- Technical Paper -

SHEAR CRACKING PREDICTION MODEL FOR REINFORCED CONCRETE BEAM WITH SIDE FACE REINFORCEMENT

Irish S. TAMBIS[*1], Tamon UEDA[*2] and Dawei ZHANG[*3]

ABSTRACT
This paper presents a method of obtaining shear cracking prediction model of both crack spacing and crack width of Reinforced Concrete (RC) beam, respectively. Essentially, the proposed prediction model for crack spacing is from the modification of the harmonized model used by the previous researcher. Whereas, crack width is from the flexural cracking caused by the combination of primary and side reinforcement and the cracking contribution of shear reinforcement. The proposed prediction model presents a more generalized approach for estimating shear crack spacing and shear crack width of RC beam with side face reinforcement.
Keywords: crack spacing, crack width, reinforced concrete, prediction model, side face reinforcement

1. INTRODUCTION

Cracking of reinforced concrete (RC) structures is considered unwanted because of the various reasons, most particularly its effects on the durability of the structure making the structure unsafe. Aesthetics and change of stiffness and force distribution may modify because of cracking. Cracks with uncontrollable widths occurring in the structures can become the cause of reduction of structural performances because it can allow corrosion of reinforcements, reduce the water/air-tightness and even deteriorate its appearance.

Design codes give guidelines for checking the amount of reinforcement required for the structure to limit the cracking to a certain value for specified loading conditions [1-3]. In connection with this, temperature bars and even the application of side reinforcements were introduced. The introduction of side bars to not so large beams is now a common practice in the construction industries for the control of cracking. That is why there is a necessity to investigate the influence of side bars and formulate a more generalized approach for the determination of shear crack spacing and crack width, respectively. The current structural codes for side-face reinforcement are meant to control flexural cracking in the webs for large concrete beams and may not provide adequate diagonal crack control under serviceability conditions [3]. This paper provides a rational method to calculate shear cracking behavior namely crack spacing and crack width for RC beam with side reinforcements.

Zakaria et.al [10] successfully predicted shear cracking behavior using the harmonized model from Collins and Mitchell [15], CEB-FIP Model code [16-17] and from the model provided by CSA-S474-04 (Canadian Standard Assoc.) [18] and NS-3473 E (Norwegian code) [19] for the crack spacing normal to the shear and longitudinal reinforcement.

$$S_{mx} = 2c_x + 0.2S_x + k_1 k_2 \frac{\emptyset_x}{\rho_x} \quad (1)$$

$$S_{my} = 2c_y + 0.2S_y + k_1 k_2 \frac{\emptyset_y}{\rho_y} \quad (2)$$

Where:
c_x and c_y are concrete covers
S_x and S_y are longitudinal and transverse reinforcement spacing
$\emptyset_x$ and $\emptyset_y$ are bar diameters
ρ_x and ρ_y are the ratio of the amount of longitudinal and transverse reinforcement to the effective concrete area
k_1 represents the bond characteristics surface of reinforcement
k_2 represents strain gradient for the strain distribution in the tension embedment depth.

This existing model is further modified considering the additional parameter i.e. the presence of side face reinforcement in the beam in the next section.

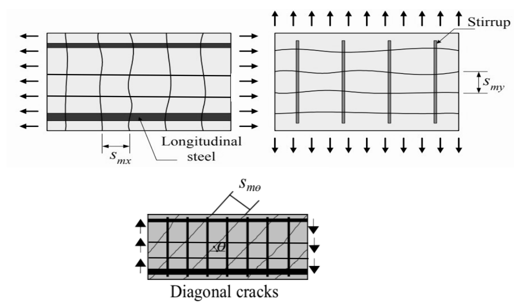


Fig.1 Characteristics of shear cracks spacing influenced by parameters

*1 Master Student, Graduate School of Engineering, Hokkaido University, Sapporo, Japan, JCI Student Member
*2 Professor, Faculty of Engineering, Hokkaido University, Sapporo, Japan, JCI Fellow Member
*3 Associate Professor, Institute of Structural Engineering, Zhejiang University, Hangzhou, China

Whereas, the crack width is primarily from the combination of the physical model of the change of strain along the web of the beam considering longitudinal reinforcement and the relative location of cracked concrete from the stirrups.

The formulation of prediction model limits only to the crack spacing and crack width of the beam and does not consider the diagonal angle of cracking, therefore the actual diagonal angle is embraced for future verification. An accurate estimate of the crack spacing and crack width for this kind of construction can result in the additional information for the designers to better understand and estimate cracking in reinforced concrete structures.

