[2112] コンクリートのクリープ予測式の比較に関する研究

COMPARISON OF CREEP PREDICTION FORMULA FOR CONCRETE

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1. INTRODUCTION

It is considered important from a practical point of view to estimate the long-term deformation behavior of concrete structures. Creep of concrete is one of the influence factors for the long-term behavior of concrete structures such as deflection and cracking. Therefore, a number of prediction formulas have been developed and proposed(1,5,6,7). Each prediction formula has its own range of application, and the precision of prediction of each formula is different. For engineering purposes, it is convenient to have a method for estimating the characteristics of each prediction formula quantitatively. Comparison of various creep prediction formulas has already been done in a statistical manner by a number of researchers(8). However, statistical comparison only gives superficial and overall characteristics, and does not necessarily provide the essential properties of each prediction formula.

In this study, an effort is made to find a method of comparison suitable to give a quantitative measure to estimate the characteristics of various prediction formulas. First, the creep compliance given by a prediction formula is converted into a mathematical form based on the generalized Maxwell chain model. Then, the relaxation spectra are obtained to give a measure of comparison. The load duration and the age of concrete at loading are selected for the parameters for this purpose.

2. Relaxation Spectrum of Creep Compliance

The creep compliance of each prediction formula is first converted into the generalized Maxwell chain model(cf. Fig.1). The stress strain law based on the Maxwell chain is as follows(2,3,4).

$$s = \Sigma_{m=1}^{n} s_m$$
$$\dot{e} = (\dot{s}_m + s_m/t_m)/E_m(t') \qquad (m = 1,\ldots,n) \qquad (1)$$

where s=stress, e=strain, s_m =stress in the m-th spring, t_m =relaxation time for the m-th spring, n=number of Maxwell models, E_m =age-dependent elastic modulus of the m-th spring, and t'=age at loading. The superimposed dots stand for time derivatives of variables. The relaxation times may be given arbitrarily. However, they are selected in this study as follows.

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$$t_m = t_1 \cdot 10^{m-1} \qquad (m = 1,\dots,n-1)$$
$$t_n = \infty \qquad (m = n) \qquad (2)$$

Then, solving a system of first-order linear differential equations, Eq.(1), for a step input of strain, i.e., e=H(t') (Heaviside step function), the relaxation function, $E_R(t,t')$, is obtained in the following form.

$$E_R(t,t') = \Sigma_{m=1}^{n} E_m(t')\exp[-(t-t')/t_m] \qquad (3)$$

where t=time(age of concrete). The plot of coefficients, E_m, versus relaxation time, t_m, is discrete and called relaxation spectrum. Since concrete is an age-dependent material, the relaxation spectrum is also dependent on the age at loading. The actual spectrum of concrete is continuous. However, if the relaxation times are sufficiently densely distributed, then the relaxation spectrum may be considered continuous approximately. The relaxation time can be determined from the experimental data. In this study, however, they are selected as in Eq.(2) to avoid a possible non-unique solution, which is inherent in the exponential(Dirichlet) series expansion.

The age-dependence of $E_m(t')$ is modeled by an arbitrary function. In order to assure small scatter of the solution(3), the following form is assumed in this study.

$$E_m(t') = E_{0m} + \Sigma_{i=1}^{3} E_{im}[\log(1+t')]^i \qquad (4)$$

The coefficients, E_{im} (i=0,1,2,3), in Eq.(4) are determined by the least square method to minimize a function, Φ, i.e., the square of deviation between the relaxation function and the given data by the creep compliance, added by two penalty functions with suitable weight functions, w_1 and w_2, as follows(3).

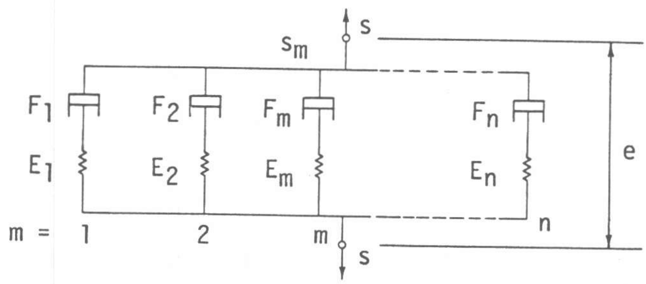

Fig.1 Generalized Maxwell Model

$$\Phi = \Sigma_{j=1}^{N} [E_R(t_j,t') - \tilde{E}_R(t_j,t')]^2 + w_1 \Sigma_{m=1}^{n-1} (E_{m-1} - E_m)^2 + w_2 \Sigma_{m=1}^{n-2} (E_{m+2} - 2E_{m+1} + E_m)^2 \quad (5)$$

where $\tilde{E}_R$ denotes the given values of the relaxation function to be fitted, and N=number of sampling points for time t.

