- Technical Paper -

PROBABILISTIC RESTORATION DESIGN FOR RC SLAB BRIDGES

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ABSTRACT

In cases of older bridges, assessing as much design information as possible is difficult. Restoration design is an important method to estimate the initial condition of structures. In this study, a probabilistic restoration design, based on latin hypercube sampling (LHS) of random variables, has been performed for RC slab bridges. Uncertainty and sensitivity analysis of random variables are also performed. From the analysis result, it is observed that measurements of total depth and compressive strength of concrete are the most influencing factors in the estimation of yield strength of steel.

Keywords: Restoration design, random variables, uncertainty, sensitivity analysis

1. INTRODUCTION

Capacity performance assessment of bridges is one of the important tasks in bridge management cycle. To accomplish this task, design documents, specifications and standards are of great importance. In the absence of these data, assessing bridge's condition is difficult. Thus, initial condition needs to be restored.

Restoration design is a method of estimating initial condition of structures and it is affected by current measurements. Since measurements are inexact and contain errors of unknown magnitude, their effects on the restoration design have to be investigated. In this study, for such investigation, a probabilistic restoration design is used. A probabilistic approach of restoration design of RC slab bridges considering different combinations of random variables, their influences and uncertainties based on the concept of sensitivity analysis are investigated.

There are different random variables which affect the restoration design process. To perform restoration design, different methods can be applied. In this study, a probabilistic approach of restoration design considering different combinations of random variables based on the Latin Hypercube Sampling (LHS) method is discussed.

The probabilistic distribution and the influence of random variables on the statistical variation of the estimated yield strength of steel, cross sectional area of steel and effective depth are investigated. Moreover, based on the statistical variation of random variables, sensitivity analysis in the vicinity of mean values is conducted.

2. RESTORATION DESIGN

Restoration design is a process of accurately describing the initial condition (design values) of a structure from its current condition (actual values). Restoration design uses design values. On the other hand, if deflection is used as additional data, it reflects the present actual values. Also an indirect method for the estimation of yield strength of steel is needed. The required dimensions of the structure are determined through measurements [1].

Restoration design, which is important to estimate the initial condition of bridges, is a basic tool for capacity performance assessment. Non-destructive tests for the estimation of current concrete strength, mid-span deflection of the bridge from load test and position of reinforcing bars using an electric magnetic device are main inputs for the restoration design [1].

Based on the flow of restoration design of RC slab bridges of known effective depth [1], the flow chart for probabilistic restoration design is shown in Fig. 1. For unknown effective depth, load tests at least at two positions should be done. For more accurate results, as many as measurements are needed.

2.1 FEM Simulation

For statistical analysis, two cases (each condition comprises of 32 combinations) with a total of 64 RC slab bridges are simulated using FEM. The mesh consists of rectangular elements of 0.05m in size with 4 nodes and SBeta material model of ATENA 2D v 4.2.2 [2] is used. The outcome of each case, effect of the random variables and the probabilistic distribution of the outcome are analyzed and plotted. Table 1 shows the two cases considered in FEM simulation.

Table1 Two cases of FEM simulation

Case 1 Normal condition	Case 2 Extreme condition
Slip is disabled at bar beginning and end points (fully anchored bars)	Slip is allowed at bar beginning and end points

The input parameters used in the FEM simulation are bridge dimensions and material properties. These random variables are continuous and contains small scatter.

