論 文

[2178] Localized Identification of Structural Parameters by Kalman Filter

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1. INTRODUCTION

The problem of system identification and parameter estimation as applied to structural engineering is complicated and expensive (in terms of computing time) especially in the analysis of structures modelled with a large number of degrees of freedom. To reduce the size of the system under consideration, the substructure approach in the analysis of structures looks attractive. In substructuring, a structure is divided into a number of smaller subsystems called substructures whose boundaries are specified. This approach when applied to system identification is efficient and practical because the analysis can be concentrated on the local and critical parts of the structure.

The identification of a local and critical part of structures is useful in the evaluation of the condition of structures. The overall performance of structures is dependent on the individual members. If one member is damaged, the load carrying capacity of the structure will certainly decrease and total collapse may occur especially if the damaged member is a critical part of the structure. In this regard, the identification of the structural parameters of the local part of structures becomes significant.

The extended Kalman filter has been applied by a number of researchers in many problems in structural dynamics [1][2][3]. However, unlike the past applications where the total structure is analysed, this paper presents a localized approach in the identification of structural parameters. By substructuring, a structure was divided into primary and secondary systems with common boundary and the identification was concentrated on the secondary system. Incorporating the state and observation equations formulated from the equation of motion of the secondary system in the EKF algorithm, the stiffness and damping parameters were estimated. To illustrate the localized identification approach, a simple shear building model subjected to ground motion was analyzed and the identification was concentrated on the first story.

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2. EXTENDED KALMAN FILTER

In order to apply the extended Kalman filter to system identification problems, an appropriate set of state vector and observational vector equations must be formulated, respectively as

$$\frac{d\mathbf{X}}{dt} = f(\mathbf{X}, t) \tag{1}$$

$$\mathbf{Y}(k) = h(\mathbf{X}(k), k) + \mathbf{v}(k) \tag{2}$$

where $\mathbf{X}(k)$ is the state vector at time $t = k\Delta t$, $\mathbf{Y}(k)$ is the observational vector at time $t = k\Delta t$, $\mathbf{v}(k)$ is the observational noise vector with covariance matrix, $\mathbf{R}(k)$, and Δt is the sampling time interval.

In the EKF algorithm [4], the initial state vector, $\hat{\mathbf{X}}(0/0)$, and its error covariance matrix, $\mathbf{P}(0/0)$ are first assumed; and then as the observational vector, $\mathbf{Y}(k)$ is processed, the state vector and the error covariance matrix are recursively updated. For convergence purposes in the parameter estimation, the extended Kalman filter with weighted global iteration (EK-WGI) developed by Hoshiya and Saito [1] is applied.

3. STATE EQUATION FOR LOCALIZED IDENTIFICATION

In substructuring, the total structure is subdivided into several substructures. Without loss of generality, the structure can be divided into two substructures in which one of them is usually smaller(in mass and/or stiffness) than the other. The smaller substructure is commonly referred to as the secondary and the larger as the primary. These two substructures, which are attached at a common boundary or interface, are referred to as primary-secondary systems or simply P-S systems (Figure 1).

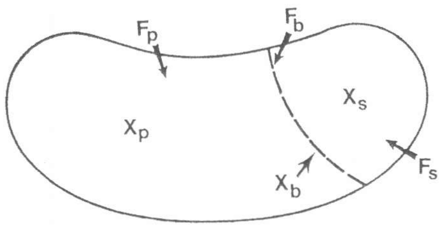

Figure 1. A Primary-Secondary System

Let the vector $\mathbf{X}_p$ denote coordinates of degrees of freedom(DOF) that belong only to the primary system, $\mathbf{X}_s$ denote solely the secondary DOF and $\mathbf{X}_b$ denote DOF of the boundary or the interface points that belong to both the primary and secondary systems. The equation of motion of the composite P-S system in partition form can then be written as follows [5]:

$$\begin{bmatrix} \mathbf{M}_{pp} & 0 & 0 \\ 0 & \mathbf{M}_{bb} & 0 \\ 0 & 0 & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_p \\ \ddot{\mathbf{X}}_b \\ \ddot{\mathbf{X}}_s \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{pp} & \mathbf{C}_{pb} & 0 \\ \mathbf{C}_{bp} & \mathbf{C}_{bb} & \mathbf{C}_{bs} \\ 0 & \mathbf{C}_{sb} & \mathbf{C}_{ss} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{X}}_p \\ \dot{\mathbf{X}}_b \\ \dot{\mathbf{X}}_s \end{Bmatrix}$$

