コンクリート工学年次論文報告集　13-2　1991

論 文

[2174] ANALYSIS OF BEAMS WITH SHEAR REINFORCEMENT BY FINITE ELEMENT METHOD

Nares PANTARATORN* and Tamon UEDA**

1. INTRODUCTION

There have been many researches relating to shear problems of reinforced concrete beams through the experimental works and analytical methods. Considering time and the latter is preferable. There has been developed a two dimensional finite element program[1] with the constitutive laws of micro-mechanical models in reinforced concrete such as concrete tension stiffening model[2], model of stress transfer at crack[3], deformational model of concrete[5] and model of reinforcing bar in concrete[5]. Although this program can be applied to shear walls but cannot predict well behavior of beams in shear under cyclic loadings[6]. The differences in micro-mechanical modeling between shear walls and beams, such as effect of anchorage slip of stirrup, have to be taken into consideration.

In this paper the micro-mechanical model for concrete tension stiffening of a beam is proposed through numerical analysis, considering the effect of anchorage slip. The model for stress transfer at crack which was derived from the condition of constant crack width path is modified, considering the analytical model for variable crack width path[8] which was observed in the experiment[9]. The computed shear reinforcement stresses are compared with the experimental ones.

2. CONCRETE TENSION STIFFNESS OF BEAM INCLUDING ANCHORAGE SLIP EFFECT

Negligence of the anchorage slip causes tension stiffness of the reinforced concrete element in the FEM program[1] to overestimate the experimental results. In order to obtain a proper tension stiffening model for beams, the bond-slip-strain relationship[10] of steel bar embedded in massive concrete where no cracking takes place is modified to be applied to stirrup which is embedded with a small cover and intersected by cracks diagonally. The modified bond-slip-strain relationship can be expressed as Eq.1.

$$\tau/f_c' = \alpha\{0.73[\ln(1+5s)]^3\}/[1+(1+\beta)(10^5\varepsilon_s)] \qquad (1)$$

where τ : bond stress, fc': concrete strength, α : bond reduction factor for deterioration near crack, $s=1000S/D$, S : local slip, D : bar diameter, β : Strain factor for small concrete cover, and ε_s : bar strain. Bond deterioration near crack is assumed to occur within the distance of $5D$ from a crack intersection and not to be affected by the angle between crack and reinforcement[8]. In the deterioration area the α factor decreases linearly and becomes zero at the crack intersection (see Fig.1). The β factor of 0.5 is determined by best fitting the measured strain

---

* Department of Civil Engineering, University of Tokyo
** Department of Civil Engineering, Hokkaido University

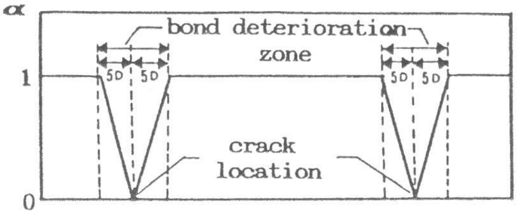


FIG.1 α diagram

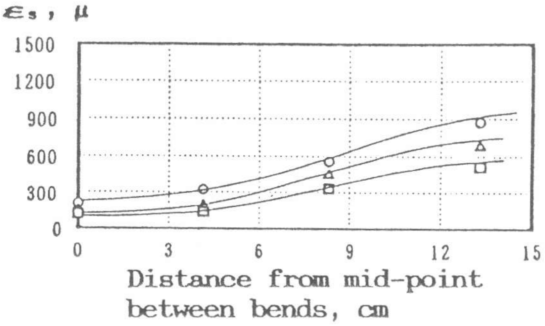


FIG.2 Strain distribution along stirrup

distributions along stirrups as shown in Fig.2.

There is an evidence of the slip of shear reinforcement at its lower bent portion as shown in Fig.3[9]. The slip is caused by the elongation of stirrup between lower bent portions and can be obtained analytically by using the modified bond-slip-strain relationship in Eq.1.

