コンクリート工学年次論文報告集 11-2 1989

論 文

[2114] Deformational Model for Solid Phase in Fresh Concrete under Compression (Single Materials)

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1. INTRODUCTION

At present there are many well-studied constitutive models for predicting deformational behavior of hardened concrete. With the use of these models, the deformational behavior of hardened concrete structures can be predicted. In the aspect of fresh concrete, one rational method is to treat fresh concrete as a multi-phase material. Analysis of dynamics of water in fresh concrete under compression is one of the example which treats fresh concrete as multi-phase material[1]. To predict the deformational and water-segregation behavior of fresh concrete as multi-phase material, the deformational models of every phase are essential especially for the complicated solid phase. Hence the idea for two-dimensional model for predicting the deformation of solid phase which is the mixture of aggregates and cementiteous material was proposed by the authors. In this paper, the idea for single materials, which are coarse aggregate, fine aggregate and powder materials separately, is proposed and that of the mixtures will be proposed following this paper. Various parameters indicating physical properties of the particles are considered. Lateral stress coefficient was derived for the case of uni-axial confined compression. Uni-axial confined compression tests of dry materials were performed to illustrate the applicability.

2. THEORETICAL IDEA OF THE MODEL

The idea for two-dimensional constitutive model of solid phase under compression is proposed in this paper. The single materials are considered composed of particles which are contacting each other. Each contact has its own contact angle (θ) and the density distribution of the contact angle is assumed to be $\Omega(\theta)$. $\Omega(\theta)$ can be explained simply as it represents the ratio of numbers of contact at contact angle θ to the total numbers of contact. At each contact plane, the force system acting on the contact is calculated and integrated over contact angle θ to obtain equilibrium. The force system contains frictional force in the tangential direction to the contact plane and normal force due to deformation normal to the contact plane. Stress-strain relation which is applied for relating the deformation normal to the contact plane with the corres-

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ponding stress is assumed. Friction is treated as Coulomb's friction. Contact area increases as the deformation progresses and it is affected by particle shape, size and grading. Re-arrangement of particles is also the significant factor especially for small sized and/or high frictional particles.

2.1 DENSITY FUNCTION FOR CONTACT ANGLE

Particles are considered having a density function for contact angle as in Fig.1. Li and Maekawa[2] proposed a different function to evaluate the effective contact area to model the shear transfer across crack. Here it is reasonable to suppose that there are negligible contacts with contact angle normal and parallel to the principal strain direction (θ equals 0 and $\pi/2$) but most are contacting such that θ is nearly or just $\pi/4$. Then the function for contact angle is assumed as

$$\Omega(\theta) = \sin(2\theta) \qquad (1)$$

which satisfies the mentioned condition.

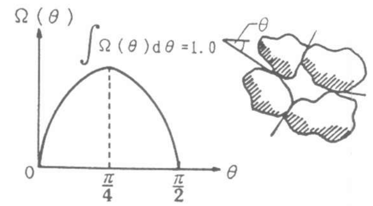

Fig.1 Density function of contact angle

2.2 DEFORMATION AT A CONTACT

Consider a unit volume of which each side has unit length so that deformations, $\omega\theta$ and $\delta\theta$, can be related to strains, ε_z and ε_y, in the analysis. Fig.2 shows 2-dimensional displacement compatibility for a contact at contact angle θ. From the geometry in Fig.2, $\omega\theta$ and $\delta\theta$ can be related to ε_y and ε_z by

$$\omega\theta = \varepsilon_z \cdot \cos\theta + \varepsilon_y \cdot \sin\theta \qquad (2)$$

$$\delta\theta = \varepsilon_z \cdot \sin\theta - \varepsilon_y \cdot \cos\theta \qquad (3)$$

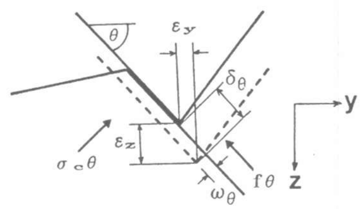

Fig.2 Initial and deformed configuration of a contact

when ε_{xy} equals zero since the co-ordinate axis coincides with the principal strain direction.