$$+\begin{bmatrix} \mathbf{K}_{pp} & \mathbf{K}_{pb} & 0 \\ \mathbf{K}_{bp} & \mathbf{K}_{bb} & \mathbf{K}_{bs} \\ 0 & \mathbf{K}_{sb} & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_p \\ \mathbf{X}_b \\ \mathbf{X}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_p \\ \mathbf{F}_b \\ \mathbf{F}_s \end{Bmatrix} \tag{3}$$

in which the subscripts p, s and b refer to primary, secondary and boundary DOF, respectively. $\mathbf{F}_p$, $\mathbf{F}_b$ and $\mathbf{F}_s$ are external forces applied to the primary, boundary and secondary DOF, respectively. It has been assumed for simplicity that $\mathbf{X}_p$ and $\mathbf{X}_s$ are not inertially coupled with $\mathbf{X}_b$. A lumped mass model satisfies this assumption.

Since our interest is on the identification of the structural parameters of a small and localized section of the structure, only the equation of motion of the secondary system will be considered, i.e.

$$\mathbf{M}_{ss}\ddot{\mathbf{X}}_s + \mathbf{C}_{ss}\dot{\mathbf{X}}_s + \mathbf{K}_{ss}\mathbf{X}_s = \mathbf{F}_s - \mathbf{K}_{sb}\mathbf{X}_b - \mathbf{C}_{sb}\dot{\mathbf{X}}_b \tag{4}$$

The state equation can be derived from the equation of motion of the secondary system. Premultiplying eq.(4) by $\mathbf{M}_{ss}^{-1}$ and introducing the following matrices $\mathbf{C}_{ss}^* = \mathbf{M}_{ss}^{-1}\mathbf{C}_{ss}$, $\mathbf{K}_{ss}^* = \mathbf{M}_{ss}^{-1}\mathbf{K}_{ss}$, $\mathbf{K}_{sb}^* = \mathbf{M}_{ss}^{-1}\mathbf{K}_{sb}$, $\mathbf{C}_{sb}^* = \mathbf{M}_{ss}^{-1}\mathbf{C}_{sb}$ and $\mathbf{F}_s^* = \mathbf{M}_{ss}^{-1}\mathbf{F}_s$ will result in

$$\ddot{\mathbf{X}}_s + \mathbf{C}_{ss}^*\dot{\mathbf{X}}_s + \mathbf{K}_{ss}^*\mathbf{X}_s = \mathbf{F}_s^* - \mathbf{K}_{sb}^*\mathbf{X}_b - \mathbf{C}_{sb}^*\dot{\mathbf{X}}_b \tag{5}$$

The current identification problem consists of finding the optimal estimates of the unknown coefficients $\mathbf{C}_{ss}^*$, $\mathbf{K}_{ss}^*$, $\mathbf{C}_{sb}^*$, and $\mathbf{K}_{sb}^*$. The secondary mass matrix $\mathbf{M}_{ss}$ is assumed to be known for simplicity. Selecting $\mathbf{X}_s$ and $\dot{\mathbf{X}}_s$ as the state variables and the coefficients of the matrices, $\mathbf{C}_{ss}^*$, $\mathbf{K}_{ss}^*$, $\mathbf{K}_{sb}^*$ and $\mathbf{C}_{sb}^*$ as augmented state variables, the state vector $\mathbf{X}$ becomes

$$\mathbf{X} = [x_{s1}\ x_{s2}\ \ldots\ x_{sm}\ \dot{x}_{s1}\ \dot{x}_{s2}\ \ldots\ \dot{x}_{sm}\ c^*_{ss11}\ \ldots\ c^*_{ssmm}\ k^*_{ss11} \\ \ldots\ k^*_{ssmm}\ k^*_{sb11}\ \ldots\ k^*_{sbml}\ c^*_{sb11}\ \ldots\ c^*_{sbml}]^T \tag{6}$$

or it can be written in compact form as

$$\mathbf{X} = [\mathbf{X}_1^T\ \ \mathbf{X}_2^T\ \ \mathbf{X}_3^T\ \ \mathbf{X}_4^T\ \ \mathbf{X}_5^T\ \ \mathbf{X}_6^T]^T \tag{7}$$

