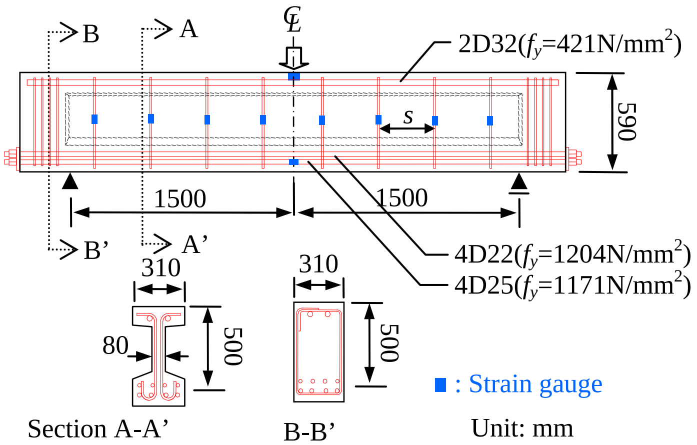

Fig. 1 Dimensions and steel layout of specimens

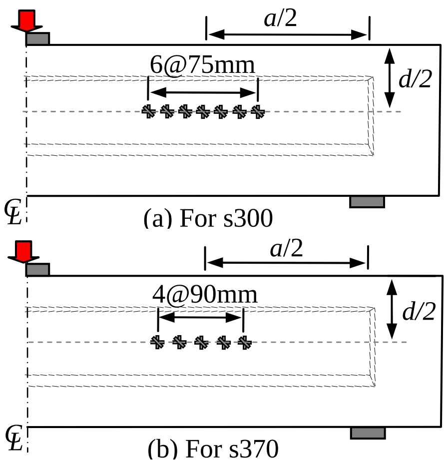

Fig. 2 Location of tri-axis strain gauges

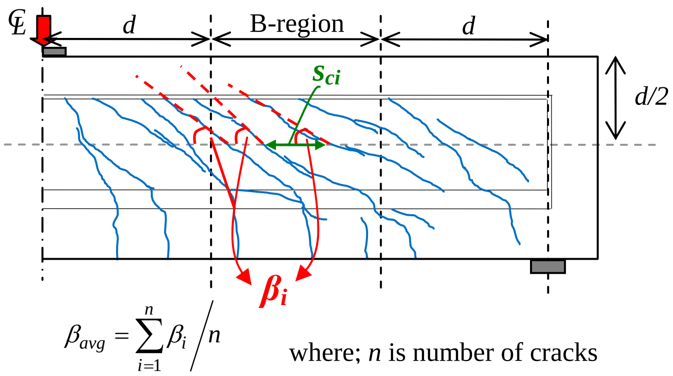

Fig. 3 Example of crack spacing and angle measurement

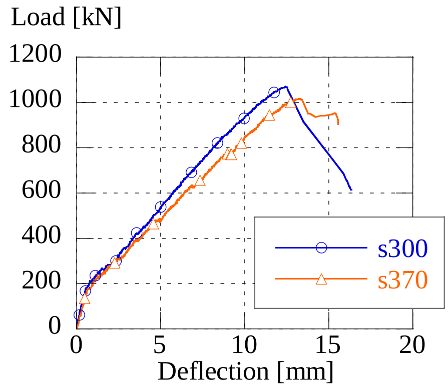

Fig. 4 Load-deflection relationship

Table 2 Experimental results

<table>
<tr><th>Specimen</th><th>f'_c [N/mm^2]</th><th>r_w [%]</th><th>s [mm]</th><th>ϕ_d [mm]</th><th>d [mm]</th><th>a/d</th><th>f_y [N/mm^2]</th><th>$\sigma_{s,max}$ *1 [N/mm^2]</th><th>f_{wy} [N/mm^2]</th><th>$\sigma_{w,max}$ *2 [N/mm^2]</th><th>$s_{c,avg}$ *3 [mm]</th><th>β_{avg} *4 [degree]</th><th>V_{exp} *5 [kN]</th><th>v_{exp} *6 $/f'^{1/2}_c$</th></tr>
<tr><td>s300</td><td>104</td><td>1.06</td><td>300</td><td rowspan="2">13.1</td><td rowspan="2">500</td><td rowspan="11">3.0</td><td rowspan="2">1171</td><td>400.8</td><td rowspan="2">1368</td><td>883</td><td>121.6</td><td>36.0</td><td>535.0</td><td>1.31</td></tr>
<tr><td>s370</td><td>105</td><td>0.88</td><td>370</td><td>724.4</td><td>904.8</td><td>144.9</td><td>33.7</td><td>508.5</td><td>1.24</td></tr>
<tr><td>C1.2-s150[2]</td><td>105</td><td>1.2</td><td>150</td><td rowspan="4">9.53</td><td rowspan="8">220</td><td rowspan="3">1204</td><td>401.4</td><td rowspan="3">953</td><td>550.8</td><td>91.0</td><td>38.8</td><td>102.7</td><td>1.14</td></tr>
<tr><td>C1.5-s120[2]</td><td>101</td><td>1.5</td><td>120</td><td>454.2</td><td>403.4</td><td>99.0</td><td>40.5</td><td>108.0</td><td>1.22</td></tr>
<tr><td>C1.8-s100[2]</td><td>107</td><td>1.8</td><td>100</td><td>613.2</td><td>549.6</td><td>92.5</td><td>36.7</td><td>134.3</td><td>1.47</td></tr>
<tr><td>C2-s90[2]</td><td>102</td><td rowspan="3">2.0</td><td>90</td><td rowspan="2">1198</td><td>601.0</td><td>954</td><td>561.2</td><td>69.9</td><td>31</td><td>137.0</td><td>1.54</td></tr>
<tr><td>C2-s50[2]</td><td>110</td><td>50</td><td>12.7</td><td>765.0</td><td>1397</td><td>721.0</td><td>60.0</td><td>36.5</td><td>168.5</td><td>1.83</td></tr>
<tr><td>C2-s160[2]</td><td>115</td><td>160</td><td>7.1</td><td>1214</td><td>420.6</td><td>955</td><td>300.8</td><td>86.0</td><td>35</td><td>96.6</td><td>1.03</td></tr>
<tr><td>C3-s60[2]</td><td>98.2</td><td>3.0</td><td>60</td><td rowspan="2">9.53</td><td rowspan="2">1198</td><td>700.0</td><td rowspan="2">954</td><td>538.6</td><td>-</td><td>-</td><td>144.9</td><td>1.66</td></tr>