2.3 STRESS-STRAIN RELATIONSHIP FOR NORMAL DIRECTION

The stress-strain relationship for relating the normal stress ($\sigma_c\theta$) to the corresponding deformation (ω) under monotonic loading condition is assumed to be linear as

$$\sigma_c\theta = E_c \cdot \omega\theta \qquad (4)$$

2.4 FRICTION (SHEAR COMPONENT)

Coulomb's friction law is assumed to be applicable for the shear component. For simplicity the frictional stress is assumed constant independent on slip δ as in the following expression

$$f\theta = \mu \cdot \sigma_c\theta \qquad (5)$$

$\sigma_c\theta$ and $f\theta$ are illustrated in Fig.2.

2.5 EQUILIBRIUM EQUATION

The local force system acting on the contact at contact angle θ can be transformed to be the force in the global coordinate system as

$$F_z\theta = (\sigma_c\theta \cdot \cos\theta + f\theta \cdot \sin\theta) \cdot A_c\theta \tag{6}$$

$$F_y\theta = (\sigma_c\theta \cdot \sin\theta - f\theta \cdot \cos\theta) \cdot A_c\theta \tag{7}$$

Equilibrium is taken by integrating the multiplication product of force with the density function of the contact angle in the global coordinate over contact angle from $\theta=0$ to $\pi/2$ and equate the integral to the external force as

$$\sigma_z \cdot A_z = \int_0^{\pi/2} \Omega(\theta) \cdot F_z\theta \cdot d\theta \tag{8}$$

$$\sigma_y \cdot A_y = \int_0^{\pi/2} \Omega(\theta) \cdot F_y\theta \cdot d\theta \tag{9}$$

Substituting (6) and (7) into (8) and (9) and since $A_y = A_z = 1$, we obtain the principal stresses

$$\sigma_z = \int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_c\theta \cdot \cos\theta + f\theta \cdot \sin\theta) \cdot A_c\theta \cdot d\theta \tag{10}$$

$$\sigma_y = \int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_c\theta \cdot \sin\theta - f\theta \cdot \cos\theta) \cdot A_c\theta \cdot d\theta \tag{11}$$

Knowing the function for contact area ($A_c\theta$), one can solve Eqs. (2),(3),(4),(5),(10) and (11) to obtain the 2-dimensional stress-strain relationship of the single materials.

The lateral stress coefficient (K_o) can be obtained once σ_z and σ_y are known by using the following expression

$$K_o = \sigma_y/\sigma_z \tag{12}$$

2.6 CONTACT AREA

The factors affecting contact area of the particles are size, shape, grading and re-arrangement of the particles. One significant phenomenon to be accounted for is the increasing of contact area as the deformation progresses. This phenomenon is taken into account in this study by assuming that contact area of a contact angle θ increases depending on the amount of slip at that contact ($\delta\theta$). Contact area at a contact angle θ can be expressed as

$$A_c\theta = (A_c\theta_o + \int dA_c\theta) \cdot \phi \tag{13}$$

Regarding geometry in Fig.2 the summation of contact area increment can be obtained from (considering unit volume so that a unit width can be applied)

$$\int dA_c\theta = 1 \cdot \delta\theta \qquad (14)$$

Considering that $A_c\theta_o$ is small comparing with the increment due to slip

$$A_c\theta = \delta\theta \cdot \phi \qquad (15)$$

Therefore

$$A_c\theta = (\varepsilon_z \cdot \sin\theta - \varepsilon_y \cdot \cos\theta) \cdot \phi \qquad (16)$$

2.7 APPLICATION TO UNI-AXIAL CONFINED COMPRESSION TEST

The stress-strain relationship for uni-axial confined compression test was derived by taking ε_y in the original model as zero. The uni-axial confined compression information is important when dealing with migration of water in fresh concrete[1].

