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ANALYTICAL STUDY OF SHEARING STRENGTH IN RC STRUCTURES WITH RIGID-PLASTIC THEORY

Zhishen WU¶, Zhitao LU¶¶, and Dajun DING¶¶¶

1. INTRODUCTION

The failure mechanism of RC in shear is one of the subjects which are yet to be investigated further[1],[2],[3],[4]. Although comprehensive study on this field has been made by many researchers, the problem of shear has not been solved satisfactorily. Among researchers in the world, the Danish group headed by Prof. Nielsen contributed much in this field of research. In 1975, Nielsen-Braestrup derived the formula to calculate shearing strength of RC beams by applying the plastic theory [5]. However, the applicability is limited only to the case of simply supported RC beams with T shaped section. In this study, the calculation method of the ultimate shear strength of RC continuous beams was discussed applying the plastic theory as well.

2. ASSUMPTIONS

2.1 Basic assumptions

(1)The structure is made of perfect rigid - plastic material.

(2)The concrete is in plane stress state.

(3)The concrete is a rigid - plastic body which complies with the Modified Mohr-Coulomb criterion for yielding and the associated flow rule (see Fig.1).

(4)The compressive yield criterion (Ra) shown in Fig.1 is reduced to Ra* , for the rational modification of the compressive plastic deformation energy of concrete.

(5)The effect of the tensile strength of the concrete is neglected, for the convenience of the analysis.

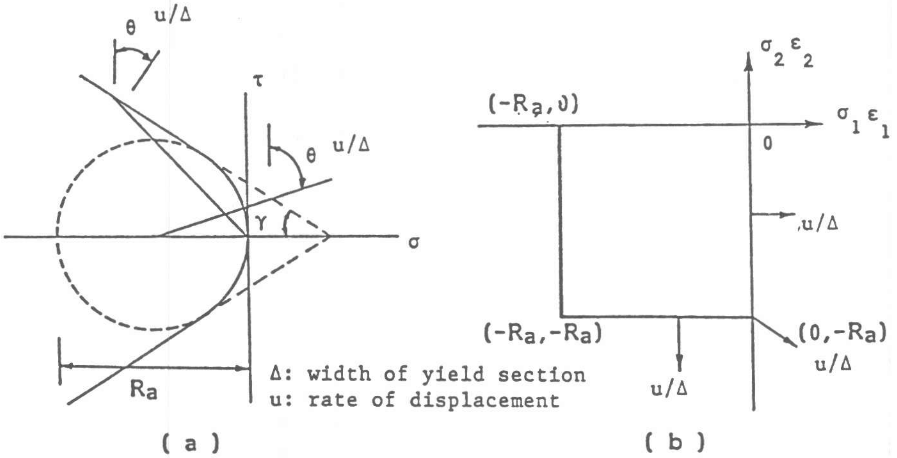
2.2 Supplemental assumptions

(1)The top and bottom longitudinal reinforcing bars are well distributed in continuous beams. The stirrups are arranged uniformly.

(2)The longitudinal reinforcement is in a state of one way tension, its compressive stress being neglected.

(3)Bond failure and other kinds of partial failure are not

¶ Graduate Student, Dept. of Civil Engineering, Nagoya University
¶¶ Associate Professor, Nanjing Institute of Technology, China
¶¶¶ Professor, Nanjing Institute of Technology, China



(a). Yield Locus (b). Yield Curve with Principal Stress

Fig.1 Yield Criterion for Concrete

considered in the collapse mechanism of a continuous beam. However, the ν factor practically takes care of these effects[6].

(4)Yielding occurs only in the reinforcement subjected to negative bending moment.

3. UPPER BOUND SOLUTION

To illustrate the problem, a two-span symmetric continuous beam as shown in Fig.2(a), in which each span is subjected to a concentrated load, is considered. The calculating formula for the ultimate shear strength is derived, and discussion is made on the half of the symmetrical structure. The failure mechanism is shown in Fig.2(b).