2. APPROACHES FROM DIFFERENT CODES

Cracking in RC structures is produced because of the tensile stress in the reinforced or prestressed concrete members. This tensile cracking is influenced by series of factors, such as reinforcement types, concrete cover thickness, effective concrete area, reinforcement diameter, reinforcement ratio, number of layers of reinforcement, and magnitude of prestressed in the case of prestressed beams [4-5].

2.1 fib Model 2010 Provision [20]

The code stated the expression for average flexural crack spacing as:

$$\ell_{s,max} = k \cdot c + \frac{f_{ctm}\emptyset_s}{\tau_{bms}\rho_{s,ef}};\ S_{rm} = \frac{2}{3}\ell_{s,max} \quad (3)$$

$$w_d = 2\ell_{s,max}(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs}) \quad (4)$$

In the equation (3), $\ell_{s,max}$ denotes the length over which slip between concrete and steel reinforcing bar occurs, k is an empirical parameter to take the influence of the concrete cover into consideration; as a simplification k = 1.0 can be assumed; c is the concrete cover; τ_{bm} is mean bond strength between steel and concrete which the code provides; f_{ctm} is the tensile strength of concrete and ε_{sm} is the average steel strain over the length $l_{s,max}$; ε_{cm} is the average concrete strain over the length $l_{s,max}$; ε_{cs} is the strain of the concrete due to (free) shrinkage. The effective reinforcement ratio is calculated as:

$$\rho_{s,ef} = \frac{A_s}{A_{c,ef}} \quad (5)$$

in which A_s = area of steel bars and $A_{c,ef}$ = effective concrete area in tension $(2.5(h - d_{eff})b_w)$, see Fig. 2.

2.2 JSCE Code (Standard Specifications for Concrete Structures) [21]

The JSCE [21] stated the value of the average crack spacing for flexure is

$$S_{cr} = 1.1k_1k_2k_3\{4c + 0.7(c_s - \emptyset)\} \quad (6)$$

$$w = S_{cr}\left[\frac{\sigma_{se}}{E_s} + \varepsilon'_{csd}\right] \quad (7)$$

In the equation (6), k_1 is a constant to take into account the effect of surface geometry of reinforcement on crack width, which is 1.0 for deformed bars and 1.3 for plain bars or prestressing steel; k_2 is a constant for the effect of concrete quality on crack width calculated using the equation $k_2 = \frac{15}{f'_c + 20} + 0.7$; f'_c is the compressive strength of concrete (N/mm^2); k_3 is a constant for the effect of multiple layers of tensile reinforcement, equal to $k_3 = \frac{5(n+2)}{7n+8}$, where n is the number of layers of tensile reinforcement; c is the concrete cover; c_s is the center-to-center distance of the tensile reinforcement; $\emptyset$ diameter of the tensile reinforcement; and ε'_{csd} is the compressive strain for evaluation of increment of crack width due to shrinkage and creep of concrete.

2.3 CSA-S474-04 and NS-3473 E [18-19]

The Canadian Standard Association [18] recommend the use for calculating the average crack spacing which is the same expression from Norwegian Standard (NS 3473) [19].

$$S_{rm} = 2.0(C_c + 0.1S) + k_1k_2d_{be}h_{ef}b/A_s \quad (8)$$

$$w_m = \varepsilon_s S_{rm} \quad (9)$$

where S_{rm} is the average crack spacing (mm); C_c is the concrete cover (mm); S is the bar spacing of the outer layer (mm); k_1 is the coefficient that characterizes bond properties of bars; k_2 is the coefficient to account for the strain gradient ($k_2 = 0.25(\varepsilon_1 + \varepsilon_2)/2\varepsilon_1$) ($\varepsilon_1$ and ε_2 are the largest and smallest tensile strains); d_{be} is the bar diameter of the outer layer (mm); h_{ef} is the effective embedment thickness (mm) ($2.5(h - d_{eff})$ [18] and the greater of $(a_1 + 7.5\emptyset)$ and $(a_2 + 7.5\emptyset)$ [19] but should not exceed the tension zone in the beam; w_m is the average crack width at the concrete surface (mm); ε_s is the averages tensile concrete strain in the effective embedment zone.

