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論 文

[1200] Size Effect for the Reinforcing Fibers of FRP Rods

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1. INTRODUCTION

FRP rods suffer some problems concerning the appropriate anchorage and the failure pattern. This affects their potential for practical application. The evaluation of rods optimum (theoretical) strength is, then, important from two points of view. First, it gives estimation to strength reduction due to improper anchorage. Second, it helps provide better understanding to the failure mechanism, through the comparison of the proposed theoretical mechanism (in calculation) and the experimental one.

Recently, Monte Carlo simulation was applied for the evaluation of the optimum strength of the FRP rods [1]. The fibers inside the rod section are given strengths at random, according to appropriate distribution, and the damage propagation across the section is studied till failure. For this purpose, a fibers strength distribution should be used. The existence of the matrix in the composite helps transfer the load of the broken fibers to the surrounding ones, across a certain transfer length, and back to the original fibers. Hence, the damaged fiber will be ineffective only along the transfer length where the damage accumulation takes place. Therefore, the relevant strength distribution for evaluating the composite is that obtained for the transfer length.

The standard testing methods for the fibers specifies a certain gage length which is usually much bigger than the transfer length. As the size and length of the fibers affect their strength, a certain size effect rule should be employed for the extrapolation of the strength distribution from the standard one. A commonly used rule is that of Weibull distribution, based on the weakest link concept. This rule, however, was subjected to some criticism in case of the glass optical fibers[2]. Therefore, in this research, the reliability of the rule is investigated for three kinds of fibers commonly used in practice for unidirectional reinforcement of FRP rods.

2. WEAKEST LINK CONCEPT

This concept was introduced by Weibull [3] in 1951, and considers the member composed of group of links connected in series. The links follow the same strength distribution and the probability of member survival is the probability of survival for all the links together. That is

$$1 - F = (1 - F_l)^n \tag{1}$$

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Where F = Cumulative probability function for member (fiber) strength.
F_l = Cumulative probability distribution for links strength.
n = Number of links in the member (fiber).

Weibull assumed the function F_l in the following form (simplified to the two-parameter form)

$$F_l = (\frac{x}{x_o})^m \tag{2}$$

Where x = link strength.
x_o = scaling factor stands for the maximum possible material strength.
m = Weibull modulus which is a material constant.

Substituting eq.2 in eq.1, and for large number of links n, one gets

$$1 - F = (1 - (\frac{x}{x_o})^m)^n \simeq \exp[-n(\frac{x}{x_o})^m] = \exp[-(\frac{x}{\alpha})^m] \tag{3}$$

Where $\alpha = n^{\frac{-1}{m}} x_o$

Since the number of links in the fiber is proportional to its length (l), Weibull's rule implies that, for different l_1 and l_2,

$$(\frac{\alpha_1}{\alpha_2})^m = \frac{n_2}{n_1} = \frac{l_2}{l_1} \tag{4}$$

In this way, the distribution parameters can be determined at any length given their values at a specified length.

In order to determine the parameters of Weibull distribution for a given set of data, there exist several methods [4,5]. Herein, two of these methods are shown and are employed for the data obtained in this research. First method is the graphical method. Taking the logarithms of both sides in eq.3 and rearranging, one gets the following equation

$$\ln(-\ln(1 - F)) = m\ln(x) - m\ln(\alpha) \tag{5}$$

Which is in the form

$$Y = mX + B \tag{6}$$

This means that the plot of $\ln(-(ln(1 - F))$ versus $\ln(x)$ is a straight line and the slope of which is m. This implies, also, that the data collected at different lengths shows parallel lines. Parameter α can be determined by searching for the point (x_i) at which $\ln(-\ln(1 - F)) = 0$, then from eq.5 $\alpha = x_i$.

The second method makes use of the distribution moments. The first moment, mean value, can be expressed as

$$\bar{x} = \alpha\Gamma(1 + \frac{1}{m}) \tag{7}$$

and the standard deviation is

$$\sigma = \alpha[\Gamma(1 + \frac{2}{m}) - (\Gamma(1 + \frac{1}{m}))^2]^{\frac{1}{2}} \tag{8}$$

Hence, the coefficient of variation can be expressed as

$$C.O.V. = \frac{\sigma}{\bar{x}} = \frac{[\Gamma(1 + \frac{2}{m}) - (\Gamma(1 + \frac{1}{m}))^2]^{\frac{1}{2}}}{\Gamma(1 + \frac{1}{m})} \tag{9}$$

The shape of the distribution, as determined by the $C.O.V.$ is a function of the parameter m only. That is why it is frequently referred to as the shape parameter, instead of Weibull modulus. Hence, for a set of data, $C.O.V.$ can be calculated and m is obtained from eq.9. Then, substitution in any of eq.7 or eq.8 gives the parameter α.