3. Comparison of Creep Prediction Formulas

Although there are a number of proposed prediction formulas for creep of concrete, ACI model(1), BP model proposed by Bažant and Panula(5), and CEB model(6) are selected in this study. The detail of each prediction formula may be found in the references listed at the end, and it is not repeated in the following context.

In order to evaluate the value of creep compliance of each prediction formula, the material parameters are selected as shown in Table 1. The parameters for BP model obtained by the material properties and the mix composition given in Table 1 are: $\bar{E}_0=6.34\times10^4$ MPa, $\bar{n}=0.130$, $\bar{m}=0.317$, and $\bar{\alpha}=0.050$.

The creep compliance of each prediction formula for these parameters are shown in Fig.2. Since ACI model is valid only for age greater than 7 days, the curve for t'=1 day is not shown. Except for the curve of BP model for t'=1 day, it is found that these prediction formulas give similar values in the practical range of load duration. However, it is also found that the shape, i.e., the curvature of each prediction formula is different.

The relaxation spectra of each prediction formula for various ages at loading are shown in Fig.3. The relaxation times (marked by circles in Fig.3) are selected by following Eq.(2). The infinite relaxation time in Fig.3 means that the last spring of the generalized Maxwell model is attached to no dashpot. For a practical purpose, the present choice of relaxation times is enough to give the characteristics of each prediction formula, although the shape of relaxation spectra obviously depends on the values of relaxation times. Comparing these relaxation spectra, it is found that BP and CEB models have a monotonically increasing spectrum in the practical range of load duration. On the other hand, the rate of increase of relaxation spectra of ACI model decreases as relaxation time increases. This aspect coincides with the shape of the creep compliance shown in Fig.2. Note that the relaxation spectra for CEB model are given only for load duration greater than 1 day because of the inherent limitation of the model. Observing these aspects related to the relaxation spectra shown in Fig.3, it is found that the relaxation spectra may be used to clarify the nature of each prediction formula and their ranges of application. Also, it may give some information for improving the accuracy of prediction formulas for various ranges of load duration in practice.

Table 1 Sample Material Properties

(1) Environmental Conditions	Relative Humidity Temperature	100 (%) 23 (°C)
(2) Specimen Size	Minimum Thickness Perimeter of Cross Section Cross-Sectional Area	100 (mm) 400 (mm) 10000 (mm^2)
(3) Concrete Properties	Slump Comp. Strength(28days) Density Cement Type	80 (mm) 36 (MPa) 23 (KN/m^3) Type I
(4) Mix Composition	Water-Cement Ratio Aggregate-Cement Ratio Sand-Cement Ratio Sand-Aggregate Ratio Air Content	50 (w%) 480 (w%) 180 (w%) 37.5 (w%) 6 (%)

(1 MPa = 10.2 Kgf/cm^2 , 1 KN/m^3 = 102 $tonf/m^3$)

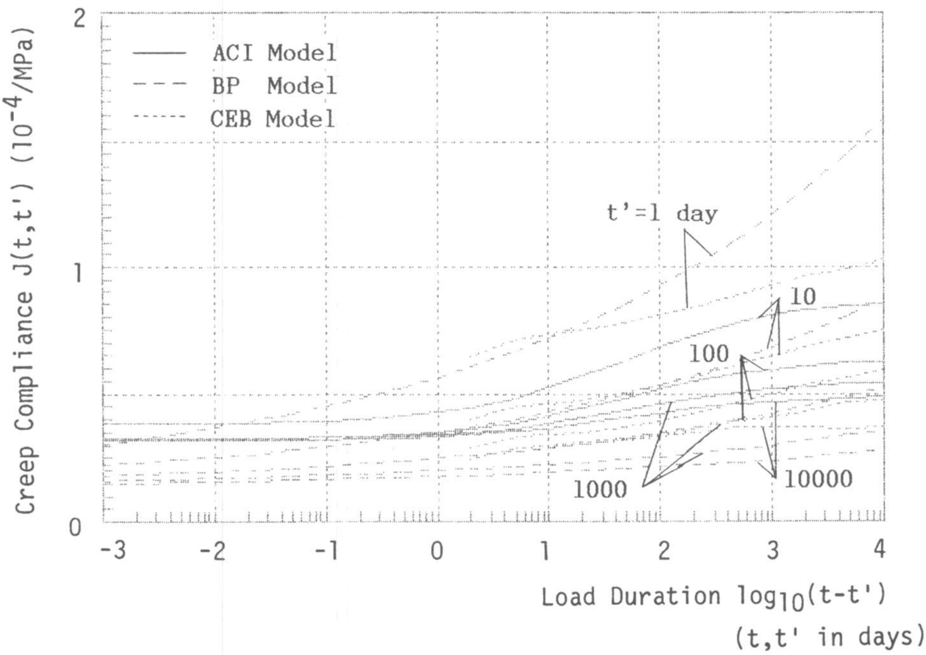

Fig.2 Creep Compliances of Various Models

(1 MPa = 10.2 Kgf/cm^2)