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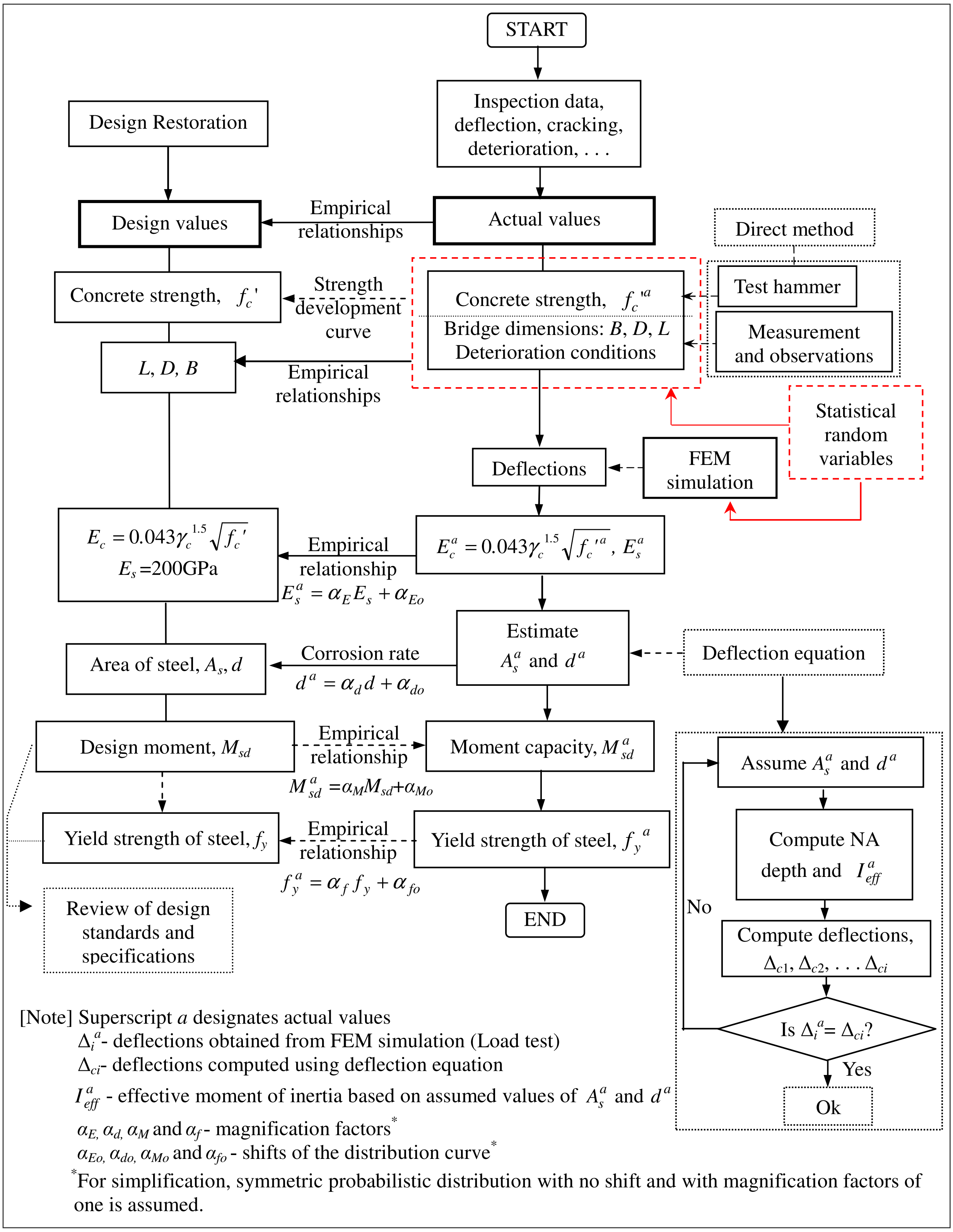

Fig.1 Flow of restoration design of RC slab bridges

2.2 Estimation of Yield Strength of Steel

Empirical formulae for the estimation of yield strength of steel by considering the effects of yielding moment with triangular section, concrete strength, Young's modulus of steel and moment capacity are obtained [3]. In reference to Fig. 2, for doubly reinforced section, the following equations hold true.

$$M^a = M_1^a + M_2^a \tag{1}$$

$$M_1^a = A_{s1}^a f_y^a (d^a - y^a/3) \tag{2}$$

$$M_2^a = A_{s2}^a f_y^a (d^a - d_c') \tag{3}$$

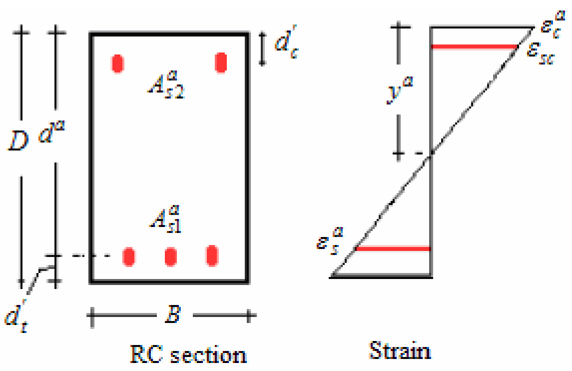

Fig.2 RC section and strain diagram

$$y^a = \frac{\varepsilon_c^a}{\varepsilon_c^a + \varepsilon_s^a} d^a \tag{4}$$

$$\varepsilon_c^a = f_c{'}^a / E_c^a, \quad \varepsilon_s^a = f_y^a / E_s^a \tag{5}$$