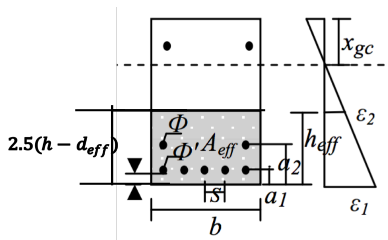


Fig. 2 fib, CSA and NS for the effective embedment thickness

3. APPLICABILITY OF THE CURRENT CODES

The application of the current codes however does not emphasize its appropriateness if additional parameters were given into consideration i.e. in the case of this study if the RC beams has the presence of additional reinforcements along sides. Investigated from several codes, the effective reinforcement ratio $(\rho_{s,ef})$

with effective concrete area in tension ($A_{c,ef}$) as one of the parameters doesn`t consider the presence of area of steel reinforcement embedded within the effective concrete area. Tendency is the inaccurate prediction for cracking behavior result. A necessity for a more general model is encouraged in this paper.

4. METHOD OF DEVELOPING PREDICTION MODEL (ANALYTICAL MODEL)

4.1 Crack Spacing

The third term of the equation (Eqs. (1) and (2)) used by Zakaria et.al [10] (harmonized with Mitchell and Collins and CEB-FIP model code [15-17]) is obtained from the basic concept in the slip theory. Strains which once were evenly distributed between concrete and steel will be localized in the steel at a crack after cracking. The local compatibility and the local equilibrium of an arbitrary section within the transfer length are shown in in Fig. 3.

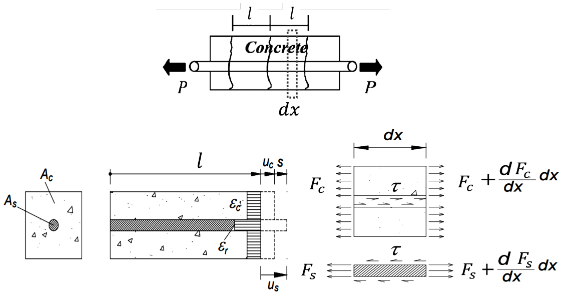


Fig. 3 Bond stress distribution and forces between reinforcing steel and concrete

The effective length between cracks is referred to as crack spacing (l) over which the slip between steel and concrete occurs and considering an element distance (dx) within the effective length, the equilibrium of forces acting on the concrete and reinforcing steel in Fig. 3 can be written as follows:

$$F + dF + C_{er}\tau_b(x)dx = F\,;\; F = \sigma_{ct}A_{eff} \quad (10)$$

The notation C_{er} represents the circumference of the reinforcement, and τ_b as the bond stress at the reinforcement-concrete interface. The redistribution of stresses gradually leads to different strains in concrete and steel ($\varepsilon_c \neq \varepsilon_r$), thus causing a physical slip to occur [6].

The same approach was introduced by Campana et.al [7] on the contribution of shear reinforcement during the shear-transfer action in crack kinematics. A simple and consistent way to investigate the behaviour of transverse reinforcement embedded in concrete is by assuming rigid plastic bond behaviour at their interface that is proved to be applicable before and after bar yielding and can be applied to a number of bond-related problems [7-8]. According to this approach, all tension force is carried by the steel at the location of the cracks and decreases in the regions where it is bonded to concrete.

If sufficient amount of stresses is transferred, so that the concrete tensile strength is exceeded, a new crack will form. Concrete tensile stresses reach their maximum in the center between cracks and vanish at cracks for stabilized cracking, thus, the tensile stress of concrete at the zero-slip point cannot be greater than the tensile strength f_{ct}, regardless of load increase. This condition corresponds to the stabilized crack spacing S_{cr} in which the maximum tensile stress $\sigma_{cmax} \leq f_{ct}$ [9-10].

From Equation (10),

$$dF = -C_{er}\tau_b(x)dx$$
$$F = -(\int_{l/2}^{0} \pi\emptyset\tau_b(x)dx)$$
$$\sigma_{ct}A_{eff} = -(\int_{l/2}^{0} \pi\emptyset\tau_b(x)dx)$$
$$f_{ct}A_{eff} = \pi\emptyset\tau_b(\tfrac{l}{2}) \quad (11)$$

Additionally, $A_{eff} = (A_{co} - A_s)$ is the effective concrete area in the tension side deducting the presence of tension steel area from the gross area of concrete. The ρ as the geometric reinforcement ratio equal to $\rho = \frac{A_s}{A_{co}}$ is introduced, A_s is the total reinforcement area in tension and A_{co} is the gross concrete sectional area in tension, adopted from the fib Model Code [20] $(2.5(h - d_{eff})b_w)$, in which h is the total depth and b_w width of the beam. The effective depth is $d_{eff} = \frac{A_{s1}d_1 + A_{sf}d_2}{A_{s1} + A_{sf}}$ with A_{s1} and A_{sf} the total area of the reinforcements for both main and side reinforcements and d_1 and d_2, are the effective distances from the top of the beam (see Fig. 4).