Due to the independence of ϕ on θ, the function ϕ can be considered as constants for each material then K_o for this case can be finally derived as

$$K_o = \frac{1-\mu}{1+\mu} \qquad (17)$$

The curves of K_o against μ from the model is given in Fig.3 and it is compared with other two curves which is calculated from $K_o = 1 - \sin\phi$, originally derived by Jaky[3] in 1948, by applying relationship between ϕ and μ. The two ϕ-μ relationships utilized are

$$\tan\phi = \pi \cdot \mu/2 \qquad (18)$$

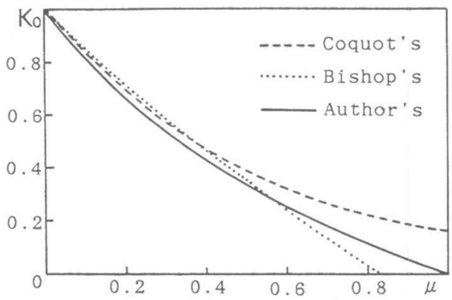

Fig.3 K_o - μ relationship

derived by Coquot[4,5] and

$$\sin\phi = \frac{15 \cdot \mu}{10 + 3 \cdot \mu} \qquad (19)$$

obtained experimentally by Bishop[5]. The curve derived from the model is in between these two for high μ and all curves are almost equivalent for μ smaller than 0.5.

3. VERIFICATION TEST OF THE MODEL

3.1 APPARATUS AND PROCEDURE

Uni-axial confined compression tests of dry materials were carried out to check the applicability of the model. Fig.4 shows the apparatus used for this test and the set-up. The materials used in the test are listed in Table.1 together with their properties. The materials were filled into the hollow steel cylinder with

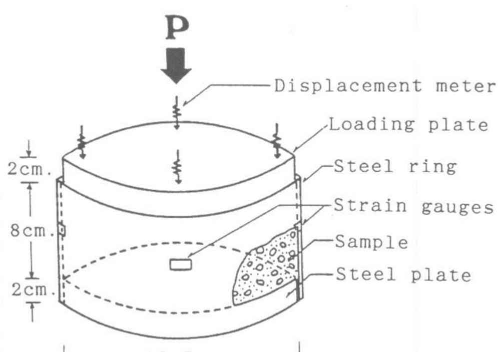

Fig.4 Uni-axial confined compression test

open top and bottom. Four strain gauges were attached to the outer side of the steel ring to measure lateral stress. Load was applied at the top of the cylinder and measured with a load cell. Stress was derived by dividing the applied load with the cross-sectional area of the steel ring. To observe volume decrease during the test, four displacement transducers were set on the loading plate at the top of the cylinder. Initial void of the material was calculated by subtracting solid volume of the material from the initial volume of the material. The solid volume of the materials was computed using specific gravity values of the materials to transform weight of the material into volume. The tested conditions are given in Table.1.

Table.1 Material properties and test condition

Test	Material	Specific gravity(g/cc)	Finess (cm^2/g)	μ	K_o	e_{max} (%)
C	Cement	3.15	3260	0.40	0.43	60.5
F	Fly ash	2.19	3000	0.23	0.63	43.2
S	River sand	2.62	(2.59)	0.32	0.52	40.2
G	River gravel	2.65	*	0.21	0.65	42.7

( ) Finess modulus
* Size ranges from 5 to 15 mm

3.2 TEST RESULTS AND DISCUSSION

Fig.5 and 6 show the analytical together with the test results of stress-strain and K_o curves. μ's were obtained from K_o's using K_o-μ relationship from the model and were given in Table.1. Since the material constants E_c and ϕ were difficult to be derived directly from experiment, the product of E_c and ϕ was derived by trial for the best fit of the test results. Since in the low stress level the effect of particle re-arrangement is dominant, all curves are normalized at ε_{11m} which is strain at a certain high stress level. It is known from the concrete practice that the stress applied to fresh concrete mixture does never exceed 50 kg/cm^2, consequently, ε_{11m} is decided to be strain at 50 kg/cm^2 in the model. It reveals from the figures that the model can well predict the results of uni-axial confined compression tests.