<tr><td>C4-s45[2]</td><td>99.1</td><td>4.0</td><td>45</td><td>724.6</td><td>372.8</td><td>56.5</td><td>33</td><td>167.6</td><td>1.91</td></tr>
<tr><td>C-s100L[2]</td><td>108</td><td>2.2</td><td>100</td><td>12.7</td><td>319</td><td>1187</td><td>641.6</td><td>931</td><td>627.2</td><td>128.4</td><td>33.7</td><td>288.7</td><td>1.50</td></tr>
</table>

*1 maximum stress in tensile bars, *2 maximum stress in stirrups, *3 average crack spacing in horizontal direction at peak load, *4 average crack angle in B-region at peak load, *5 diagonal compressive capacity, *6 $v_{exp}=V_{exp}/(b_w d)$

reinforcing bars as both of the tensile and shear reinforcements. In addition, the combination of thin web cross section with dense stirrups will cause specimens to exhibit the diagonal compression failure. In order to avoid the local failure, the web width outside support was increased to that of the bottom flange. Anchor plates and nuts were used to ensure the sufficient anchorage of the tensile bars and prevent anchorage failure.

3.2 Instrumentation and test procedures

For all specimens, applied load, mid-span deflections and strains of concrete, longitudinal bars and stirrups were measured. Concrete strain gauges were attached at the top fiber of the mid span. Strain gauges were attached at the mid span to measure the strain of longitudinal bars whereas at the distance of d/2 from compression fiber for all stirrups in the shear spans. Angle of principle strain of web concrete was measured by tri-axis strain gauges. The locations of these strain gauges are illustrated in Fig. 2. Besides, surfaces of all specimens were painted by white color to ease the drawing and observing of cracks during the experiments. Pictures were taken by two digital single-lens reflex cameras for both shear spans. From the pictures taken at the peak load, the crack spacing in horizontal direction (s_{ci}) and the crack angle (β_i) were measured at the middle height of the web. The example of s_{ci} and β_i measurement of a crack is presented in Fig. 3. The average of s_{ci} of cracks in the shear span ($s_{c,avg}$) and the average of β_i of cracks in B-region (β_{avg}) will be used in the later discussion since it was observed that the crushing area, which is corresponding to the failure region, was mostly found in B-region (the portion outside the distance approximately d away from the loading point and supports).

