- Technical Paper -

EVALUATION METHOD FOR SHEAR CAPACITY OF TAPERED RC BEAMS WITHOUT SHEAR REINFORCEMENT

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ABSTRACT
This paper presents the evaluation method for shear capacity of tapered RC beams without stirrups. The nonlinear FEM analysis was found to be appropriate to simulate the shear behavior of tapered RC beams without stirrups. The simple relationship between the slope of taper and the inclination of the compressive strut was also proposed through FEM simulations. By determining the location of the critical section, the shear capacity of tapered RC beams without stirrups can be calculated. The calculated value showed good correspondence with the experimental results.
Keywords: Critical section, nonlinear FEM analysis, inclination of compressive strut, variable depth

1. INTRODUCTION

Reinforced concrete (RC) members in which the depth of the cross section varies along its axis are frequently used in structural portal frames, cantilevers and bridge structures. These members can be designed according to the required resistance against external load. The cantilever or tapered RC beam in the Fig. 1 has larger depth at the middle part to resist large flexural moment. Consequently, it can efficiently use the concrete and steel reinforcements, considerably reducing the structure's weight, and contribute to an aesthetic design. However, it is insufficient of the experimental data to predict the shear behavior of tapered RC beams. Moreover, rational and economical design method for RC members with variable depth in the JSCE specifications for concrete [1] has not been completed. Engineers are using such beams based on the empirical background. Therefore, it is necessary to propose a convenient evaluation method with high accuracy and compatibility for tapered RC beams to ensure the reasonable design.

In some previous researches, Ishibashi et al. [2] and the authors (Iwanaga et al. [3]) reported that for deep tapered RC beams (*a/d* ratio from 0.33 to 1) and short tapered RC beams (*a/d* ratio from 1 to 2.5), the effect of taper is not influential to the shear capacity. The reason may be attributed to the occurrence of the arch action. Nevertheless, from the experimental results of Kakuta et al. [4] and MacLeod et al. [5], the shear capacity of tapered RC beams with *a/d* in range of 3.0 to 4.0 can increase compared to the constant depth beams; even if the amount of concrete was reduced due to the tapered slope. However, they did not give a clear explanation about how the slope of taper affects the shear failure processes. According to the recent research of the authors (Iwanaga et al. [6]), the similar conclusion was obtained for tapered RC beams (*a/d* equals to 2.71) without stirrups. As the elongation of the previous research, the objective of this study is to propose an evaluation method for shear capacity of tapered RC beams without stirrups through experiments and FEM analysis.

Fig. 1 Tapered RC beam in the pier structure

2. NONLINEAR FEM ANALYSIS

2.1 Critical Section

From the experimental observation on shear behavior of tested tapered RC beams in the previous paper [6], it was confirmed that the slope of the taper affects the shear capacity of tapered RC beams in which the main shear resistance mechanism is the beam action. Based on the experimental shear behavior [6], the modification with the concept of critical section from MacLeod et al. [5] and Stefanou [7] was conducted. The forces acting at the shear crack before the failure in the free body of tapered RC beams without stirrups is shown in Fig. 2. The shear capacity of tapered RC beams without stirrups can be calculated as the following equations:

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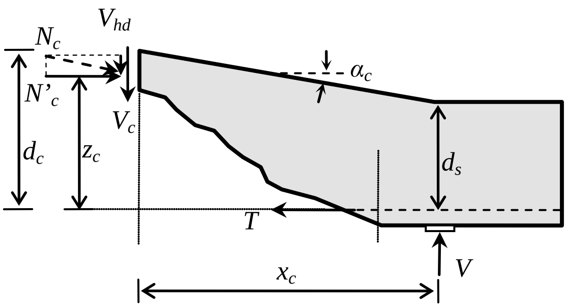

