論 文

[2173] BOND EFFECT ON THE CRACK PROPAGATION OF MASSIVE CONCRETE STRUCTURES UNDER THERMAL STRESS FIELDS

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ABSTRACT

Thermal cracks in massive concrete structures due to hydration heat of cement during construction period are very important in the design practice. Recently, the concept of fracture mechanics became to be introduced in the design codes instead of tensile strength criterion, when the structure is analyzed for fracture. With this phenomenon a massive reinforced concrete structure was analyzed for crack propagation and the energy loss due to bond slip between reinforcement and concrete was discussed.

1. INTRODUCTION

Thermal cracks due to hydration heat of cement are very common in massive concrete structures even during construction period. In the design of these types of structures, thermal crack widths, lengths and spacings should be considered since they have much influence on the structural performances. In the past, tensile strength criterion was used in the design codes and since it is understood that it does not give good agreement with fracture test data, energy criterion became more reasonable in real design practice when fracture is considered. This research work is concentrated on the analysis of behaviour of massive concrete structures at early ages, and the determination of effect of bond between reinforcing bars and surrounding concrete on the fracture propagation.

In the analysis, concrete is treated as a heterogeneous material and crack formation is considered as a band of micro - cracks smearedly distributed within the band. However, crack initiation is determined by the well known tensile strength criterion, and crack propagation is governed by the energy criterion. In the analysis of massive concrete structures with initial thermal stress fields, the total energy supplied to propagate the crack band is calculated as the sum of the total strain energy released by the cracked element and the total potential energy released by the uncracked elements of the rest of the structure. An interlayer bond element was proposed to consider the effect of bond slip between steel and surrounding concrete, on the crack propagation.

Based on these considerations, a massive concrete structure at different ages was analyzed using finite element method and the behaviour of the structure was discussed.

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2. FINITE ELEMENT FORMULATION

2.1. CONSTITUTIVE RELATION FOR UN-CRACKED AND CRACKED CONCRETE

The elastic constitutive relation for homogeneous material with uncracked elastic stiffness matrix $[D_{C,uc}]$ can be written as

$$\{\sigma_C\} = [D_{C,uc}]\{\varepsilon_C\} \tag{1}$$

where, $\sigma_C = \{\sigma_p, \sigma_q, \sigma_r\}^T$ and $\varepsilon = \{\varepsilon_p, \varepsilon_q, \varepsilon_r\}^T$ are the principal stress and strain components in p, q and r directions respectively (Fig.1(a)).

Now, when only cracks to the direction p are permitted, the appearance of cracks at constant stress increases only the overall strain ε_p normal to the cracks and has no effect on the lateral strains ε_q and ε_r. Hence, the orthotropic stiffness matrix of cracked concrete $[D_{C,cr}]$ can be achieved by the gradual reduction of elastic stiffness matrix with the progressive development of micro-cracks. Therefore, the compliance matrix after the appearance of partial discontinuous cracks should have the form[1]

$$[C(\mu)_{C,pcr}] = \begin{bmatrix} \mu^{-1}C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ sym. & & C_{33} \end{bmatrix} \tag{2}$$

where, μ is called cracking parameter varies from 1 to 0 when concrete gets fully cracked. Hence, the final form of the stiffness matrix of the fully cracked concrete becomes

$$[D_{C,cr}] = \lim_{\mu \to 0}[C(\mu)_{C,pcr}]^{-1} \tag{3}$$

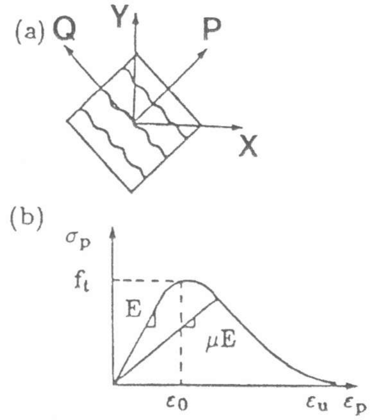

Fig. 1. (a). Co-ordinate System
(b). Stress-Strain Relation

2.2. ELEMENT STIFFNESS MATRIX FOR CONCRETE

The stiffness matrix for isoparametric concrete element (Fig.2(a)) can be obtained from the principal of virtual work as

$$[K_C] = \int_V [B]^T[D_{C,cr}][B]dV \tag{4}$$

where, $[B]$ is the strain displacement matrix for concrete.