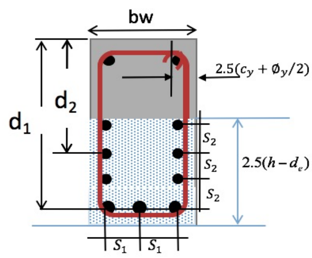


Fig.4 Detailed representation of variables

In the case of transverse reinforcement which is the stirrups, the total effective concrete area is $2.5(c_y + \emptyset_y/2) \cdot S_w$ or $\left({b_w}/{2}\right) s_y$ whichever is lesser (fib model code [20]); c_y is the clear cover from the outer fiber to the face of stirrups; $\emptyset$ is the diameter and S_w is the stirrup spacing (see Fig 5).

From Equation (11),

$$l = 2\frac{f_{ct}(A_{co} - A_{st})}{\pi\emptyset\tau_b}$$
$$l = 2\frac{f_{ct}A_{st}(\frac{A_{co}}{A_{st}} - 1)}{\pi\emptyset\tau_b}$$

Manipulating the relation, yields to:

$$l = \frac{f_{ct}\emptyset(1-\rho)}{2\tau_b\rho} \quad (12)$$

Adopting the two other terms from the model used by Zakaria et.al [10] harmonized from the codes for the influence of concrete cover (non-slip theory) and spacing between reinforcements considering equivalent parameter and with corresponding adjusting coefficients from the previous model, the average crack spacing normal to shear reinforcement becomes:

$$S_{mx} = 2C_x + 0.2S_{eqx} + k_1 k_2 \frac{f_{ct}\phi(1-\rho_x)}{2\tau_b \rho_x} \quad (13)$$

And the stabilized crack spacing influenced by stirrups normal to longitudinal reinforcement is expressed as:

$$S_{my} = 2C_y + 0.2S_{eqy} + k_1 k_2 \frac{f_{ct}\phi(1-\rho_y)}{2\tau_b \rho_y} \quad (14)$$

where C_x and C_y are the concrete covers from the outer fiber to the reinforcement and according to AIJ (Association of Institute of Japan) [22] is the average concrete covers if different concrete covers on the side and bottom of the beam; $S_{eq} = (\frac{(n_1-1)S_1{}^2+n_2S_2{}^2}{(n_1-1)S_1+n_2S_2})$ is the equivalent spacing of the main reinforcement and side reinforcement (Fig. 4); $\phi = (\frac{n_1\emptyset_1{}^2+n_2\emptyset_2{}^2}{n_1\emptyset_1+n_2\emptyset_2})$ equivalent diameter of the main reinforcement and side reinforcement for different diameters from fib Model Code [20]; k_1 the coefficient for bond characteristics of the bars as 0.4 for deformed bars and 0.8 for plain bar and $k_2 = \frac{0.25(\varepsilon_1+\varepsilon_2)}{2\varepsilon_1}$ is the coefficient for strain gradient both from Canadian Standards Association (CSA-S474)[18] (ε_1 and ε_2 are the largest and smallest tensile strains respectively in the embedment effective zone) (see Fig. 2).

Adopting the fib Model Code [20] for the relationship of the mean bond strength between concrete and steel (τ_{bms}) and the mean tensile strength of concrete (f_{ctm}); the mean bond strength is $\tau_{bms} = 1.8f_{ctm}$ for the stabilized cracking stage. Hence, the proposed model becomes,

$$S_{mx} = 2C_x + 0.2S_{eqx} + k_1 k_2 \frac{\phi(1-\rho_x)}{3.6\rho_x} \quad (15)$$

$$S_{my} = 2C_y + 0.2S_{eqy} + k_1 k_2 \frac{\phi(1-\rho_y)}{3.6\rho_y} \quad (16)$$

The shear crack spacing is embraced from the fib Model Code [20] ($S_{m\theta}$), in which the shear crack spacing is related to the crack control spacing of both the longitudinal and transverse reinforcement, represented by S_{mx} and S_{my} from the previous model:

$$S_{m\theta} = \frac{1}{\frac{sin\theta}{S_{mx}}+\frac{cos\theta}{S_{my}}} \quad (17)$$

in which θ is the shear crack angle; and S_{mx} and S_{my} are the horizontal and vertical crack spacing can be calculated from Eq. (15 and 16).