3. EXPERIMENTAL WORK

In the experimental program, three kinds of fibers were tested in axial tension, namely: aramid, carbon and glass fibers. Three lengths were specified for testing: 25 (standard), 12 and 6 mm. The effect of strain rate on the distribution shape was investigated by applying two strain rates to the fibers. The same strain rate was adjusted for different lengths by changing the machine crosshead speed. Tested lengths and the corresponding crosshead speeds and strain rates are shown in Table 1. The testing machine was Autograph with load capacity of 5000 grams. It monitors both stroke and load level on LCD screens and prints their final values by a built-in printer within ±1% error.

A total of about 3000 fibers were tested: 100 fibers of 25 mm length per strain rate per material and 200 fibers per strain rate per material for other lengths. Each fiber were mounted in a standard rectangular (20mm x 45mm) carton frame with an opening in the center across which the tested fiber is mounted. The experiments were conducted in an average temperature of 20°C.

Fibers diameters were determined in a previous study [6] using the scanning electron microscope. The reliability of the experiments were confirmed through plotting the correlations between the strength and both Young's modulus and maximum strain. Fig.1 shows sample of these plots; where a linear correlation between strength and maximum strain, and absence of correlation between strength and modulus can be observed.

Table 1 Strain Rates forDifferent Fiber Lengths

Length (mm)	Crosshead Speed (mm/min)	Strain Rate (/min)
6	0.5	0.083
	1.0	0.167
12	1.0	0.083
	2.0	0.167
25	2.0	0.080
	5.0	0.200

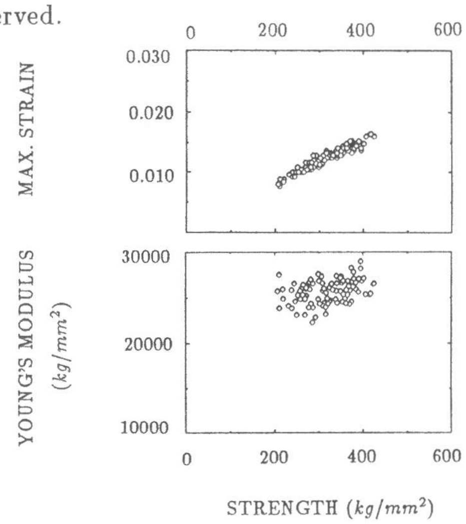

Fig.1 Tensile Parameters Correlations

(Carbon Fibers, Length : 25 mm, Loading rate : 0.08/min.)

4. RESULTS AND DISCUSSION

The tested fibers statistical parameters are given in Table 2 for the two strain rates (0.083 , 0.167/min). It is obvious from the values of the $C.O.V.$ that Weibull modulus can be considered as a material constant.

Further confirmation of the constancy of Weibull modulus for a certain material is made by