Fig. 2 Forces acting in tapered RC beams

$$V = V_c + V_{hd} \tag{1}$$

$$V_{hd} = N_c' \tan \alpha_c \tag{2}$$

$$N_c' = M_c / z_c = 8Vx_c / 7d_c \tag{3}$$

$$x_c = (d_c - d_s) / \tan \alpha_c \tag{4}$$

$$V = 7V_c / (8d_s / d_c - 1) \tag{5}$$

Where, V is the shear capacity of tapered RC beam without stirrups, V_c is the shear capacity of the critical section which uses the effective depth d_c of critical section to calculate, V_{hd} is the vertical component of the compression force N_c, x_c is the distance from support to the critical section, z_c is the internal lever arm (=jd_c) with j = 7/8, α_c is the slope of the taper, and d_s is the effective depth at the support. The Eq. (5) is obtained by substituting Eq. (2)-(4) into Eq. (1). However, how to accurately determine the position of the critical section where shear capacity should be evaluated is still uncertain as the crack patterns and stress flow of the test specimens were not carefully investigated. Therefore, in this paper, FEM analysis was carried out as the supplementary of the previous experimental research [6] to clarify the shear resistance mechanism and identify the critical section.

2.2 Nonlinear FEM Analysis Model

The two dimensional nonlinear FEM analysis using DIANA system was conducted to simulate the shear behavior of RC beams. 6-node triangular and 8-node quadrilateral isoparametric plane stress elements were used for all concrete elements. The embedded reinforcement elements which have perfect bond with concrete were selected for steel reinforcements. The mesh size was approximately 50 mm for the squared mesh. However, due to the specimen's geometry, meshes may expect to have different sizes. In order to eliminate mesh size effect, concrete constitutive models which consider crack band width h was selected. And the crack bandwidth h was calculated equal to $\sqrt{A}$, where A is the total area of the element.

The total strain fixed smeared crack model was applied as the crack model in this analysis. It is developed along the lines of the Modified Compression Field Theory [8]. As shown in Fig. 3 (a) and (b), parabolic curve model considering compressive fracture energy G_{Fc} and Hordijk model considering tensile fracture energy G_F were used for compressive and tensile behavior of concrete, respectively. The compressive fracture energy G_{Fc} was obtained from the equation proposed by Nakamura and Higai [9].

$$G_{Fc} = 8.8\sqrt{f_c'} \quad \text{(N/m)} \tag{6}$$

The tensile fracture energy G_F was obtained from JSCE specifications [1]:

$$G_F = 10\,(d_{\max})^{1/3} f_c'^{1/3} \quad \text{(N/m)} \tag{7}$$

Where, d_{max} is the maximum aggregate size (mm), f_c' is compressive strength of concrete (N/mm^2).

For the shear model, the constant shear retention model with the value of shear retention factor β, equals to 0.1, was applied for all specimens. The steel plates at the loading point and support of the specimen were assumed to be elastic bodies, while the perfect elastic-plastic model was applied for steel rebars as shown (Fig. 3 (c)).

Displacement control with Quasi-Newton method (also called "Secant method") was adopted to solve equilibrium equations. In each step with 0.02 mm displacement increment, when the variation of internal energy has become less than 0.01% of the internal energy of the first iteration in the step, the iteration process was terminated to move to the next step. Figure 4 shows the finite element mesh of specimen