$$E_c^a = 0.043 \gamma_c^{1.5} \sqrt{f_c{'}^a} \tag{6}$$

$$d_t^{'} = d_c^{'} \rightarrow d^a - d_c^{'} = 2d^a - D \tag{7}$$

$$A_s^a = A_{s1}^a + A_{s2}^a \tag{8}$$

$$A_{s2}^a = \beta A_s^a \rightarrow A_{s1}^a = (1-\beta) A_s^a \tag{9}$$

where,

M_1^a : bending moment carried by the concrete and a part of tensile steel (N-mm/m)

M_2^a : bending moment carried by A_{s2}^a and a rest of tensile steel (N-mm/m)

M_{sd} : ultimate design moment (N-mm/m)

M^a : actual flexural capacity of the section (N-mm/m)

A_{s1}^a : actual area of tensile steel (mm^2/m)

A_{s2}^a : actual area of compression steel (mm^2/m)

f_y^a : actual yield strength of steel (MPa)

$f_c{'}^a$: current concrete strength (MPa)

E_s^a : Young's modulus of steel (GPa)

E_c^a : Young's modulus of concrete (GPa)

γ_c : unit weight of concrete (24kN/m^3)

β : ratio of A_{s2}^a to A_s^a

m_d $= M_{sd} / A_s^a d^a$ (N/mm^2)

α_M $= M^a / M_{sd}$

Upon substitution and simplification, Eq. 1 is simplified to the following form.

$$A(f_y^a)^2 + Bf_y^a + C = 0 \tag{10}$$

where,

$A = (1 - \beta + \beta\eta)$

$B = 13.19\alpha_E (2 - 2\beta + 3\beta\eta)\sqrt{f_c{'}^a} - (1+\alpha_M) m_d$

$C = -39.56\alpha_E (1+\alpha_M) m_d \sqrt{f_c{'}^a}$

$\eta = (2d^a - D)/d^a$

$\alpha_E = E_s / E_s^a$ ($E_s = 200$GPa)

To obtain the value of A_{s2}^a, additional load test at different position should be conducted. As per AASHTO LRFD Bridge Design Specification [3], the minimum amount of top reinforcements for shrinkage and temperature for RC slab bridges to be provided is given as follow.

$$A_{s2}^a \geq \frac{\phi A_g}{f_y^a}, \ \phi = 0.75\text{MPa} \tag{11}$$

From Eqs. 9 and 11, β can be computed.

$$\beta = \frac{\phi A_g}{A_s^a f_y^a}, \ \phi = 0.75\text{MPa} \tag{12}$$

Substituting Eq. 12 into Eq. 10, gives the following simplified form.

$$(f_y^a)^2 + B' f_y^a + C' = 0 \tag{13}$$

where,

$B' = 26.29\alpha_E \sqrt{f_c{'}^a} - \phi\alpha_a\xi - (1+\alpha_M) m_d$

$C' = -39.56\alpha_E (\phi\alpha_a\varphi - (1+\alpha_M) m_d)$

α_a : ratio of A_g to A_s^a

A_g : gross area of the section=BD (mm^2)

ξ $= (D - d^a)/d^a$

φ $= (4d^a - 3D)/d^a$

For singly reinforced section, the value of β is zero. Use a regression analysis to obtain a general empirical equation for the estimation of actual yield strength of steel. Thus, the following empirical relationship using quadratic interpolation [1] has been obtained.

$$\begin{aligned} f_y^a = &-0.000215(1+\alpha_M)^2 m_d^{\,2} + (0.00099 f_c{'}^a \\ &+ 0.053\alpha_E + 1.141)(1+\alpha_M) m_d + 7.956 \end{aligned} \tag{14}$$

3. STATISTICAL ANALYSIS

3.1 Probabilistic Analysis

To consider the probabilistic distribution of random variables on effect of the restoration design result, the concept of normal distribution is used. The assumptions considered in the analysis are random variables are independent of each other and they follow normal distribution. The probability function, $f(x)$, using normal distribution [4] is given in Eq. 15.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \tag{15}$$

where,

x : random variable

μ : mean value

σ : standard deviation

The distribution of cover thickness of main reinforcing bars and a probabilistic analysis of random variables of a standard RC slab bridge, by using LHS method, are performed.