When a crack width is small enough, tensile stress can be transferred by concrete(tension softening). The relationship between tensile stress of concrete and crackopening proposed by Reinhardt et al.[11], as expressed in Eq.2, is applied in this study.

$$\sigma_c / f_t = [1+(c_1 w/w_0)^3]\exp(-c_2 w/w_0) - (w/w_0)(1+c_1^3)\exp(-c_2) \qquad (2)$$

where σ_c : concrete stress, f_t : tensile strength of concrete, $w=S_{cr}-\Delta_{total}$: crack width (crack opening), S_{cr} : slip at crack, $\Delta_{total}=\int \varepsilon_c(x)dx$: total concrete deformation, ε_c : concrete strain, w_0 : stress-free crack width($\simeq 160\mu m$), and c_1 and c_2 : material constants ($=3$ and 6.93).

The average stress-strain relationships of both concrete and shear reinforcement in an element as shown in Fig.3 can be obtained from Eqs.1 to

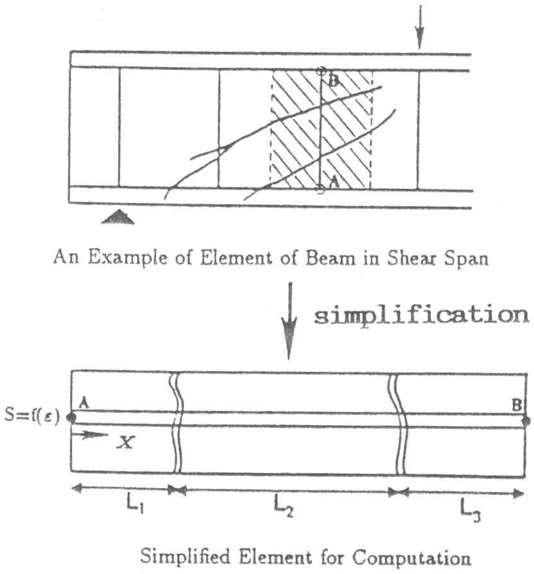


FIG.3 Simplified element for computation

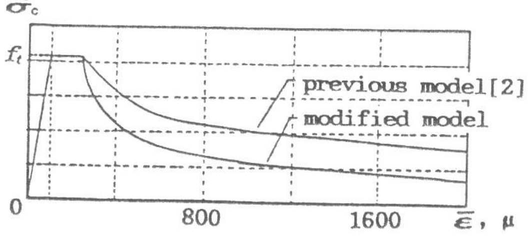


FIG.4 Proposed model compared with previous model[2]

5 and the equilibrium among σ_c, σ_s and τ.

$$\bar{\varepsilon}_c = \bar{\varepsilon}_s = \bar{\varepsilon} = [\int^L \varepsilon_s(x)dx + S_\theta]/L \tag{3}$$

$$\bar{\sigma}_c = \int^L \sigma_c(x)dx/L \tag{4}$$

$$\bar{\sigma}_s = \int^L \sigma_s(x)dx/L \tag{5}$$

where S_θ : slip at lower bend, σ_s : bar stress, L : length of element and $\bar{\ }$: average. For the same average strain of bar, its average stress computed with the slip at bend is slightly smaller than that without the slip. The difference is negligible. However, the average stress-strain relationship of concrete with the slip obviously differs from that without the slip, which is close to the model[2] in the FEM program for shear walls (see Fig.4). Effects of number of cracks (one or two) and beam height (*300* to *600mm*) on the average stress-strain relationship of concrete are small. Based on the computed average stress-strain relationships, a simplified unique relationship with the effect of the slip at bend is proposed as follows.

$$\bar{\sigma}_c = f_t \ (\varepsilon_{tu}/\bar{\varepsilon})^m \ \exp[-n(1-\varepsilon_{tu}/\bar{\varepsilon})] \tag{6}$$

where ε_{tu} : cracking strain ($\simeq 0.02\%$), $m=0.3$ and $n=1$.