4.2 Crack Width

For the formulation of shear crack width equation, the author considers the combination of the physical model in the change of strain in flexure and the contribution of shear reinforcement at particular location of crack to be analyzed. The proposed model can be said that it combines the idea of Frosch [14] and Zakaria et.al [10] adopted from the CEB-FIP model [16-17].

The study of Frosch [14] extended the available equations for determining the crack width at any other location along the height of the beam by considering the effect of strain gradient (Fig. 6a). This study proposes a strain gradient factor β that needs to be multiplied by crack width (w_s) for determining the crack width (w_z) at any depth z [11]. This relationship in crack width was found to be in good agreement with the test data obtained from RC beams by Frantz and Breen [1] and Kaar and Mattock (1963) [11]. So the prediction model for crack width in flexure at web of beam location,

$$w_z = \varepsilon_x S_c \quad (18)$$

where ε_x is the linear strain at the location of considered crack ($\frac{y_{cr}-C_{na}}{y_s-C_{na}}\varepsilon_s = \beta\varepsilon_s$); and S_c is the crack spacing in RC beam (see Fig. 6b).

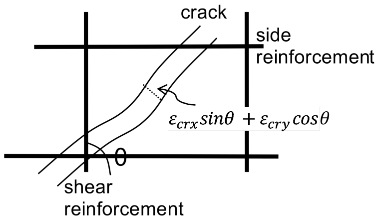
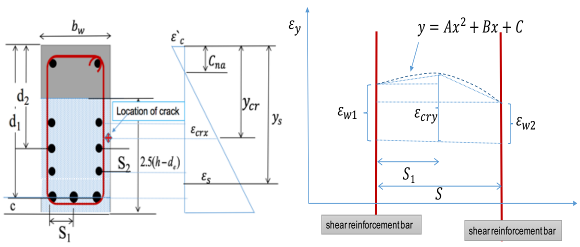


Fig. 5 The equivalent strain in the orthogonal direction of the crack

The ε_{crx} and ε_{cry} are the distributed strains of crack at any depth of the beam and at distance between shear reinforcements considering the direction of the width of the crack by the diagonal angle as shown in Fig. 5.



(a) (b)

Fig. 6 Representation for the behavior of strain (a) influenced by side face reinforcement; (b) influenced by stirrups

For the effect of stirrups, Aguilar G. et.al. [12] indicates in their study of the experimental evaluation of design procedures for shear strength of RC beams that the strain readings in the stirrup legs were sensitive to the relative location of the strain gages with respect to the diagonal cracks within the shear span [12-13]. This is also in agreement with the findings of Anderson and Ramirez [13], indicating that the efficiency of the transverse reinforcement is highly dependent on its relative location with respect to the crack pattern. The CEB-FIP model [16-17] calculated the average shear crack width $w_{avg} = s_{rm}\varepsilon_w$; s_{rm} is the average crack spacing and ε_w is the shear reinforcement strain. Zakaria et.al [10] used ε_w as the shear reinforcement strain intersection with shear crack considered for calculating the shear crack width.

The above concept was gathered proposing prediction model for crack width at any location in the web which combines the strain affected by longitudinal reinforcements and stirrups. Neglecting the average strain of concrete, the proposed crack width model becomes,

$$w_z = S_{m\theta}(\varepsilon_s \beta sin\theta + \varepsilon_{cry} cos\theta) \quad (19)$$

where, $\beta = \frac{(y_{cr} - c_{na})}{(y_s - c_{na})}$, the strain gradient at a particular location of crack; ε_s is the primary reinforcement strain at d_e; and ε_{cry} is the crack strain at distance from the shear reinforcements and θ is the cracking angle.

To get the value of ε_{cry} or the crack strain not along the shear reinforcement, the location of the maximum crack strain is assumed as shown in Fig. 6b. S_1 and $S_2 = (S - S_1)$, are the distances of the considered crack from the stirrups in which the crack is in between. The function of crack strain distribution between stirrups is presumed to be quadratic ($\varepsilon_{cry} = y = Ax^2 + Bx + C$), where A, B and C are coefficients and can be obtained depending on the location of the crack to be analyzed (see Fig. 6b). The specific crack width between the stirrups can then be determined.