Quantitatively, for the RC slab bridge, sensitivity of the parameters in the estimation of actual yield strength of steel from Eq. 14 is analyzed.

In the LHS sampling method, the cumulative distribution function of each factor is divided into intervals with equal probability, and then sampling is done by only from each interval [5]. The arrangement of sampling intervals and sampling of random variables is shown in Fig. 3 [5]. The 32 combinations of random variables of LHS table are sufficient in this simulation.

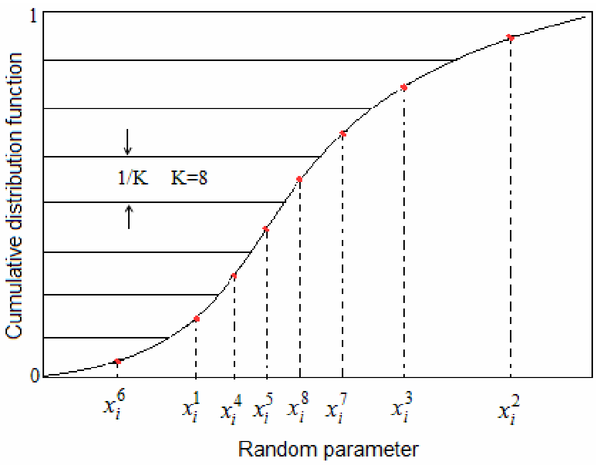

Fig. 3 Arrangement of sampling intervals and sampling of random variables

A standard RC slab bridge with center to center length of 10.40m is designed as per AASHTO LRFD Bridge Design Specification [3] and simulated using FEM. To obtain the maximum live load effect on the structure, the concept of influence lines is used. The random variables used in this study are span length, effective strip width, total depth and compressive strength of concrete.

The allowable limits of measurements by AASHTO LRFD Bridge Construction Specification [6] are considered. In the simulation, half of the allowable limits permitted are considered as a standard deviation (σ). The statistical parameters of random variables are shown in Table 2.

The material properties used are: f_y=400MPa and E_s=200GPa. Diameter 32mm reinforcing bars with c/c spacing of 180mm and cover thickness of 35mm are used. A two-point load with a c/c spacing of 2.0m and three load positions (location of rear loads are at 2.0m, 3.0m and 4.2m from the left support) are considered.

Based on the results, incremental instantaneous mid-span deflection due to applied load, the actual area of reinforcing bar and effective depth are computed using the elastic deflection equation by a trial and error procedure.

The restoration design scheme is shown in Fig. 4. Finally, the actual yield strength of steel is estimated using Eq. 14. For the estimation of f_y^a, the ultimate design moment, M_{sd}, of 705kN-m/m is used.

Table 2 Statistical parameter of random variables

No.	Random variables	Mean values	Std. dev.
1	Span length (mm)	10400	5.20
2	Effective width (mm)	3250	1.62
3	Total depth (mm)	540	6.00
4	Compressive strength of concrete (MPa)	28	1.40

Results of statistical analysis showing mean value, standard deviation and coefficient of variation of the set of random variables is shown in Table 3.

Table 3 Results of statistical analysis

	Estimated values (Case 1)			Estimated values (Case 2)		
	d^a (mm)	A_s^a (mm^2/m)	f_y^a (MPa)	d^a (mm)	A_s^a (mm^2/m)	f_y^a (MPa)
Mean	489.47	4636.45	377.51	490.50	3566.03	340.13
Std. dev.	5.73	393.64	30.09	3.39	426.99	36.44
COV	0.012	0.085	0.078	0.007	0.119	0.107

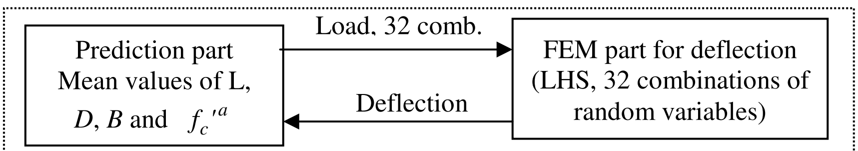

Fig. 4 Restoration design scheme

The ratio (A_s^a of Case 1) to (A_s^a of Case 2) is assumed to be equal to the ratio of the actual to ultimate moment capacity of the section, M^a/M_{sd}. Thus, for Case 2, α_M of -0.30 is used and the corresponding values of d^a , A_s^a and f_y^a are estimated accordingly.