For reversed cyclic loading, the previous model[2] in which stress carried by concrete is determined as a sum of the stress transmitted by crack contact surface, σ_{cc} and that by bond action, σ_{sb}. The same expression as in Eq.7 is applied in this study.

$$\sigma_c = \sigma_{cc} + \sigma_{cb} \tag{7}$$

3. FORCE TRANSFER AT CRACK

The equations for stresses transferred at a crack, which are derived from the contact density model[3], can be well applied to loading paths with constant crack widths. For monotonic loading, they are as follows.

$$\tau_c = 3.83 f_c'^{1/3} \ \lambda^2/(1+\lambda^2) \tag{8}$$

$$\sigma_n' = 3.83 f_c'^{1/3} \ [\pi/2 - \cot^{-1}\lambda - \lambda/(1+\lambda^2)] \tag{9}$$

where τ_c : shear stress along crack, σ_n' : compressive stress normal to crack, $\lambda=\delta/w=\gamma_{xy}/\varepsilon_x$, δ : shear slip and w : crack opening. However, some experimental observations on the displacements at shear crack in beams indicate that the crack opening and shear slip vary at the same time as shown in Fig.5. For an envelope curve, Eq.8 gives slightly higher shear stress transferred at a crack than a recent numerical model for stress transfer at a crack for any concrete deformation path[8]. However, this difference is neglected considering the effects of reinforcement crossing the crack which are to provide dowel forces and to make crack deformations at a concrete surface less than those at reinforcement because of bond action (see Fig.6). At unloading and reloadings the previous model gives quite different values of transferred stresses from the recent model[8]. To compute the transferred stresses, the measured crack deformations as in Fig.5[9] are modified by considering the effect of bond of reinforcement (see Fig.6). Because of this difference, modified shear-compressive stress relationships for unloading and reloading are proposed as follows (see Fig.7).

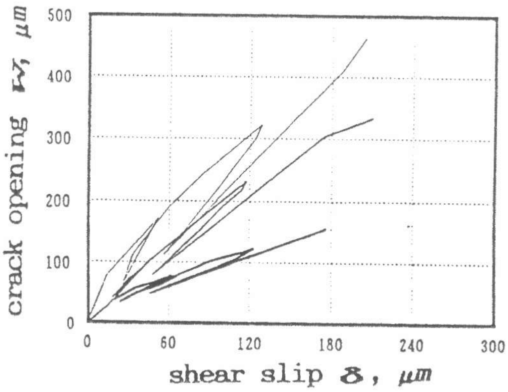


FIG.5 Observed crack opening - shear slip relationship [9]