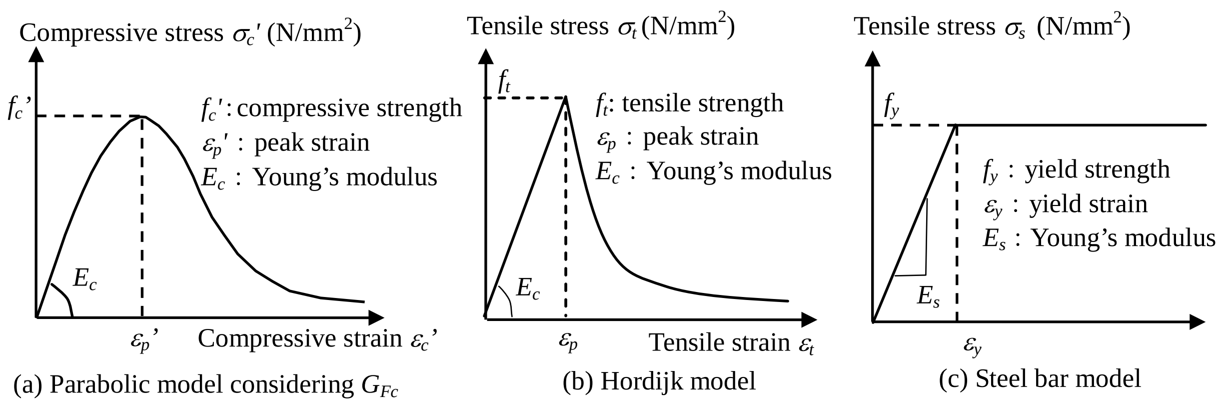

Fig. 3 The material model for concrete and steel bar

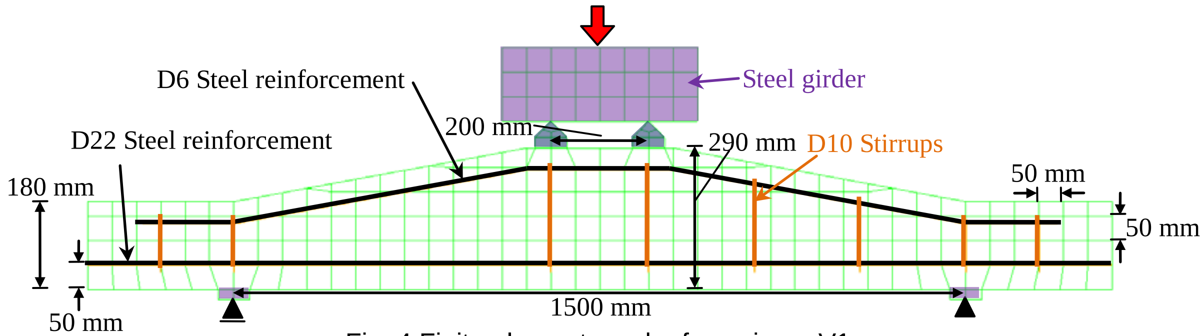

Fig. 4 Finite element mesh of specimen V1

V1 in the previous paper [6] as one example.

2.3 Verification of the Model

For the two RC beams without stirrups C1 and V1 in the authors' previous paper (Iwanaga et al. [6]), FEM analysis was conducted. The load-deflection relationship of these two specimens obtained in the experiment and FEM analysis is shown in Fig. 5. Similar to the experimental results, in the FEM analysis, the shear capacity of the tapered RC beam without stirrups is larger than that of the RC beam with same constant depth. The distribution of the principal tensile strain just before and after the peak load is comparable with those obtained from experimental observation and image analysis (Fig. 6). According to such analytical results, especially the results until the peak that are more important in this study, the FEM model using DIANA system is supposed to be appropriate to simulate the shear behavior of tapered RC beams