The probabilistic distributions of the estimated actual yield strength of steel and area of steel, for both conditions, following the normal distribution function are shown in Fig. 5 and Fig. 6, respectively. Moreover the cumulative percent distributions are plotted.

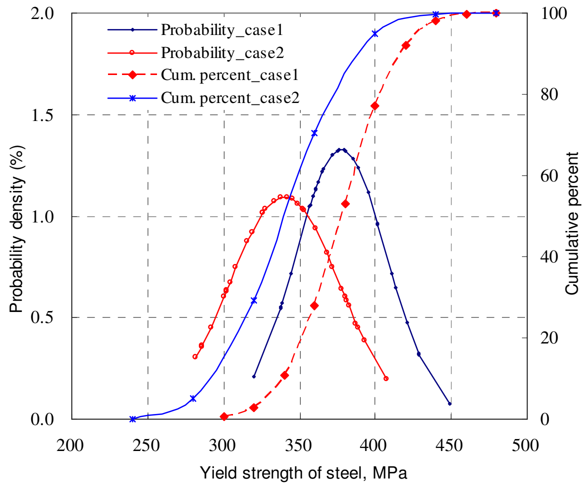

Fig. 5 Probabilistic distribution of f_y^a

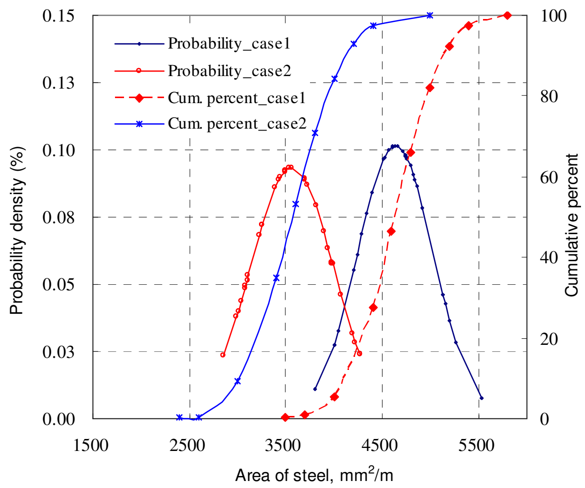

Fig. 6 Probabilistic distribution of A_s^a

Based on the results, confidence limits for the mean values with 95 % confidence levels are estimated. Thus, the confidence intervals for the mean value of f_y^a are shown in Table 4.

Using the estimated effective depth, the cover thickness of the simulated bridge for both cases are computed and their cumulative percent distributions are plotted in Fig. 7.

For Case 1, an average concrete cover of 33.75mm with a standard deviation of 3.12mm and COV of 9.28% is obtained, which has a variation of -3.57% from the actual cover thickness. Almost the same result is obtained for Case 2.

Table 4 Confidence intervals for f_y^a

Cases	f_y^a (MPa)	
	Lower limit	Upper limit
Case 1	367.08	387.93
Case 2	327.51	352.76

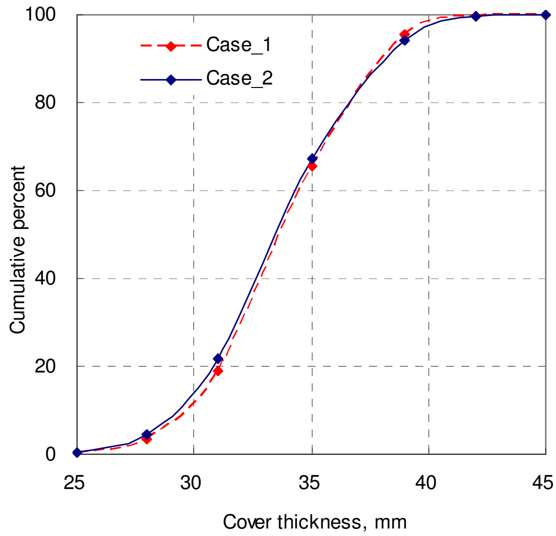

Fig. 7 Distribution of cover thickness

3.2 Sensitivity Analysis

Sensitivity analysis is the study of how the variation (uncertainty) in the output of a statistical model can be attributed to different variations in the inputs of the model.