4. EXPERIMENTAL RESULTS

4.1 Load-deflection relationship

Load-deflection relationships are illustrated in Fig. 4. Firstly, specimens behaved in elastic manner until the first flexural crack occurred in the bottom flange near the mid span, which is reflected in the graph as a rate of inclination decreases. After the first flexural crack, the load-deflection curve remained to advance almost linearly with the continuous initiation of diagonal cracks at the web concrete. In the pre-peak

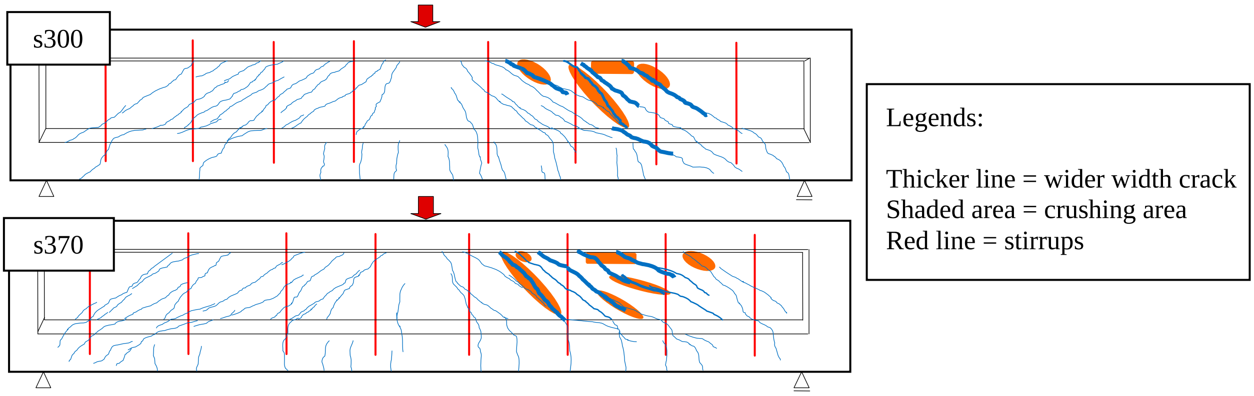

Fig. 5 Crack patterns just before the peak load

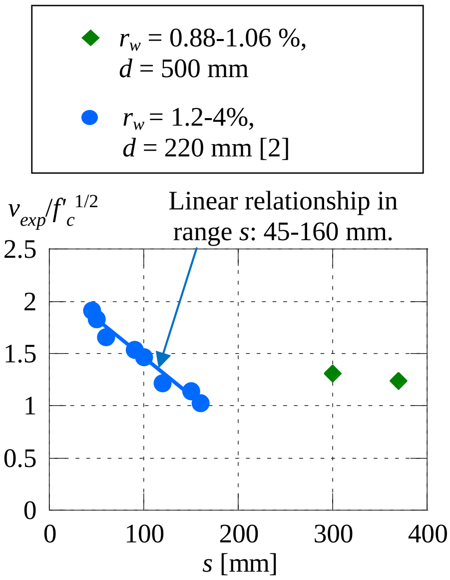

Fig. 6 Effect of s

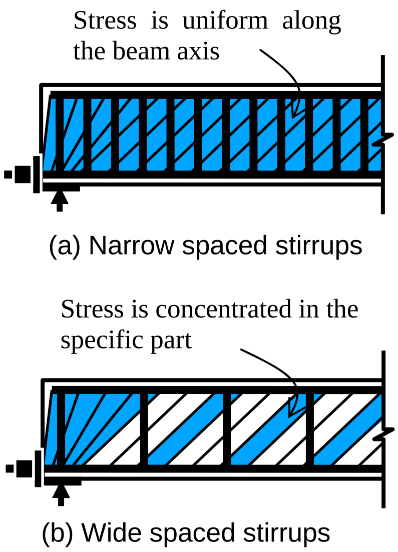

Fig. 7 Effect of spacing of stirrups

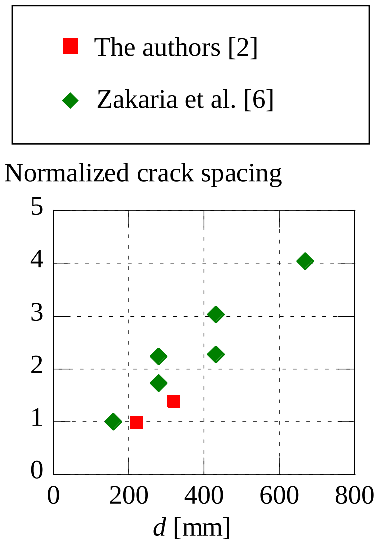

Fig. 8 Effect of d on diagonal crack spacing

region, the deflection increased with a relatively small increase in applied load as the web concrete began to crush. Afterwards, the applied load reached to the peak. After the peak load, the applied load rapidly decreased. The experimental results are summarized in Table 2. Data of the stresses of longitudinal bars and stirrups revealed no yielding at the peak load. It implies that the failure mode was neither the flexure failure nor the diagonal tension failure. The web concrete crushed at the peak load and splitting cracks along the member axis near tensile bars did not initiate at that time; hence, the cause of failure was not by anchorage failure of both tensile bars and stirrups. It can be concluded by considering these observations that the failure mode of all specimens was designated as the diagonal compression failure. The diagonal compression failure in which the web concrete crushed before the yielding of stirrup exhibited the brittle mode. Crack patterns just before the peak load are demonstrated in Fig. 5. The thicker lines and the shaded areas represent the wider width crack and the crushing areas, respectively.