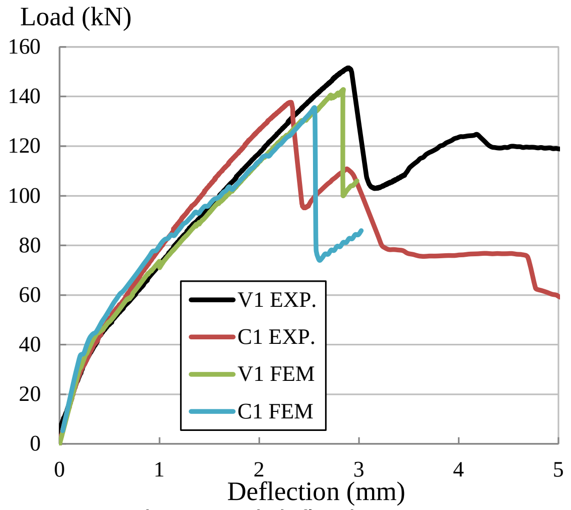

Fig. 5 Load-deflection curves

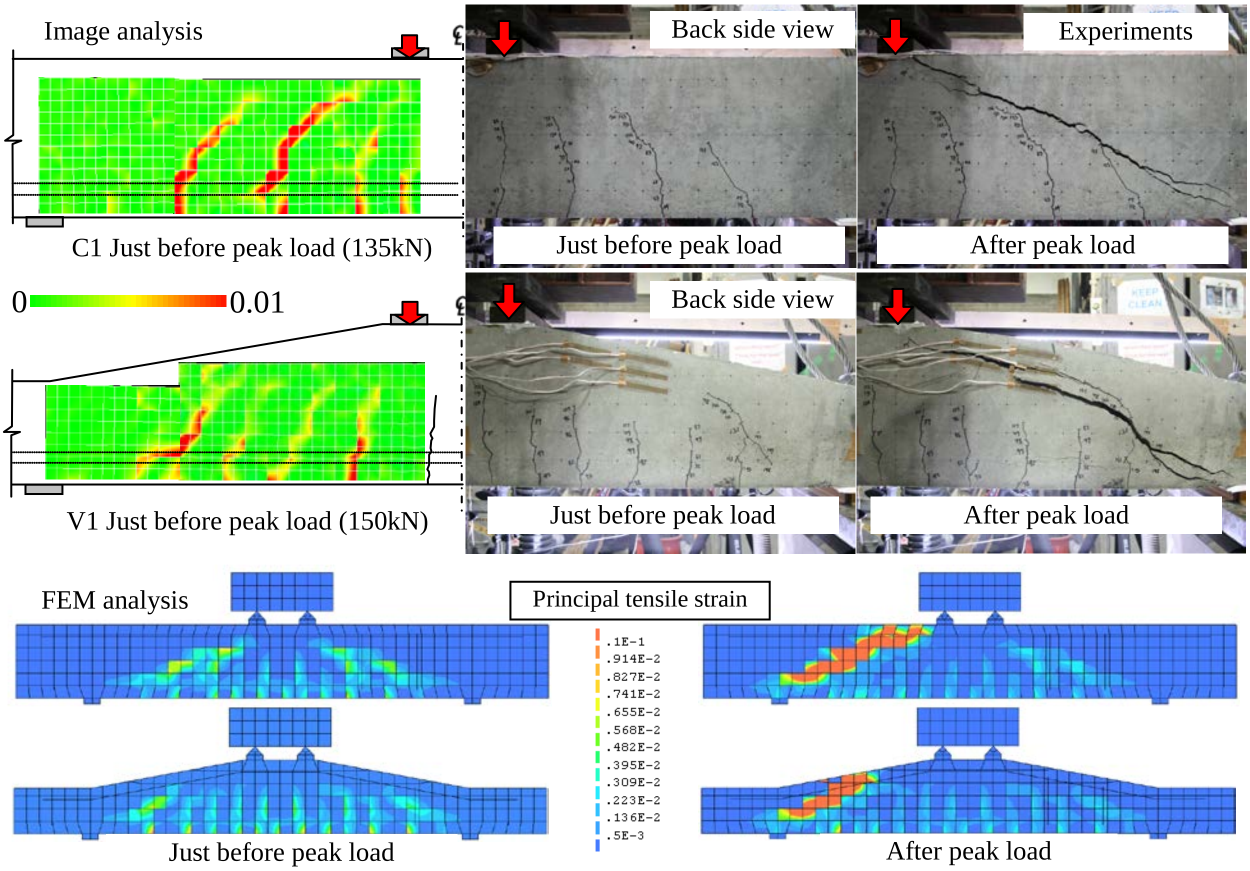

Fig. 6 Crack patterns just before and after peak

without stirrups.

2.4 Inclination of Compressive Strut Flow

In order to obtain an accurate method to determine the position of the critical section in tapered RC beams, the parametric study was conducted by changing the shear span and effective depth at the support and loading point in FEM analysis.