2.3. ELEMENT STIFFNESS MATRIX FOR REINFORCING BAR ELEMENTS

In the analysis, reinforcing bars are arranged along the grids of the finite element mesh and in the formulation it was considered as a bar element having one degree of freedom system along the bar at each ends. Then, the stiffness matrix for steel bars becomes

$$[K_S] = \frac{E_S A_S}{2w} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{5}$$

where, E_S and A_S are the Young's modules and area of the reinforcing bars while $2w$ is the length of the bar element (Fig.2(b)).

2.4. BOND ELEMENT STIFFNESS

The bond or interlayer element between reinforcement and concrete is considered to contain simple springs parallel and perpendicular to the reinforcement having spring stiffness k_h and k_v to respective directions. This element has length $2w$, but zero width as the nodal point pairs (1,4) and (2,3) are identical initially[2] (Fig.2(c)).

The bond stress displacement relation for interlayer element can be written as

$$\{\tau\} = [k_b]\{u_b\} \tag{6}$$

in which $\{u_b\}$ = the relative displacement vector and, $[k_b]$ = diagonal material property matrix expressing the bond stiffness and can be written as

$$\{u_b\} = \begin{Bmatrix} u_h \\ u_v \end{Bmatrix} = \begin{bmatrix} u_h^{top} - u_h^{bottom} \\ u_v^{top} - u_v^{bottom} \end{bmatrix}, \qquad [k_b] = \begin{bmatrix} k_h & 0 \\ 0 & k_v \end{bmatrix} \tag{7}$$

with

$$\begin{Bmatrix} u_h^{bottom} \\ u_v^{bottom} \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} 1-\frac{x}{w} & 0 & 1+\frac{x}{w} & 0 \\ 0 & 1-\frac{x}{w} & 0 & 1+\frac{x}{w} \end{bmatrix} \{u^t\},$$

$$\begin{Bmatrix} u_h^{top} \\ u_v^{top} \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} 1+\frac{x}{w} & 0 & 1-\frac{x}{w} & 0 \\ 0 & 1+\frac{x}{w} & 0 & 1-\frac{x}{w} \end{bmatrix} \{u^b\}$$

Here, $\{u^t\}^T = \{u_1, \ v_1, \ u_2, \ v_2\}$ and $\{u^b\}^T = \{u_3, \ v_3, \ u_4, \ v_4\}$.

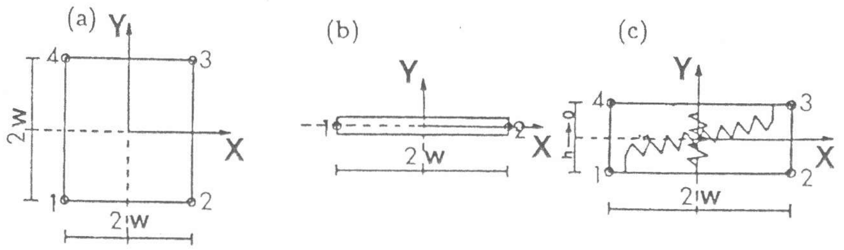

Fig. 2. Isoparametric Elment (a). Concrete, (b). Steel Bar, (c). Bond

Now, beginning from the energy equation and minimizing the potential energy Φ, with respect to nodal point displacements, the element stiffness for four-nodal-point bond element will be derived from

$$\Phi = \frac{1}{2}\int_{-w}^{w} \{u_b\}^T[k_b]\{u_b\}dx, \qquad [K_B] = \int_V B_b^T[k_b]B_b dV \tag{8}$$

where, $[B_b]$ is the matrix which gives the relation between relative displacement and nodal displacement of bond element.