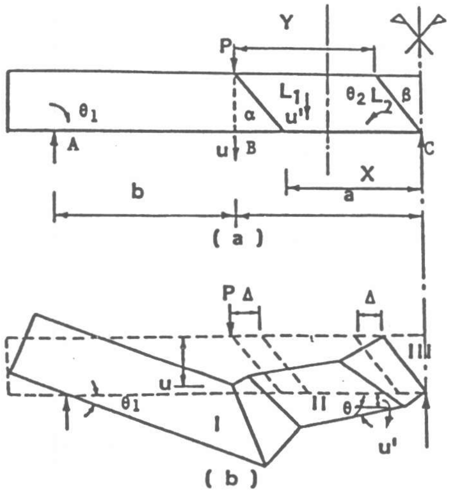


Fig.2 Collapse Mechanism of Continuous Beam

In Fig.2, the notations are as follows.

u: Virtual displacement rate of the point B.

x,y: Dimensions of the portion II and variables.

$\theta 1$, $\theta 2$: Angular displacement rates depending on u and $\theta 1 = u/b$.

u': Virtual displacement rate of the point C depending on u.

Here, concerning the plastic deformation, following assumption is made.

$$u' = u - uY^2/a^2 , \qquad \theta 2 = uY/a^2$$

This meams that, when Y=0, u'=u and $\theta 2 = 0$, i.e., the body 2 has vertical displacement only, when Y=a, u'=0 and $\theta 2 = u/a$, i.e., the body 2 has angular displacement around the central support only.

The structure is divided into the following three rigid bodies;
the body 1, which has angular displacement around the side support only, and its angular displacement rate is $\theta 1$,
the body 2, which has vertical displacement rate (u') and angular displacement rate ($\theta 2$) around the central support, and
the body 3, which remains without displacement.

The local coordinate system (Z1, Z2) along the yield line and the directions of the local strain rates are shown in Fig.3.

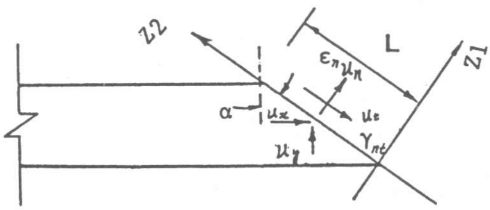


Fig.3 Local Coordinate System

Referring to Fig.3, the virtual relative displacements rate of the two opposite points of plastic zone, u_n and u_t are expressed as,

$$\begin{Bmatrix} u_n \\ u_t \end{Bmatrix} = T \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \qquad (1)$$

Here, $$T = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{pmatrix}$$

The following equations are derived from Eq.(1).

$$\varepsilon_n = u_n/\Delta , \quad \varepsilon_t = 0 , \quad \gamma_{nt} = u_t/\Delta \qquad (2)$$

Thus, the principal strains are expressed as

$$\varepsilon_1, \varepsilon_2 = (\varepsilon_n + \varepsilon_t)/2 \pm \sqrt{\{(\varepsilon_n - \varepsilon_t)/2\}^2 + (\gamma_{nt}/2)^2} \qquad (3)$$

Assuming $\varepsilon 2 \leqq 0$, by the associated flow rule and the yield criteria, we have $\sigma 1 = 0$ and $\sigma 2 = -Ra$. Now we can calculate the internal virtual work done by concrete, stirrups, and longitudinal steel of the structure, and the rate of the external virtual work done by the external load. For example, the virtual work done by concrete is given as,

$$Wc = \int_0^L B\Delta\varepsilon_2\sigma_2 dZ \tag{4}$$

By the principle of virtual work, we obtain a non-dimensional formula for the ultimate load as follows:

$$\begin{aligned} P/BHR_a = \sum_{i=1}^{2} \nu/2\mu\,((M_i/2+(1-k_i)N_i)M + 1/2M_i((M_i+(1-K_i)N_i) \\ M_i+(1-K_i)N_i)^2M^2+N_i^2 - (1-K_i)N_i\sqrt{(1-K_i)^2N_i^2M^2 + N_i^2} \\ + N_i^2/M \cdot \ln \frac{(M_i+(1-K_i)N_i)M + \sqrt{(M_i+(1-K_i)N_i)^2M^2 + N_i^2}}{(1-K_i)N_iM + \sqrt{((1-K_i)N_i)^2M^2+N_i^2}})) \\ +\phi_1 \cdot M/2\mu\,(1-K_1)(1-K_1+K_2\mu\,(2K_2-K_1-1)) \\ +\phi_1 \cdot M\,(1-K_2)^2(2-K_2)/2 + \phi_2K_2/l_2M^2 \end{aligned} \tag{5}$$

We easily obtain the shear strength of simple beams (see Fig.4), referring to the case of continuous beams.

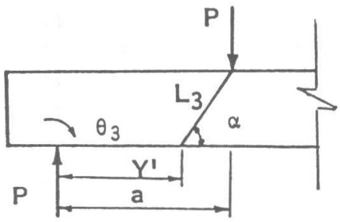


Fig.4 Collapse Mechanism of Simple Beam

$$\begin{aligned} P/BHR_a = \nu/2\mu\,((M_3/2+(1-k_3)N_3)M + 1/2M_3((M_3+(1-K_3)N_3) \\ (M_3+(1-K_3)N_3)^2M^2+N_3^2 - (1-K_3)N_3\sqrt{(1-K_3)^2N_3^2M^2 + N_3^2} \\ + N_3^2/M \cdot \ln \frac{(M_3+(1-K_3)N_3)M + \sqrt{(M_3+(1-K_3)N_3)^2M^2 + N_3^2}}{(1-K_3)N_3M + \sqrt{((1-K_3)N_3)^2M^2+N_3^2}})) \\ +M^2\phi_1(1-K_3)^2(2+K_3) \end{aligned} \tag{6}$$

Where : $K_1=X/a$, $K_2=Y/a$, $K_3=Y'/a$, $M=a/H$, $\mu=b/a$, $\nu=Ra^*/Ra$

$\phi_1=Ags\ Rgs/BSR_a$, $\phi_2=Agr\ Rgr/BHR_a$, $l_1=\sqrt{(1-K_1)+M_{-2}}$

$l_2=\sqrt{(1-K_2)^2+M^{-2}}$, $l_3=\sqrt{(1-K_3)^2+M^{-2}}$, $M_1=(1+K_2\mu)l_1^2$, $M_2=l_2^2K_2$,

$M_3=l_3^2K_3$ $N_1=K_2\mu(K_1-K_2)-1+K_1$, $N_2=1-K_2^2$, $N_3=-(1-K_3^2)$

Ra: Compressive strength of concrete

ν : Plastic factor for considering concrete ductility

B,H : Beam width and effective height, respectively.

S : Spacing of stirrups.

Ags, Agr : Cross sectional areas of stirrups and longitudinal reinforcing bars in negative bending moment, respectively.

Rgs, Rgr : Yield strength of stirrups and longitudinal reinforcing bars in negative moment, respectively.

Formulas (5)and(6) give upper bound general solutions for the ultimate shear strength of beams, which is the function of parameters K1, K2, K3, X, Y, and Y'. However, a general expression for the lowest upper bound solution can not be derived. Therefore, we have computed the minimum point in terms of the variables X and Y, for each special point of M, $\Phi 1$ and $\Phi 2$ to get the lowest upper bound solution.