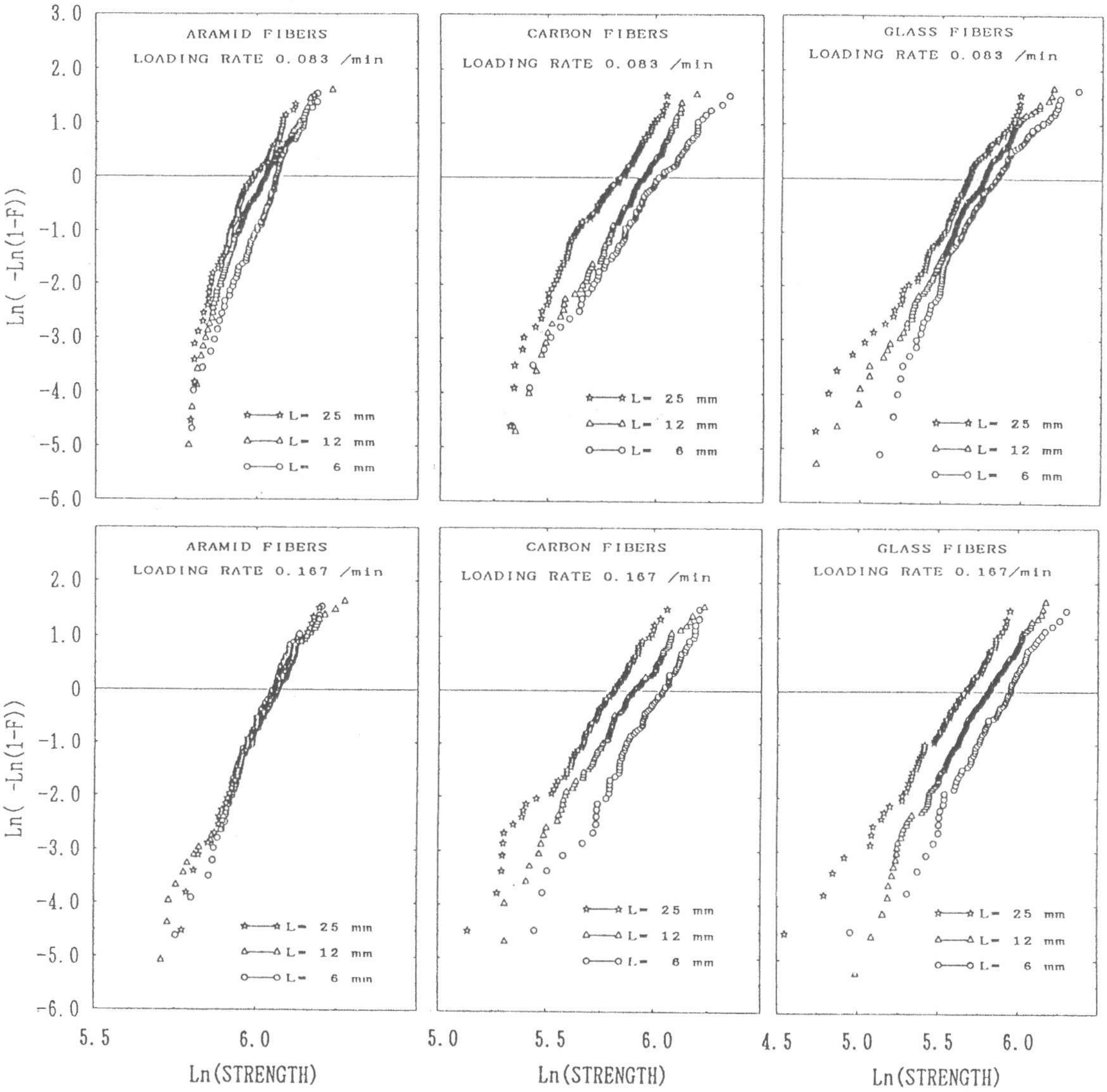

Fig.2 Tensile Strength Distributions for Different fibers lengths (in the form of eq.5)

Table 2 Statistical Parameters of Fibers Tensile Strength

Material	Length (mm)	Mean (kg/mm^2)		Standard Deviation (kg/mm^2)		C.O.V.		Predicted Mean (kg/mm^2)	
		0.083*	0.167*	0.083*	0.167*	0.083*	0.167*	0.083*	0.167*
Aramid	6	416	411	32	33	0.078	0.081	429	458
	12	402	414	36	40	0.089	0.096	409	435
	25	390	413	32	36	0.082	0.088	390	413
Carbon	6	388	392	74	61	0.190	0.157	389	386
	12	357	347	58	63	0.162	0.183	351	347
	25	317	311	55	56	0.173	0.179	317	311
Glass	6	322	352	75	74	0.234	0.210	359	359
	12	295	309	66	68	0.224	0.220	312	309
	25	271	266	61	63	0.225	0.240	271	266

* Strain rate (1/min.)

plotting the results in forms of X and Y of eq.5. The plots, shown in Fig. 2, show essentially parallel lines. The parameters of Weibull distribution were calculated using the aforementioned methods and the results are shown in Table 3. It is obvious that m can be considered a material constant.

In actual practice, the distribution is obtained for the test results of the standard length (25 mm) and those for shorter fibers are predicted from it. Hence, the parameters of the distributions at 25 mm long fibers, the average in Table 3, were used for predicting other distributions, and the results were tested for goodness of fit. The test conducted herein, to evaluate the goodness of fit, is the K-S (Kolomogrov-Smirnov) test that calculates the maximum absolute difference between the proposed and actual cumulative distributions. Then compares it with a standard value dependent on the degree of significance (5% is the common practice) and the number of