Based on the concept of compressive force path proposed by Kotsovos [10] and the experimental observation in this study, the critical section is assumed to be the location where the compressive force changes the direction (Fig. 7). Therefore, the inclination of the compressive strut flow became much significant in determining the position of the critical section. The tendency of the inclination of the compressive strut flow initiated from the support was investigated using ideas by Lertsamattiyakul [11]. Figure 8 shows the contour figures of principal stress σ_2 of analytical resistance just before the peak load of the tapered RC beam V1. To estimate the slope of the concentrated stress flow, the values of principal stress σ_2 at each Gauss's point was considered. The solid circles indicate the location where the value of principal compressive stress at the Gauss's point was the maximum in the horizontal layer. As shown in Fig. 8, the points having the maximum principal compressive stress tended to be positioned in liner line. Therefore, liner approximation was conducted to find the angle of the compression strut flow.

Figure 9 shows some cases simulated in FEM analysis which indicates that, as the slope of the taper increases, the inclination of the compressive strut also increases. Table 1 shows all the simulated cases' information and the corresponding angles of the compression struts, including the five specimens in Fig. 9. In order to obtain a simple relation between the slope of the taper and the inclination of the compressive strut, the method of tracing a linear regression curve was used. Since the slope of the taper has more direct effect on the angle of the compressive strut than the other factors such as shear span or *a/d* ratio when the tapered RC beam without stirrups is belonging to slender beams region (2.5<*a/d*<5.0), the linear regression relationship was fitted only between the slope of the taper and the inclination of the compressive strut. Finally, by tracing a linear regression curve (Fig. 10) the equation was derived as:

$$\tan\theta_{strut} = 0.75\tan\alpha_c + 0.409 \quad (8)$$

Once the inclination of the compressive strut was determined, the location of the critical section can be obtained.

$$x_c = z_c / \tan\theta_{strut} \quad (9)$$

By substituting Eq. (8) and Eq. (9) into Eq. (4), the following equation can be obtained:

$$d_c / d_s = \tan\theta_{strut} / (\tan\theta_{strut} - 0.875\tan\alpha_c) \quad (10)$$