4.2 Effect of wide stirrup spacing

The authors [2] adopted a method to eliminate the variation of f'_c by normalizing the obtained shear capacities ($v_{exp}=V_{exp}/(b_wd)$) by $f'_c{}^{1/2}$ which is used in the design equation of JSCE [1] and the predictive equation by Placas et al. [3]. This method is also applied in this study. The relationships between s and $v_{exp}/f'_c{}^{1/2}$ including the specimens in the authors' previous experiment [2] are demonstrated in Fig. 6. These specimens had f'_c approximately 100 N/mm^2. The specimens in the previous research [2] had s in the range from 45-160 mm and d of 220 mm while beams in this study had s = 300 mm and 370 mm and d = 500 mm. Figure 6 shows that the diagonal compressive capacity had a linear relationship with s when 45 mm $\leq s \leq$ 160 mm regardless of its diameters as reported by the authors [2]. They explained that this result came from two mechanisms caused by the confinement effect by stirrups. One is smaller diagonal crack width (w) caused by providing closer shear reinforcements; therefore, the critical average stress in web concrete (σ_{2max}) would be greater because w affects the diagonal compressive capacity as reported by Schäfer et al. [4] and Reineck [5]. The other is the localization of compressive strut. Figure 7 explains the model of compressive strut formation under different stirrup

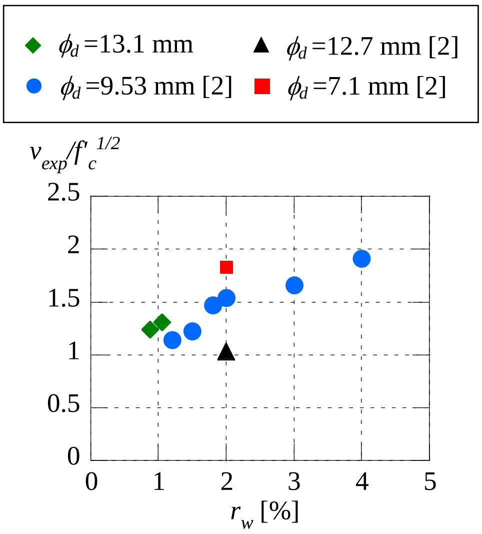

Fig. 9 Effect of stirrup ratio (r_w)