From the derived element stiffness matrices for concrete, reinforcing bars and bond elements, the global structural stiffness matrix can be formulated and was used in the finite element analysis.

3. ENERGY CONCEPT FOR FRACTURE PROPAGATION

3.1. FRACTURE ENERGY

The fracture energy can be defined as the energy required to produce a crack per unit area of the crack plane. For uniaxial stress state system where the uniaxial tensile stress in $p-$direction exceeded the tensile strength, the fracture energy G_f can be related with the total strain energy W_f with crack band width w_c as follows (Fig.3).

$$G_f = w_c W_f \tag{9}$$

Since at present, there is no any appropriate formula to estimate the exact value of fracture energy for early age concrete, the empirical formula (given in Ref. [1]) proposed by Bazant was used with the consideration of change of Young's modules[3] with time and is written as

$$G_f = 1.697(f_t + 8.95) f_t^2 d_a / E_C(t) \tag{10}$$

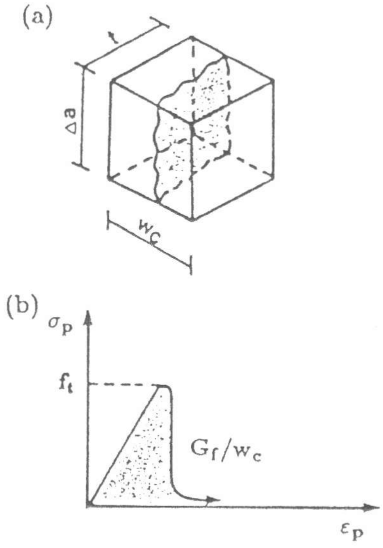

Fig. 3. (a). Cracked Element
(b). Sudden Stress Drop

in which, f_t is the tensile strength in kgf/cm^2, d_a is the maximum aggregate size in cm, E_C is the Young's modules in kgf/cm^2 and G_f is the fracture energy in kgf- cm/cm^2.

In the analysis of massive concrete structures, since the size of elements are comparatively large, according to the eqn. (9) to preserve the fracture energy an idealized stress-strain curve as sudden stress drop (Fig.3(b)) was used.

3.2. ENERGY RELEASED FOR FRACTURE

In general, the amount of energy supplied for cracking can be calculated as the difference of the amount of total potential energy before and after cracking. But, in the analysis of thermal stress problems, this gives inconveniences in the calculations and the remedy which is given in Ref. [3] was taken on the basis of Rice's work.

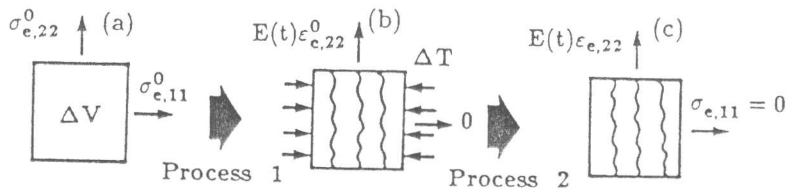

Fig. 4. (a). Before Cracking, (b). Intermediate Stage, (c). Cracked Stage

The total energy supplied was calculated by dividing the whole process of cracking into two processes as follows (Fig.4) [3]. In the first process, the cracks are created inside the volume ΔV of the element of the crack in the direction perpendicular to the principal tensile stress when it reaches to its tensile strength, while at the same time, the deformations, stresses and strains in the rest of the body are imagined to remain fixed. This is achieved by introducing an external virtual surface traction to the cracked surface. Next, the second process is performed by releasing of the applied virtual force gradually by applying the opposite and equal forces reaching in this way the final state. Hence, the total change in potential energy $\Delta\Pi$ is[3]

$$\Delta\Pi = \Delta\Pi_{1^{st}Process} + \Delta\Pi_{2^{nd}Process} \tag{11}$$

3.3. CRITERION FOR CRACK PROPAGATION

Crack initiation occurs when the maximum principal stress reaches to the tensile strength of concrete and but, this is not the criteria for crack propagation of concrete when fracture of concrete is concerned. Hence, using fracture energy the criterion for crack propagation can be given as follows.