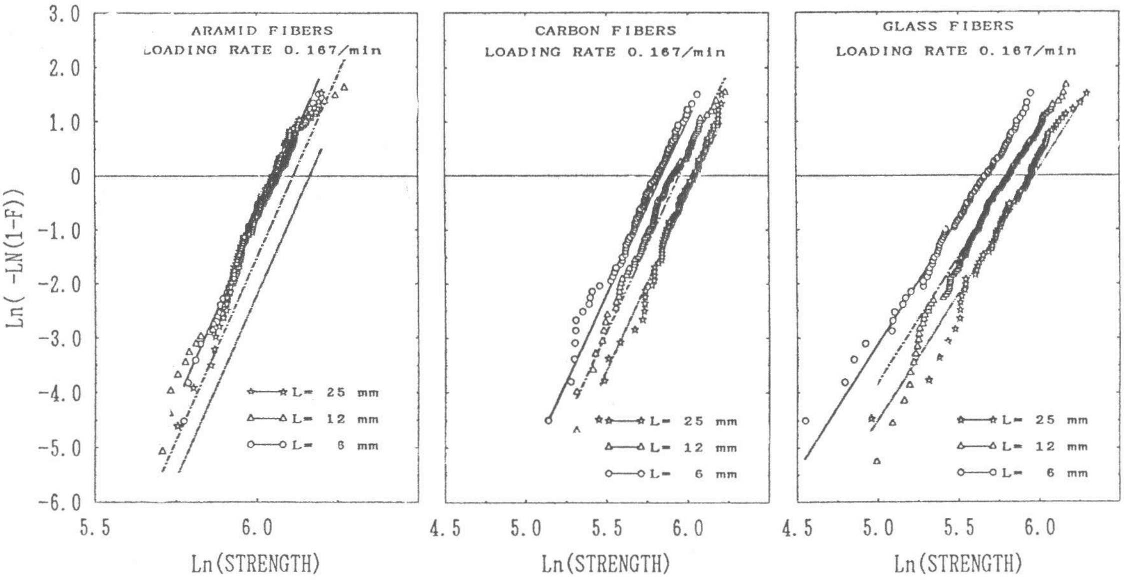

Fig.3 Actual Data and Predictions according to Weibull's Weakest Link Concept.

Table 3 Weibull Distribution Parameters for Different Fibers

Mterial	Strain Rate (/min)	Length (mm)	Method of Moments		Graphical Method		Average	
			m	α^{-1}	m	α^{-1}	m	α^{-1}
Aramid	0.083	6	16.00	0.00232	15.05	0.00233	15.53	0.00233
		12	13.50	0.00239	13.46	0.00242	13.48	0.00241
		25	15.00	0.00247	14.20	0.00252	14.60	0.00250
	0.167	6	15.00	0.00235	14.53	0.00236	14.76	0.00235
		12	12.60	0.00232	12.37	0.00235	12.49	0.00234
		25	13.60	0.00233	13.26	0.00233	13.43	0.00233
Carbon	0.083	6	6.00	0.00240	5.97	0.00244	6.07	0.00242
		12	7.10	0.00262	7.03	0.00263	7.06	0.00263
		25	6.84	0.00294	6.67	0.00292	6.76	0.00293
	0.167	6	7.50	0.00239	7.17	0.00238	7.34	0.00239
		12	6.44	0.00268	6.33	0.00274	6.38	0.00271
		25	6.60	0.00299	6.20	0.00298	6.40	0.00299
Glass	0.083	6	4.90	0.00285	5.16	0.00292	5.03	0.00288
		12	5.00	0.00311	5.14	0.00314	5.07	0.00312
		25	5.00	0.00383	4.82	0.00347	4.91	0.00365
	0.167	6	5.55	0.00277	5.26	0.00259	5.41	0.00268
		12	5.50	0.00299	5.30	0.00300	5.40	0.00300
		25	4.80	0.00344	4.46	0.00344	4.63	0.00344

pieces of data. The significance of this test arises from the fact that it tests the similarity of cumulative distributions which are employed for the assignment of fibers strengths, randomly, in the simulation technique.

When each strain rate was considered separately, carbon and glass fibers passed the test showing reliable prediction. On the contrary, aramid fibers prediction curves shows overestimation of fibers strengths at shorter lengths, for both strain rates. Moreover, the average parameters for both rates were employed for prediction but only carbon fibers could pass the K-S test showing the least sensitivity to strain rate. Examples of the prediction curves plotted with the actual data are shown in Fig.3 that confirms the reliability of Weibull distribution for predicting strength at different lengths of both carbon and glass fibers, and shows the overestimation in case of aramid fibers. According to eqs. 4 & 7, the mean strength is proportional to the reciprocal of the mth. root of length ratio. Then, considering the mean strength at length 25 mm as a reference, predictions of the mean strengths are given in the last two columns of Table 2. These values confirm the reliability of Weibull distribution for both carbon and glass fibers, and the overestimation for aramid fibers. The size effect rule of aramid fibers, according to this data, is complicated and has strain rate dependence. At the high strain rate, strength distribution remains unchanged for all the lengths. The explanation of this phenomena, however, is not within the scope of this paper and it might be dealt with elsewhere.

5. CONCLUSIONS

1. For the three kinds of fibers tested in this research and for the two applied strain rates, Weibull modulus is a material constant in axial tension.
2. For the case of carbon fibers,the weakest link concept can be used for predicting the strength distributions of shorter fibers regardless of the strain rate.
3. In case of glass fibers, the weakest link concept is still applied but with strain rate dependence.
4. For aramid fibers, the size effect cannot be described by the weakest link concept that gives an overestimation to the strength although distribution shape is efficiently described.

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