$$d_c / d_s = (6\tan\alpha_c + 3.27)/(3.27 - \tan\alpha_c) \quad (11)$$

With Eq. (11), the effective depth d_c of critical

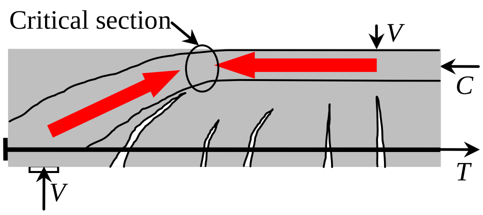

Fig. 7 Compressive force path

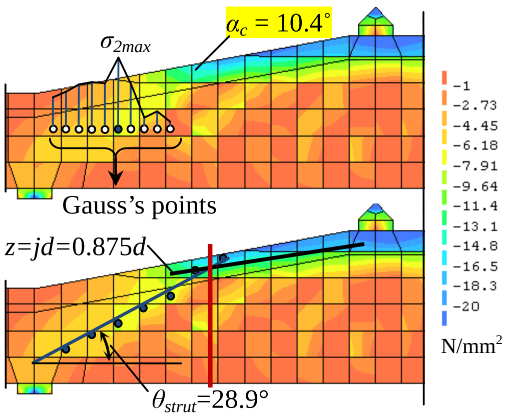

Fig. 8 Inclination of compressive strut in V1

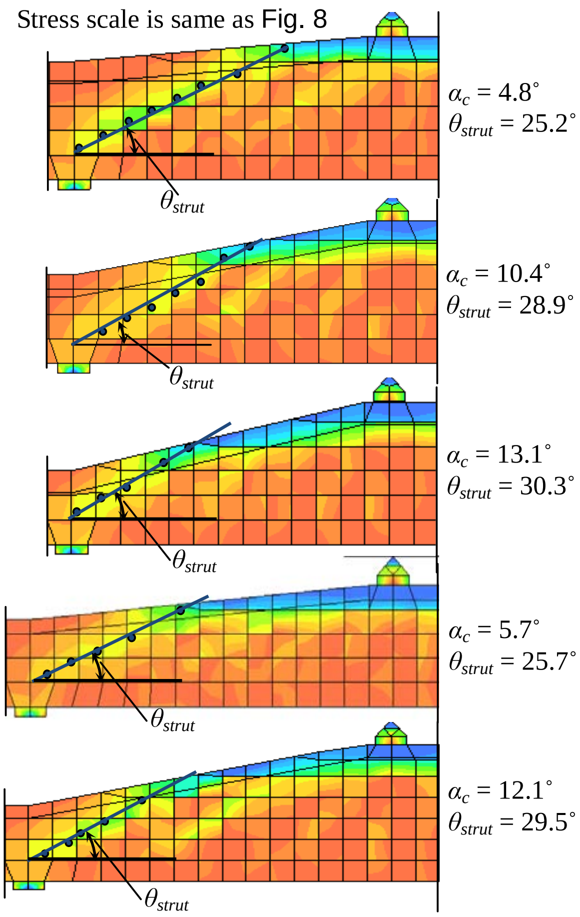

Fig. 9 Compressive struts in FEM cases

Table 1 Cases simulated in FEM analysis

a (mm)	d_s (mm)	d (mm)	a/d	α_c (Degree)	θ_{strut} (Degree)
650	190	240	2.71	4.8	25.2
650	130	240		10.4	28.9
650	100	240		13.1	30.3
750	130	200	3.75	5.7	25.7
750	130	260	2.88	8.9	27.5
750	130	280	2.68	12.1	29.5
825	130	240	3.44	8.1	27.2
900	130	240	3.75	7.4	27.1
1250	100	250	5	7.1	26.8

a: shear span; d_s: effective depth at the support; d: effective depth at the loading point; α_c: slope of the taper; θ_{strut}: inclination of the compressive strut.