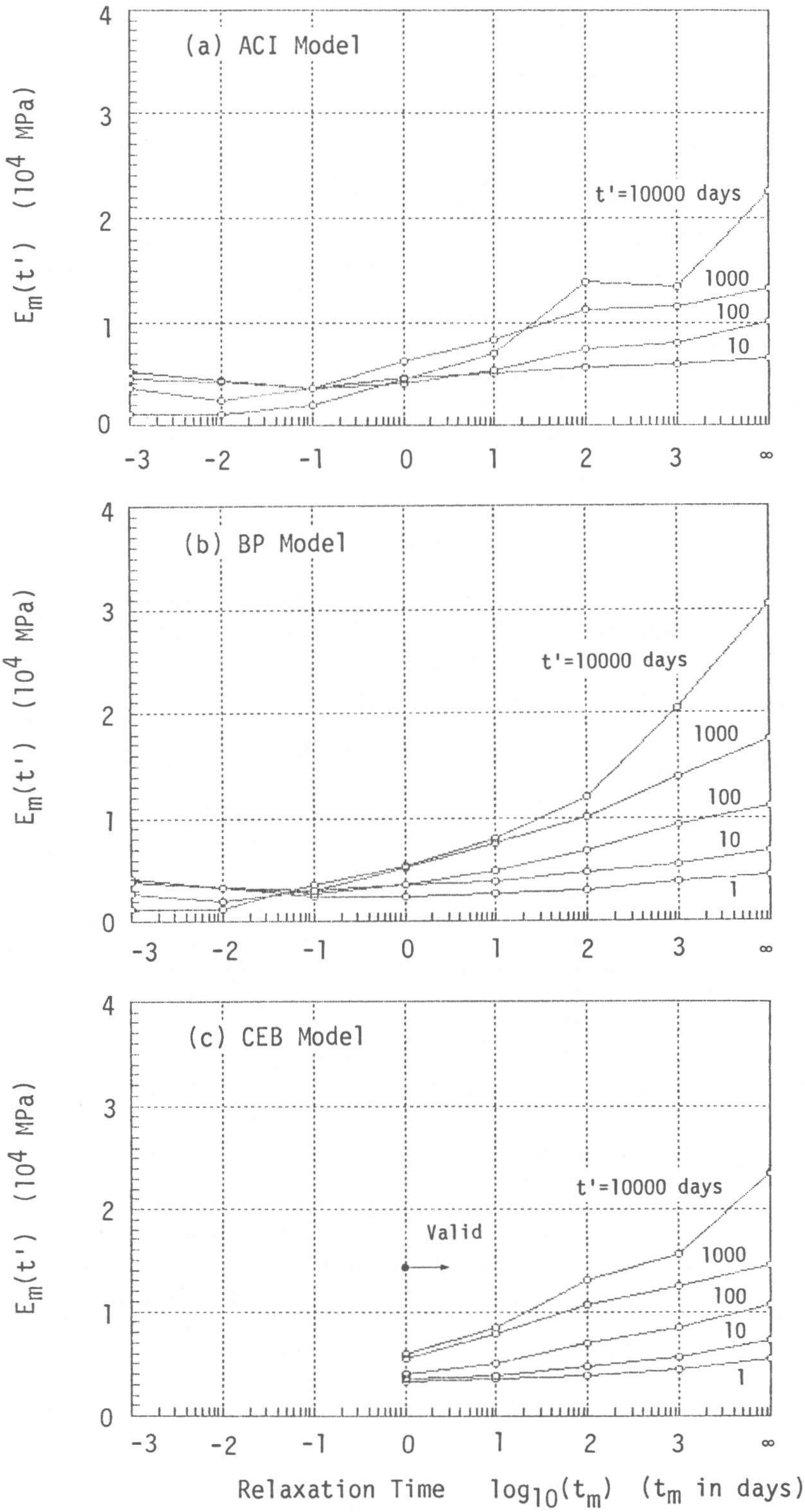

Fig.3 Relaxation Spectra of Various Models
(1 MPa = 10.2 Kgf/cm²)

4. Conclusion

A method to convert a creep compliance function given by a prediction formula into a generalized Maxwell chain model is tried to use to give a relaxation spectrum in order to obtain a suitable measure for comparison of various creep prediction formula. It is found that this method provides a useful information on the characteristics and the limit of application of each prediction formula the validity and the accuracy of which have already been verified by comparing with experimental data. Although comparison is made only for one particular set of material parameters in this study, the usefulness of the method is considered to be verified. It is, of course, desirable to compare the prediction formula for various sets of material parameters. In this study, the age at loading is selected as a parameter for the relaxation spectrum, however, other parameters such as parameters for material properties may be selected.

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