in which

$$\mathbf{X}_1 = \{x_{s1}\ x_{s2}\ \ldots\ x_{sm}\}^T,\ \mathbf{X}_2 = \{\dot{x}_{s1}\ \dot{x}_{s2}\ \ldots\ \dot{x}_{sm}\}^T,\ \mathbf{X}_3 = \left\{c^*_{ss11}\ \ldots\ c^*_{ssmm}\right\}^T,$$
$$\mathbf{X}_4 = \left\{k^*_{ss11}\ \ldots\ k^*_{ssmm}\right\}^T,\ \mathbf{X}_5 = \left\{k^*_{sb11}\ \ldots\ k^*_{sbml}\right\}^T,\ \mathbf{X}_6 = \left\{c^*_{sb11}\ \ldots\ c^*_{sbml}\right\}^T$$

where c^*_{ssij}, k^*_{ssij}, k^*_{sbij}, c^*_{sbij} are ij elements of matrices $\mathbf{C}_{ss}^*$, $\mathbf{K}_{ss}^*$, $\mathbf{K}_{sb}^*$ and $\mathbf{C}_{sb}^*$, respectively; m is the number of secondary DOF and l is the number of boundary DOF. Equation (6) or (7) is a $[2m + 2m^2 + 2ml]$ dimensional state vector.

With the aid of eq.(7), the differential equation given by eq.(5) can be written into a state equation of the system as

$$\dot{\mathbf{X}} = \begin{Bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \\ \dot{\mathbf{X}}_3 \\ \dot{\mathbf{X}}_4 \\ \dot{\mathbf{X}}_5 \\ \dot{\mathbf{X}}_6 \end{Bmatrix} = \begin{Bmatrix} \mathbf{X}_2 \\ -\mathbf{C}_{ss}^*(\mathbf{X}_3)\mathbf{X}_2 - \mathbf{K}_{ss}^*(\mathbf{X}_4)\mathbf{X}_1 + \mathbf{F}_s^* - \mathbf{K}_{sb}^*(\mathbf{X}_5)\mathbf{X}_b - \mathbf{C}_{sb}^*(\mathbf{X}_6)\dot{\mathbf{X}}_b \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

Equation (8) is a continuous state equation of the dynamic system and corresponds to eq.(1). Using an appropriate observation equation and the response time histories of the secondary system, the structural parameters can be identified. Having identified the elements of the matrices $\mathbf{C}^*_{ss}$, $\mathbf{K}^*_{ss}$, $\mathbf{K}^*_{sb}$ and $\mathbf{C}^*_{sb}$ and multipying by $\mathbf{M}_{ss}$, an estimate of $\mathbf{C}_{ss}$, $\mathbf{K}_{ss}$, $\mathbf{K}_{sb}$ and $\mathbf{C}_{sb}$ can be obtained. It must be understood that in order to implement the identification procedure described, the external force vector $\mathbf{F}_s$, the boundary displacement vector $\mathbf{X}_b$ and the boundary velocity vector $\dot{\mathbf{X}}_b$ must be known.

4. APPLICATION TO A SHEAR BUILDING

To illustrate the application of the localized identification of structures, a shear building will be analyzed. Consider the model of a four DOF shear building subjected to ground motion as shown in Figure 2.

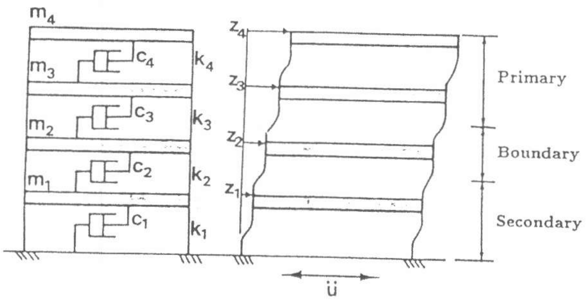

Figure 2. A four DOF shear building model considered as a composite P-S system

The equation of motion for the shear building considered as a composite P-S system equivalent to eq.(3) can be written as

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \begin{Bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \\ \ddot{z}_3 \\ \ddot{z}_4 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & 0 \\ 0 & -c_3 & c_3+c_4 & -c_4 \\ 0 & 0 & -c_4 & c_4 \end{bmatrix} \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{Bmatrix}$$

$$+ \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{Bmatrix} = - \begin{Bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{Bmatrix} \ddot{u} \tag{9}$$