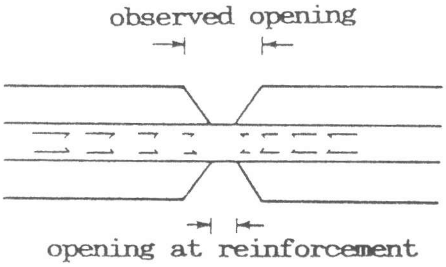


FIG.6 Bond effect on crack opening

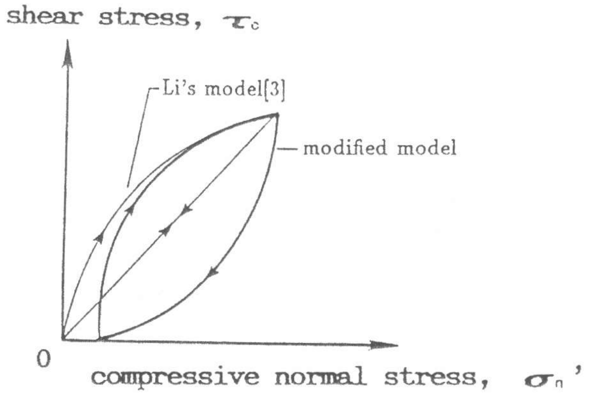


FIG.7 Shear stress - compressive normal stress relationship

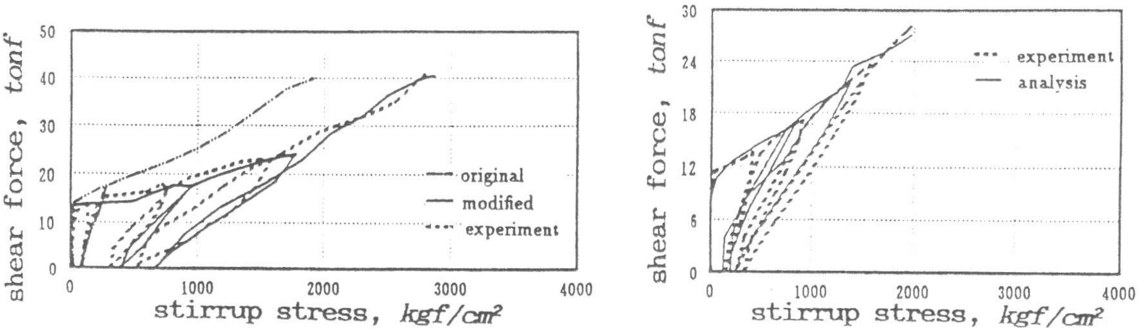
$$\tau_c/\tau_{max} = (\sigma_n'/\sigma_{max}')^k \quad (10)$$

$$(\sigma_n' - \sigma_{min}')/(\sigma_{max}' - \sigma_{min}') = [(\tau_c - \tau_{min})/(\tau_{max} - \tau_{min})]^k \quad (11)$$

where $k=5$, τ_{max}, σ_{max}', τ_{min} and σ_{min}' : maximum and minimum shear and normal compressive stress at the last unloading.

4. SHEAR FORCE-STIRRUP STRESS RELATIONSHIP AND CONCRETE COMPRESSIVE STRESS FLOW IN SHEAR SPAN

An example of the measured and computed relationships between shear force applied to a beam and stirrup stress are shown in Fig.8. In Fig.9 the layout of one of the specimens is given. For the computation with the modified tension stiffening model only, the predicted curves are close to those of the experimental ones only on the envelope but not at unloading and reloading. With the both modified models of tension stiffening and stress transfer at crack, the predicted curves agree well with the experimental ones for all the paths (see Fig.8a). For further confirmation, a test of beams with stirrups welded to longitudinal bars at the top and bottom was conducted, where the effect of anchorage slip of stirrup was eliminated. The prediction with only the modified model of



(a) (b)

FIG.8 Shear force - stirrup stress relationships

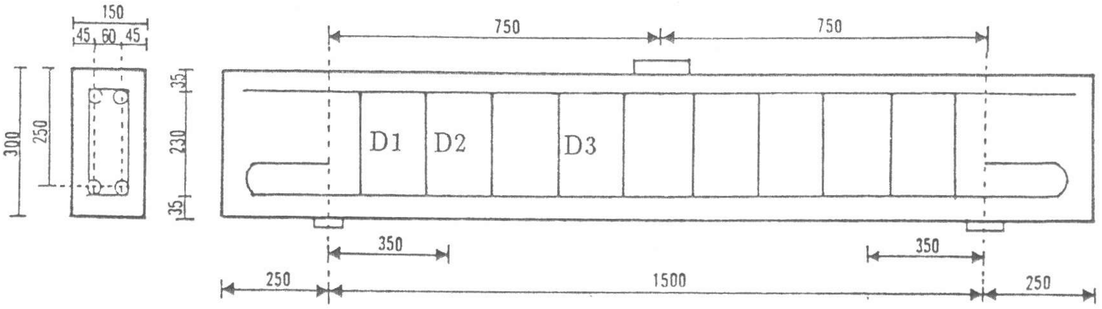


FIG.10 Beam layout

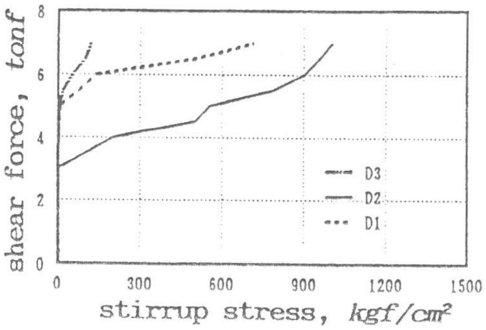


FIG.10 Stress of stirrups at different locations

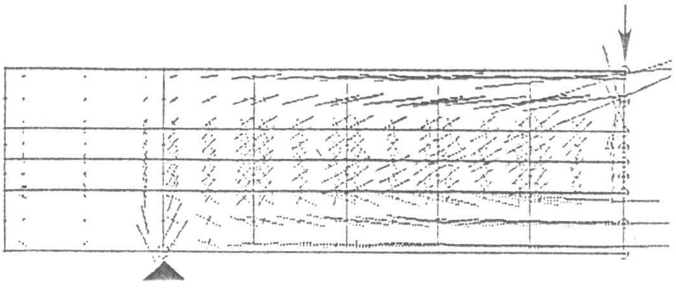


FIG.11 Principal stresses in a shear span

concrete tension stiffening gives good agreement with theexperimental results as seen in Fig.8b.