The total energy released to extend the crack band by length Δa is equal to the difference of energy released and consumed by crack formation and this can be expressed as

$$\Delta U = \Delta\Pi_{1^{st}Process} + \Delta\Pi_{2^{nd}Process} - (G_f\Delta a + \Delta U_b) \tag{12}$$

in which, the energy loss due to bond slip ΔU_b is given by

$$\Delta U_b = w_c \int_{-w}^{w} \tau dx \tag{13}$$

where, $w_c = 2w$.

Now from the energy criterion of fracture propagation, if the net amount of energy released ΔU is positive, the crack band is unstable and, without supplying external energy to the system, the crack propagation can be occurred in a dynamic manner. If it is equal to zero, no energy is released, and crack band propagate in static manner without any energy supply. If it is negative, no crack propagation occurs without supplying external energy to the structure, and the crack band is stable.

4. NUMERICAL CONSIDERATION ON CRACK PROPAGATION

With this consideration, a massive concrete structure was analyzed for crack propagation in the early ages. A block of concrete having size 1550 × 30 × 100 cm. (L × W × H) was cast on a hardened concrete block having same length and height but 100 cm width (Fig.5(a)). Reinforcement are arranged along the horizontal grids of the mesh having its ratio equal to 2.0 % and 25 × 25 cm, 25 × 50 cm mesh sizes was used for the newly cast and old concrete blocks respectively. First this was analyzed to determine the temperature distribution of the structure at every 3 hr. up to 1^{st} day, every 12 hr. up to 4^{th} day, every day up to 7^{th} day and every 7 days up to 28^{th} day, two-dimensionally. Using the results of the temperature analysis, stress analysis was performed.

In the analysis, a high value for the vertical stiffness k_v for the bond element was used. The value of k_h was assumed to have 200 kg/cm^2/cm. The maximum principal stress was found to occurs on the top layer of the mesh and with time it moved from both ends towards to the center. At the third day, tensile stress of the elements on the top layer at about 325cm from both ends reached to the tensile strength and cracks were propagated to the interface of the newly cast and hardened concrete block. Then, the stress at the middle section also reached the strength and penetrated to the interface of the blocks (Fig.5(b)). The same structure was analyzed experimentally by Ishikawa et.al.[4], and similar crack patterns were observed.

The bond forces obtained were very small compared to the stress of concrete and the loss of energy was comparatively small. Hence, at this time step there was no much influence of the bond slip to the crack propagation and the crack patterns were same as in the case where the effect of bond was not included. This may be due to the very high value of stiffness assumed for the bond element in the analysis. Hence, the consideration of the values of the bond stiffness of early age concrete is necessary.

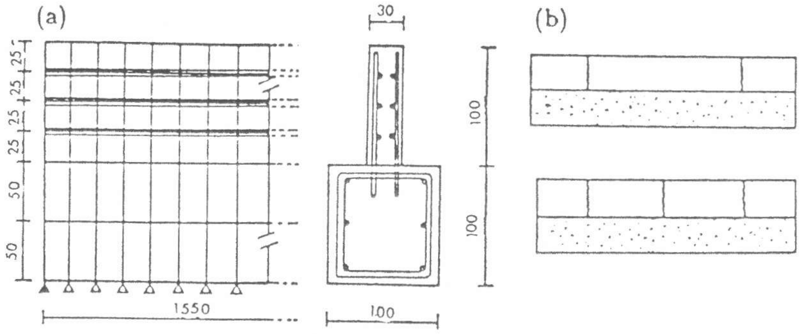

Fig. 5. (a). Finite Element Discritization, (b). Crack Pattern

5. CONCLUSION

In the present analysis, since very high values of stiffness was used for the bond element, the energy loss due to bond slip was comparatively small and the crack patterns was almost same where this effect was not considered. When the interlayer bond element is included in the finite element mesh, the number of element was increased. Hence, the analysis was not able to continue with low memory levels of the computer. This gives much difficulties in the computation and hence, alternation with optimum finite element discritization is necessary and will be considered as the continuation of this work.

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