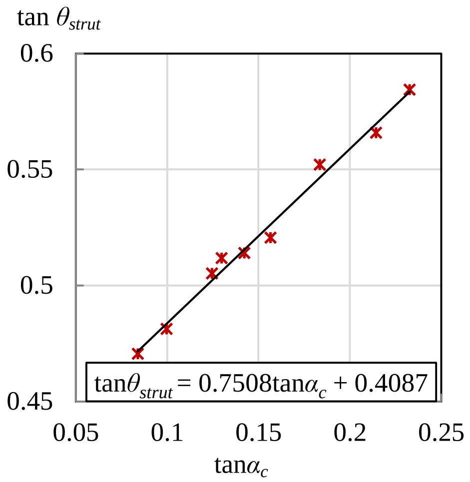

Fig. 10 Relationship between slope of taper and inclination of compressive strut

Table 2 Summary of calculations with proposed method

No.	α_c (°)	f_c' (N/mm^2)	a (mm)	d_s (mm)	d (mm)	a/d	d_c (mm)	A_s (mm^2)	b_w (mm)	V_c (kN)	V_{calc} (kN)	V'_{cal} (kN)	V_{exp} (kN)	V_{exp}/V_{calc}
Kakuta et al. [4]														
D-1	0	33.0	700	170	170	4.12	170	397.2	100	24.6	24.6	24.6	28.6	1.16
D-2	4.40			120			140			21.4	25.6	19.3	28.6	1.12
D-3	8.75			70			94			16.6	23.5	13.9	27.4	1.17
E-1	0		550	170		3.24	170			26.7	26.7	26.7	30.3	1.13
E-2	5.71			120			146			23.8	30.0	20.6	30.8	1.03
E-3	11.31			70			102			18.4	28.6	14.5	30.8	1.08
F-1	0		850	270	270	3.15	270			32.6	32.6	32.6	32.3	0.99
F-2	5.36			195			235			29.4	36.6	25.6	36.3	0.99
F-3	10.62			120			171			23.3	35.4	18.5	36.8	1.04
G-1	0		1150	370	370	3.11	370			37.4	37.4	37.4	37.7	1.01
G-3	10.30			170			240			27.1	40.6	21.6	42.0	1.03
H-1	0		1450	470	470	3.09	470			41.4	41.4	41.4	42.0	1.01
H-2	5.10			345			413			37.5	46.2	32.8	46.9	1.02
H-3	10.12			220			309			30.3	45.1	24.1	46.9	1.04
MacLeod et al. [5]														
B3	0	38.0	960	220	220	4.36	220	741.0	150	45.4	45.4	45.4	43.5	0.96
B4	6.34	33.5	1050	170	270	3.89	212			41.3	53.3	35.7	50.6	0.95
This study [6]														
C1	0	38.8	650	240	240	2.71	240	774.2	200	69.0	69.0	69.0	68.8	1.00
V1	10.39	34.4		130			184			53.7	80.8	41.7	75.8	0.94

A_s: cross section area of tensile reinforcement bars; b_w: width of the beam; V_{calc}: calculated shear capacity with proposed method in this study; V'_{cal}: calculated shear capacity with current JSCE method and minimum effective depth d_s; V_{exp}: shear capacity from the experiments.

section can be calculated. By substituting the value of d_c into the equations by Niwa et al. [12] (Eq. (12) and Eq. (13)) which was slightly modified by JSCE and adopted in the standard specifications for design shear capacity of linear members without shear reinforcing steel, the shear capacity V_c of the critical section can be calculated.

$$V_c = \alpha \, f_c'^{\frac{1}{3}} \, p_w^{\frac{1}{3}} \left(\frac{1000}{d} \right)^{\frac{1}{4}} b_w \, d \qquad (12)$$

$$\alpha = 0.20 \left(0.75 + \frac{1.4}{a/d} \right) \qquad (13)$$

By substituting the values of d_c and V_c into Eq. (5), the shear capacity V of the tapered RC beam without stirrups can be obtained. Since the equation of V will be same as the V_c when the slope of taper is zero, it has a perfect compatibility with the current equation.

2.5 Verification of the proposed method in RC tapered beams without stirrups

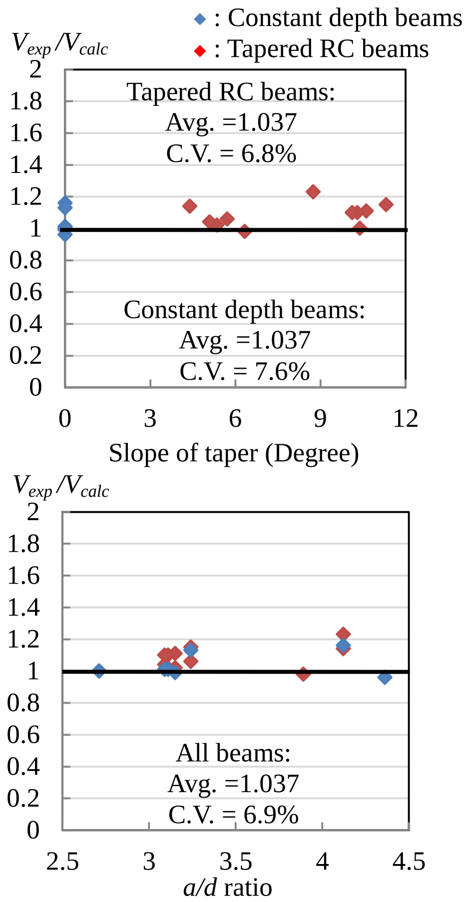

Fig. 11 Accuracy of the proposed method

In this section, the proposed methodology to identify the location of the critical section and calculate shear capacity is verified against the experimental results. A total of 18 experimental data conducted by Kakuta et al. [4] and MacLeod et al. [5] including this study [6] were collected. The summary of the calculations is tabulated in Table 2. The obtained results show a reasonable agreement with the experimental results. Comparing with the conservative shear capacity calculated with the minimum effective depth d_s, the proposed method shows much more accuracy. The ratio of the shear capacity obtained in the experiment and calculation is also plotted against the slope of taper and *a/d* ratio in Fig. 11. The average values and accuracies for tapered RC beams and constant depth beams are almost same. It shows the high accuracy and compatibility of the proposed method. However, when stirrups are used in tapered RC beams, more researches are needed to modify the proposed method and verify the validity.