5. VERIFICATION OF THE MODEL, (S_{mx})

The proposed model was verified using the existing result of Adebar and Leeuwen [3] who experimentally investigated the large RC beam with side-face reinforcement for flexural cracking (Table 1). However, due to limited data of the RC beams with side face reinforcements, only the crack spacing normal to shear reinforcement (S_{mx}) is verified.

Table 1. Tested beams considered

Specimen	depth (h), mm	width (bw), mm	fc` (MPa)	cover, c (mm)	side bar diam. (db), mm	side bar spacing (sb),	side rein. Ratio, psk
FS1	1200	180	41	40	10	450	0.25
FS2	1200	180	41	40	10	300	0.37
FS3	1200	180	41	40	10	160	0.69
FS4	1200	180	41	30	10	160	0.89
FS5	1200	180	41	30	10	110	1.30
FS6	1200	180	41	30	10	80	1.79

Table 2 shows the calculation results of the average crack spacing using different codes and the proposed prediction model. The table also includes the ratio of the mean value of crack spacing S_{cal}/S_{exp} and ratio of S_{cal}/S_{exp} for standard deviation. As shown in the table, the proposed model has the least value of ratio of S_{cal}/S_{exp} for standard deviation the same with JSCE [21], but when compared with the ratio for mean crack spacing, the proposed model has the best prediction among the codes.

Table 2. Summary of results from Adebar and Leeuwen [3] experiment with proposed prediction model and existing codes

Beam	Experiment (mm)	Proposed (mm)	fib (mm)	JSCE (mm)	NS (mm)	CSA (mm)
1	339.0	195.9	528.1	257.0	323.3	516.2
2	190.0	166.6	503.7	243.7	296.4	363.5
3	158.0	138.5	423.0	230.1	235.3	256.2
4	169.0	128.5	423.0	230.1	235.3	256.2
5	103.0	124.0	342.8	180.5	181.3	188.2
6	87.0	106.7	306.9	175.8	154.6	140.2
Mean of Scal/Sexp	-	0.8	2.4	1.3	1.4	1.6
StanDev of Scal/Sexp	-	0.4	1.0	0.4	0.7	1.5

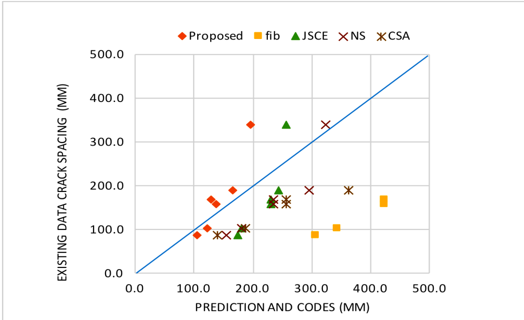


Fig. 7 Accuracy of the proposed prediction model for horizontal crack spacing, S_{mx}

The graph in Fig. 7 shows the evaluation of the prediction model through the existing experimental result of Adebar and Leeuwen [3]. For RC beam with additional parameter as side reinforcement, the values using the proposed model agrees reasonably with the existing experimental values compared to the results from the codes hence, indicates reliability. This is perhaps due to the effect of side reinforcement not considered in the calculation with codes. It can be seen also in the graph, most codes result in the overestimation of crack spacing than the proposed model and can be judged either slightly underestimated or predicted reasonably the crack spacing (S_{mx}).

6. CONCLUSIONS

The modification of the existing model especially the third term (bond-slip approach) in the existing equation takes into account the area of the embedded reinforcements in the effective area of concrete unlike the effective concrete area defined in the previous model that does not consider the presence of tension steel cross-section area. This term in the equation makes the most contribution in the reduction of crack spacing computed using the proposed equation as the area of primary and side reinforcements were deducted making the third term as small as possible. Aside from that are the coefficient formulated in the third term during the modification considering the relationship of bond strength (τ_b) and tensile strength of concrete (f_{ct}) given by the fib Model Code 2010, plus the other two terms (concrete cover and reinforcement spacing contribution). Hence, reducing the crack spacing.

In the verification of the crack spacing normal to the shear reinforcement (S_{mx}); despite the very limited data to verify, among other model presented, it can be said that the proposed prediction reasonably predicted the crack spacing in the available literature. While the crack width section shows the method of obtaining crack width following the proposed model from the combination of the effect of longitudinal and shear reinforcements.

As there is not enough data for verifying the diagonal crack spacing and crack width in RC beams with side face reinforcements, a demand for further experiment with a variety of parameters is recommended towards the judgment of the reliability of the proposed prediction model for both crack spacing and crack width.

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