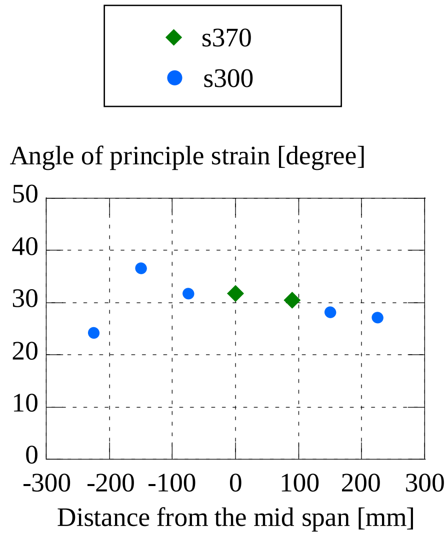

Fig. 10 Distribution of angle of principle strain

spacing. Figure 7(a) demonstrates that the diagonal stress generates uniformly along the beam axis with close-spacing stirrups. In contrary, as shown in Fig. 7(b), the diagonal stress is concentrated in a local portion of the beam with wide-spacing stirrups. This stress concentration causes early crushing in the web concrete; hence the diagonal compressive capacity decreases. The localization of compressive strut is induced by a lack of the confinement effect. It is because the presence of confinement effect by stirrups can prevent the excessive crack opening so that the stress can distribute along the beam axis. In the previous experiment [2], it was indicated that in case of the closer-spacing specimens, cracks distribute more finely and the crushing area at web distributes more widely than that of the wider-spacing specimens. As well, $s_{c,avg}$ shows a decreasing trend with closer stirrup spacing and it was implied that the failure localization occurred when wider $s_{c,avg}$ were observed [2]. Zakaria et al. [6] reported that larger beams caused greater diagonal crack spacing. The relationship between crack spacing normalized by crack spacing of the smallest specimens and effective depth is illustrated in Fig. 8. It can be observed that the normalized crack spacing increases proportionally with the increase in the size of specimens.

The experimental results obtained in this study do not correspond to the previous experiment as can be seen from Fig. 6. Even though s was greatly increased, the diagonal compressive capacity increased. Cracks distributed finely and the crushing area at web distributed widely as can be observed from Fig. 5. Considered from the previous results that $s_{c,avg}$ should be proportional with the increase of s and d, $s_{c,avg}$ of both specimen were relatively narrow. From these evidences, it implies that the confinement effect by stirrups was sufficient in this case and the diagonal stress did not concentrate in a local area of the specimens even wide stirrup spacing was used. It is because the usage of two-legged stirrups of ϕ_d =13.1 mm in this study resulted in higher effective area in which the confinement effect by the stirrup can control excessive crack opening while the usage of single-legged stirrup in the previous experiment, which the effective area was comparatively smaller, induced the localization of compressive strut. When the diagonal stress is uniform, stirrups would only affect the diagonal compressive capacity by the former mechanism, which is influenced by stirrup ratio. Therefore, the same set of experimental results is plotted against r_w in Fig 9. It can be noticed that r_w can represent the diagonal compressive capacity in the range of s from 300-370 mm, although it was discovered by the authors [2] that s is the governing factor rather than r_w in the range of s = 45-160 mm.

4.3 Angle of principle strain and diagonal crack

The results of measured angle of principle strain at the peak load are exhibited in Fig. 10. The location of the measurement was shown in Fig. 2. Some gauges were broken during the experiments. The angle of principle strain varies from 24.3 to 36.6 degrees. The average angle of principle strain of s300 and s370 are 29.6 and 31.2 degrees, respectively.

As for the angle of diagonal crack, the measurement was focused in B-region since it was observed that the crushing areas, which are corresponding to the failure regions, were mostly found in that region. In the previous research [2], a clear tendency of β_{avg} and each possible factor influencing the crack angle cannot be found. β_{avg} was varied from 27 to 47 degrees and the average values β_{avg} for all specimens was 36.6 degrees. From Table 2, β_{avg} of s300 and s370 are 36.0 and 33.7 degrees, respectively. These values correspond to those observed in the previous experiment.

5. COMPARISON WITH THE EQUATIONS

Table 3 presents ratios between the diagonal compressive capacities from the experiments including by the authors' previous experiment [2] and results obtained by the equations reviewed in chapter 2. The average of these ratios (*avg.*) with a coefficient of