3. CONCLUSIONS

(1) The inclination of the compressive strut increases as the slope of taper increases. The simple linear relationship between slope of the taper and inclination of the compressive strut for tapered RC beams without shear reinforcements was proposed ($2.5<a/d<5.0$).

(2) Using the proposed method, the location of critical section can be determined easily, and the calculated shear capacity showed a reasonable agreement with the experimental results.

REFERENCES

[1] JSCE, "Standard Specifications for Concrete Structures-2007, Design", JSCE Guidelines for Concrete, No. 15, Dec. 2010.

[2] Ishibashi, T., Saitou, K., Terada, T. and Watanabe, T., "Shear Strength of Reinforced Concrete Beams with Variable Effective Depth", Proceedings of the JCI, Vol. 9, No. 2, 1987, pp. 305-310. (in Japanese)

[3] Iwanaga, T., Matsumoto, K. and Niwa, J., "Shear Resistance Mechanism of RC Beam with Variable Depth in Shear Span", Proceedings of 65th JSCE Annual Meeting, Vol. 78, Sep. 2010, pp. 155-156. (in Japanese)

[4] Kakuta, Y. and Matsui, T., "Shear Failure Test of RC Beams with Variable Cross Section", Proceedings of 32nd JSCE Annual Meeting, Vol. 78, 1977, pp. 170-171. (in Japanese)

[5] MacLeod, I.A. and Houmsi, A., "Shear Strength of Haunched Beams without Shear Reinforcement, ACI Structure Journal, Vol. 91, No. 1, 1994, pp. 79-89.

[6] Iwanaga, T., Matsumoto, K. and Niwa, J., "Evaluation of Shear Failure Mechanism for Variable Cross-section RC Beams that Focuses on the Formation Process of Cracks", Proceedings of JCI, Vol. 33, No. 2, July. 2011, pp. 601-606. (in Japanese)

[7] Stefanou, G. D., "Shear Resistance of Reinforced Concrete Beams with Non-Prismatic Sections", Engineering Fracture Mechanics, Vol. 18, No. 3, 1983, pp. 643-666.

[8] Vecchio, F.J. and Collins, M.P., "The modified compression field theory for reinforced concrete elements subjected to shear", ACI Journal, Vol. 83, No. 22, 1986, pp. 219-231.

[9] Nakamura, H. and Higai, T., "Compressive Fracture Energy and Fracture Zone Length of Concrete", JCI-C51E Seminar on Post-peak Behavior of RC Structures Subjected to Seismic Loads, Vol. 2, 1999, pp. 259-272.

[10] Kotsovos, M.D., "Compressive Force Path Concept: Basic for Reinforced Concrete Ultimate Limit State Design", ACI Structural Journal, Vol. 85, No. 1, Jan.-Feb. 1988, pp. 68-75.

[11] Lertsamattiyakul, M., Niwa, J., Tamura, S. and Hamada, Y., "The Proposal of Simplified Truss Model for Shear Carrying Capacity of Prestressed Concrete Beams", Proceedings of First International Conference on Urban Earthquake Engineering, Mar. 2004, pp. 429-436.

[12] Niwa, J., Yamada, K., Yokozawa, K, and Okamura, H., "Revaluation of the Equation for Shear Strength of Reinforced Concrete Beams without Web Reinforcement", Concrete Library of JSCE, No. 9, June 1987, pp. 65-84.