The sensitivity of each random variable is represented by the squared value of the partial coefficient of correlation (r_p^2). The sensitivity factor α_i based on the first-order approximation second-moment method [7] is used.

The effect of random variables on the estimated actual yield strength of steel is investigated. To determine their effects, sensitivity analysis is performed.

$$\alpha_i = \frac{\partial F}{\partial x_i}\frac{\overline{x_i}}{\overline{F}} \qquad (i = 1,2,3,. ..,n) \qquad (16)$$

where,

α_i : sensitivity factor of random variable i
$\overline{x}$, $\overline{F}$: means of x_i and F, respectively
F : function with statistical variations
x_i : random variable i

The sensitivity factor α_i is a kind of index to estimate the contribution of the uncertainty of x_i to the uncertainty of F. Using the central difference approximation equations, the uncertainty of each random variables can be obtained [7].

$$\frac{\Delta F}{\Delta x} = \frac{F(\bar{x}+\Delta x) - F(\bar{x}-\Delta x)}{2\Delta x} \tag{17}$$

where,
Δx : infinitesimal part of each random variable

In this analysis, 1/1000 of the mean value is taken as Δx. The contributions of the uncertainty of each random variable are obtained by multiplying the sensitivity factor by its coefficient of variation.

$$U_{i.F} = \alpha_i (COV_i) \tag{18}$$

where,
$U_{i.F}$: the contributions of the uncertainty of random variable i
COV_i : the coefficient of variation for random variable i
α_i : sensitivity factor of random variable i

For sensitivity analysis of random variables, the central approximation equation given in Eq. 17 is used. This sensitivity measure gives the change in f_y^a due to a change of one of the random parameters.

The sensitivity and contributions of the uncertainty of random variables on f_y^a are computed using Eqs. 16 and 17, respectively. The comparison of uncertainty of random variables of the two cases is shown in Fig. 8.

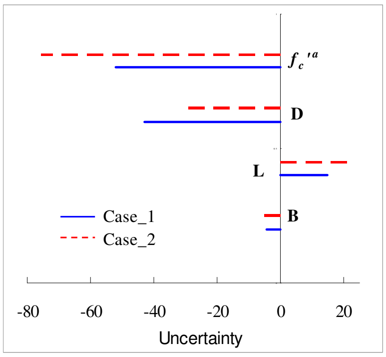

Fig. 8 Uncertainty of random variables

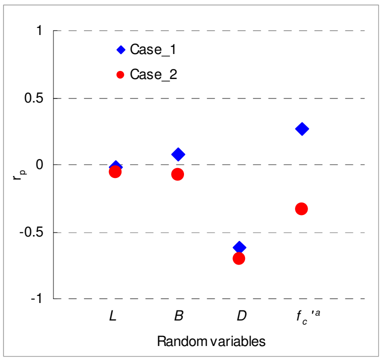

Fig. 9 Sensitivity of random variables

As shown in Fig. 8, the largest contributions to the estimated value of yield strength of steel are the change in comprehensive strength of concrete and total depth of the bridge. The contribution of effective width is insignificant and it is negligible as compared to other parameters.

The sensitivity of each random variable, the squared value of the partial coefficient of correlation (r_p^2), is calculated and it is shown in Fig. 9. From the result, it is observed that the influence of total depth has a dominant factor for the estimated value of yield strength of steel. The influences of span length and strip width are small and can be negligible.

4. CONCLUSIONS

(1) A probabilistic design restoration scheme for RC slab bridges is presented.
(2) With the probabilistic design restoration scheme, not only the mean values but also the confidence limits are obtained.
(3) For the typical RC slab bridge case, with 95 % confidence levels, confidence intervals of $\pm$ 2.76% and $\pm$3.71% of the mean values of f_y^a for Case 1 and Case 2 are obtained respectively.
(4) From the sensitivity analysis, it is observed that change in comprehensive strength of concrete and total depth of the bridge are the most influencing factor in the estimation of yield strength of steel.
(5) A latin hypercube sampling (LHS) method is used to improve the computational efficiency and the accuracy in the estimation of f_y^a.

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