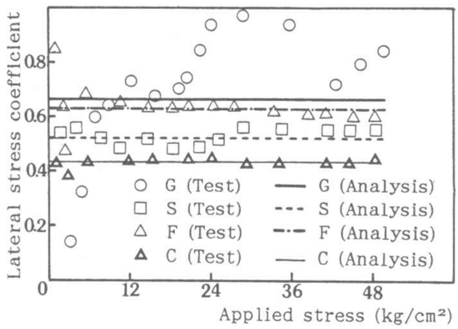

Fig.5 K_o versus applied stress relationship

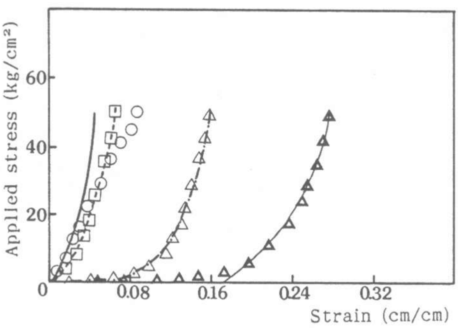

Fig.6 Stress versus strain relationship

Consider the results of K_o in Fig.5, it can be seen that K_o is higher for fly ash than for cement and river sand. This means that the inter-particle friction of fly ash is the lowest among the three materials since the higher the μ is, the lower the K_o will be obtained (see Eq.18). This is real as it is known that particle shape of fly ash is the roundest and cement is the most angular among the three.

In Fig.5 and Fig.6, the predicted and test results of gravel are consistent only in the low stress level. This is because crushing of the gravel occured at high stress and the crushing phenomenon is not encountered in the model. However crushing does hardly occur in real concrete mixture when particle finer than gravel are included in the mixture.

4. CONCLUSIONS

Conclusions can be made on the basis of the proposed model as:

1) A 2-dimensional constitutive model for single materials considering various parameters indicating physical properties of particles was proposed. The model can be applied to predict the stress-strain behavior of the single materials.

2) Inter-particle friction of the materials can be derived from the lateral stress coefficient.

3) The material constants are required in order to make the model more practical.

NOTATIONS

K_o : coefficient of lateral stress
θ : contact angle
$\Omega(\theta)$: probability density function for contact angle
$\omega\theta$: deformation normal to contact plane at contact angle θ
$\delta\theta$: deformation tangential to contact plane at contact angle θ (slip)
ε_y : normal deformation in the y direction of the global coordinate
ε_z : normal deformation in the z direction of the global coordinate
ε_{xy} : shear deformation in the global coordinate
E_c : stiffness of the stress-strain relation normal to contact plane
$A_c\theta$: contact area at angle θ
$\sigma_c\theta$: contact stress in the normal direction to contact plane θ
$f\theta$: frictional stress of the contact plane
$F_z\theta$: force acting in z direction in the global coordinate system
$F_y\theta$: force acting in y direction in the global coordinate system
σ_z: stress in z direction in global coordinate system
σ_y: stress in y direction in global coordinate system
A_z : area normal to z direction in global coordinate system
A_y : area normal to y direction in global coordinate system
$A_c\theta_o$: initial contact area of contact angle θ
$dA_c\theta$: increment of the contact area at contact angle θ
μ : coefficient of physical friction between grain
K_o : lateral stress coefficient
ϕ : macroscopic frictional angle
ϕ : function for effect of particle size, shape, grading and re-arrangement on contact area

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