The computed stresses of stirrups at different locations, which are indicated in Fig.9, are shown in Fig.10. The stresses of stirrups near the loading point or the support are smaller than those of a stirrup at the center of shear span.

The computed diagonal concrete stress flows in a shear span are rather uniform with an angle to the member axis about 30° as shown in Fig.11.

5. CONCLUDING REMARKS

The finite element analysis program for shear walls[1] is modified in order to be applied to beams. The modification is conducted on (1) concrete tension stiffening model considering stirrup slip at lower bend and (2) stress transfer model at crack considering difference in typical crack deformation path between shear walls and beams. The modified finite element analysis program predicts well relationships between applied shear force and stirrup stress for stirrups at different locations in beams under cyclic loading and indicates uniform diagonal concrete compressive stress stress flows with an inclination about 30° in a shear span.

ACKNOWLEDGEMENT

The authors would like to express their gratitude to Prof. H. Okamura, the University of Tokyo, for his valuable advices towards this study.

REFERENCES

1) Okamura, H., Shin, H. and Maekawa, K. : Development of Analytical Models for Reinforced Concrete, Proc. of the Sessions Related to Design, Analysis and Testing at Structures Congress '89, ASCE, May 1989, pp.49-58.
2) Okamura, H., Maekawa, K. and Izumo, J. : Reinforced Concrete Plate Element Subjected to Cyclic Loading, Proc. of IABSE Colloquium Delft 1987, August 1987, pp.575-589.
3) Li, B., Maekawa, K. and Okamura, H. : Contact Density Model for Stress Transfer Across Cracks in Concrete, Journal of the Faculty of Engineering, the University of Tokyo(B), Vol.40, No.1, March 1989, pp.9-52.
4) Maekawa, K. and Okamura, H. : The Deformational Behavior and Constitutive Equation of Concrete Using Elasto-Plastic and Fracture Model, Journal of the Faculty of Engineering, the University of Tokyo(B), Vol.37, No.2, September 1983, pp.253-328.
5) Izumo, J., Shima, H. and Okamura, H. : Analytical Model for Reinforced Concrete Panel Elements Subjected to In-Plane Forces, Concrete Journal, Vol.25, No.9, September 1987, pp.107-120 (in Japanese).
6) Koshiji, T. and Okamura, H. : The Effect of Cyclic Loading on Shear Resisting Mechanism in RC Beams, Proc. of JCI, Vol.11, No.2, 1989, pp.339-344 (in Japanese).
7) Pantaratorn, N. : Finite Element Analysis on Shear Resisting Mechanism of RC Beam, Dissertation Submitted to the University of Tokyo, March 1991.
8) Bujadham, B. and Mishima, T. : Cyclic Discrete Crack Modeling for Reinforced Concrete, Proc. of SCI-C 1990, Second International Conference, April 1990, pp.1225-1236.
9) Hassan, H. M. : Shear Cracking Behavior and Shear Resisting Mechanism of Reinforced Concrete Beams with Web Reinforcement, Dissertation Submitted to the University of Tokyo, March 1988.
10)Shima, H., Chou, L. and Okamura, H. : Micro and Macro Models for Bond Behavior in Reinforced Concrete, Journal of the Faculty of Engineering, the University of Tokyo(B), Vol.39, No.2, September 1987, pp.133-194.
11)Reinhardt, H. W., Cornelissen, H. A. W. and Hordijk, D. A. : Tensile Tests and Failure Analysis of Concrete, Journal of Structural Engineering, ASCE, November 1986, pp.2462-2477.