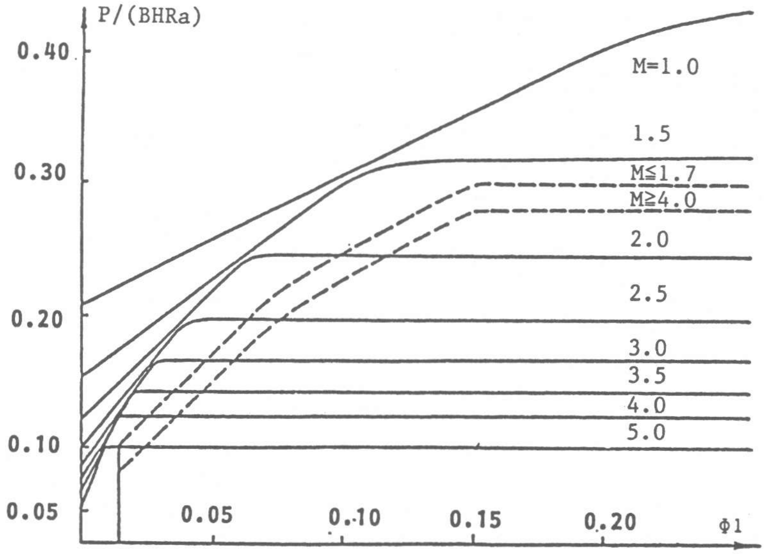


Fig.5 P/(BHRa) —— Φ1 Curves

Fig.5 shows the theoretical curves calculated from Eq.(6) (solid lines, where $\nu=1$) compared with the curves from the Chinese code TJ 10-74 (dotted lines).

The lower bound and upper bound solutions are the same at some special points. This fact has been used, by one of the authors [6], to determine indirectly the stability of the formulas (5) and (6).

In order to prove the correctness of the theory described above, we have carried out an experiment. Test results are shown in Table 1.

4. RESULTS OF ANALYSIS AND EXPERIMENT

In Table 1, the four specimens of CBP1 to CBE2 are continuous beams and the two specimens SB2-a and SB2-b are simply supported beams. The failure loads obtained by the test are shown in the column (4) and calculated values are shown in the column (5). Here, ν is giving as the test failure loads to the calculated loads.

Meanwhile, analyses are carried out on the mechanism of the stress redistribution of the continuous beams, and finally on the equivalent indirectly simply supported beams.

The result of the analysis indicates that the strength of the continuous beam is lower than that of the simply supported beam which has the same generalized shear span ratio m/VH (where m and V are the moment and shear force at the failure section, respectively), but is not lower when it is compared with the simply supported beam which has the same computed shear span ratio a/H.

The shear strength of the continuous beam was found to be lower than that of the simply supported beam. This fact may be the result of the failure of the bond between tensile reinforcement and concrete, and the

Table 1 Test Results

(1)	(2)	(3)	(4)	(5)	(6)		(7)
Element No	Failure Type	M	Failure Load(T)	ν	Horizontal projective Length of Inclined Crack		Condition of Plastic Hinge
					side L1	side L2	
CBP-1	Shear-Comp.	4.5	7.74	0.90	40	42	Occuring at the Central Support only
CBE-1	Shear-Comp.	4.5	9.25	0.93	45	45	No
CBP-2	Shear-Comp.	3	10.70	0.77	30	34	Occuring at the Central Support only
CBE-2	Shear-Comp.	3	11.75	0.80	30	39	No
SB2-a	Shear-Comp.	3	9.76	0.85	58	—	——
SB2-b	Shear-Comp.	2.3	12.36	0.86	37	—	——

redistrbution of the internal stress of top and bottom longitudinal reinforcing bars and concrete. In the experiment, the failure of the bond for continuous beams was found to be severe. It releases the restriction of the the reinforcement for the opening of the diagonal crack, reduces the dowel action of the reinforcement, and even makes compressive reinforcement into tensile reinforcement[6], hence makes the compressive stress of concrete increase, and compressive zone decrease.

In the thesis [6] a discussion is made on the appropriate value for the plastic coefficient, conversion of P/(BHRa) to shear strength, effect of loading manner, and the applicable range ($M > 1$) of the formulas (5) and (6).

5. CONCLUSION

The applicability of the solutions of the shear strength of RC members obtained by rigid-plastic theory is investigated. Good agreements were, so far, obtained between the shear strengths predicted by the formulas and test results. However, as the number of test specimens are limited and more experimental data are necessary to assure its full applicability.

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