Table 3 Comparison between the experiment and calculated results

Specimen	f'_c [N/mm^2]	b_w [mm]	d [mm]	a/d	r_w [%]	ϕ_d [mm]	s [mm]	$V_{exp}/V_{Tantipidok}$	V_{exp}/V_{Placas}	V_{exp}/V_{JSCE}
s300	104	80	500	3.0	1.06	13.1	300	*	1.05	1.04
s370	105				0.88		370	*	0.99	1.01
C1.2-s150[2]	105	40	220		1.2	9.53	150	1.03	0.84	0.91
C1.5-s120[2]	101				1.5		120	0.96	0.86	0.98
C1.8-s100[2]	107				1.8		100	1.07	0.99	1.18
C2-s90[2]	102				2.0		90	1.08	1.00	1.23
C2-s50[2]	110				2.0		50	1.12	1.19	1.46
C2-s160[2]	115				2.0	12.7	160	0.97	0.67	0.88
C3-s60[2]	98.2				3.0	9.53	60	1.05	0.95	1.33
C4-s45[2]	99.1				4.0		45	1.15	0.97	1.53
C-s100L[2]	108				2.2	12.7	100	1.09	0.95	1.20
							avg.	-	0.95	1.16
							C.V.	-	13.8%	18.8%

*: Out of the application range

variation (*C.V.*) is also provided in Table 3. The average of V_{exp}/V_{Placas} equals to 0.95 with a *C.V.* of 13.8 %. It implies that Placas's equation (Eq. 2) can evaluate an average value of the diagonal compressive capacity even f'_c approximates 100 N/mm^2. On the contrary, this equation exhibits large variation as *C.V.* equals to 13.8 %. JSCE standard specifications (Eq. 3) demonstrates the average of V_{exp}/V_{JSCE} = 1.16. The specifications may be conservative because of safety reason. The results calculated by Eq. 3 gives larger variation than Eq. 2 as *C.V.* = 18.8%.

In the case of s300 and s370, the equation by Placas et al. (Eq. 2) and JSCE equation (Eq. 3) are accurate while the equation by the authors (Eq. 1) cannot predict the diagonal compressive capacity accurately. This is an expected outcome since Eq. 1 is an empirical equation which it may not be applicable outside of its application range. Within its range of application, the authors' equation results precise prediction (*avg.* = 1.06, *C.V.* = 6.0%). In addition, the fact that Placas's equation is accurate against s300 and s370 implies that r_w is an appropriate indicator for the diagonal compressive capacity in this case. From these reasons, further investigation is still required.

6. CONCLUSIONS

The experiment of practical size and thin web reinforced concrete beams with wide stirrup spacing was carried out. The research contributes to more general use of high strength materials and further development of rational design approach for diagonal compressive capacity. As a result, even though wide stirrup spacing was provided, the diagonal cracks distributed finely, the crushing area at web distributed widely and the average crack spacing in horizontal direction of both specimens was relatively narrow compared to the previous experiments. It implies that the diagonal stress did not concentrate in a local area of the specimens. It is because sufficient confinement effect provided by two-legged stirrups can prevent the localization of compressive stress in struts. Although it was reported by the authors that the stirrup spacing is important for evaluating the diagonal compressive capacity when using stirrup spacing from 45 mm to 160 mm; on the other hand, the stirrup ratio was a better representative for predicting the diagonal compressive capacity in the range of *s* from 300-370 mm.

REFERENCES

[1] Japan Society of Civil Engineers (JSCE), "Standard Specifications for Concrete Structures-2007, Design," 2007

[2] Tantipidok, P. et al., "Proposed Predictive Equations for Diagonal Compressive Capacity of Reinforced Concrete Beams," Journal of Japan Society of Civil Engineers, Ser. E2 (Materials and Concrete Structures), Vol. 67, No. 4, 2011, pp. 535-548.

[3] Placas, A. and Regan, P. E., "Shear Failure of Reinforced Concrete Beam", ACI Journal, Vol. 68, No. 10, 1971, pp. 763-774.

[4] Schäfer, K., Schelling, G. and Kuchler, T., "Compression and Transverse Tension in Reinforced Concrete Elements," Deutscher Ausschuss für Stahlbeton No. 408, Beuth Verlag, GmbH, Berlin, 1990, pp. 1-85.

[5] Reineck, K. H., "Theoretical Considerations and Experimental Evidence on Web Compression Failures of High Strength Concrete Beams," CEB Bulletin D'Information No. 193, Dec. 1989, pp. 59-74.

[6] Zakaria, M. et al., "Experimental Investigation on Shear Cracking Behavior in Reinforced Concrete Beams with Shear Reinforcement", Journal of Advanced Concrete Technology, Vol. 7, No. 1, Feb. 2009, pp. 79-96.