Let us divide the composite system such that z_1 is the secondary DOF, z_2 is the boundary DOF and both z_3 and z_4 are the primary DOF. It would be practical to concentrate the identification on the first story which is usually a critical section for a building. Hence only the first equation in eq.(9) will be considerd and is given by

$$m_1\ddot{z}_1 + (c_1+c_2)\dot{z}_1 + (k_1+k_2)z_1 = -m_1\ddot{u} + k_2 z_2 + c_2\dot{z}_2 \tag{10}$$

and this can be rewritten as

$$\ddot{z}_1 + \left\{\frac{(c_1+c_2)}{m_1}\right\}\dot{z}_1 + \left\{\frac{(k_1+k_2)}{m_1}\right\} z_1 = -\ddot{u} - \left\{-\frac{k_2}{m_1}\right\} z_2 - \left\{-\frac{c_2}{m_1}\right\}\dot{z}_2 \tag{11}$$

This equation corresponds to the general equation of motion of the secondary system given by

$$\ddot{\mathbf{X}}_s + \mathbf{C}^*_{ss}\dot{\mathbf{X}}_s + \mathbf{K}^*_{ss}\mathbf{X}_s = \mathbf{F}^*_s - \mathbf{K}^*_{sb}\mathbf{X}_b - \mathbf{C}^*_{sb}\dot{\mathbf{X}}_b \tag{12}$$

in which $\mathbf{X}_s = z_1$, $\mathbf{X}_b = z_2$, $\mathbf{F}^*_s = -\ddot{u}$, $\mathbf{C}^*_{ss} = \{(c_1 + c_2)/m_1)\}$, $\mathbf{K}^*_{ss} = \{(k_1 + k_2/m_1)\}$, $\mathbf{K}^*_{sb} = \{-\ k_2/m_1\}$ and $\mathbf{C}^*_{sb} = \{-\ c_2/m_1\}$. Having derived the equation of motion of the secondary system, the state vector given as eq.(7) and the state equation given as eq.(8) can be derived. In our present problem, the state vector consists only of 6 elements with $\mathbf{X}_1 = \{z_1\}$, $\mathbf{X}_2 = \{\dot{z}_1\}$, $\mathbf{X}_3 = \mathbf{C}^*_{ss}$, $\mathbf{X}_4 = \mathbf{K}^*_{ss}$, $\mathbf{X}_5 = \mathbf{K}^*_{sb}$ and $\mathbf{X}_6 = \mathbf{C}^*_{sb}$.

Depending on the observed data such as displacement or velocity, the measurement or the observation equation is given by eq.(13) or (14), respectively.

$$\mathbf{Y}(k) = [1\quad 0\quad 0\quad 0\quad 0\quad 0]\,\mathbf{X}(k) + \mathbf{v}(k) \tag{13}$$

$$\mathbf{Y}(k) = [0\quad 1\quad 0\quad 0\quad 0\quad 0]\,\mathbf{X}(k) + \mathbf{v}(k) \tag{14}$$

The augmented state variables $\mathbf{X}_3$, $\mathbf{X}_4$, $\mathbf{X}_5$ and $\mathbf{X}_6$ are the parameters to be identified. It must be understood that the input data which consist of the ground acceleration $\ddot{u}$, the boundary displacement z_2 and the boundary velocity $\dot{z}_2$ must be known so that the identification can be implemented.

5. NUMERICAL EXAMPLE

The identification method was applied to the four DOF shear building described in Figure 2 using the structural parameters given in Table 1.

Table 1. Parameters of Structural Model (m_i: mass in kgf-sec^2/cm, c_i: damping coefficient in kgf-sec/cm, k_i: story stiffness in kgf/cm)

Parameter	mass 1	mass 2	mass 3	mass 4
m_i	20	10	10	10
c_i	32	18	14	14
k_i	8000	4500	3500	3500

From Table 1, the parameters of the secondary system represented by the first story can be computed as $\mathbf{M}_{ss} = 20$, $\mathbf{C}_{ss} = 50$, $\mathbf{K}_{ss} = 12500$, $\mathbf{K}_{sb} = -4500$ and $\mathbf{C}_{sb} = -18$. Dividing each parameter by $\mathbf{M}_{ss}$ results in $\mathbf{C}^*_{ss} = 2.5$, $\mathbf{K}^*_{ss} = 625.0$, $\mathbf{K}^*_{sb} = -\ 225.0$ and $\mathbf{C}^*_{sb} = -\ 0.90$.

The El Centro (1940) acceleration time history was used as the input ground motion, $\ddot{u}$. Using the linear acceleration method, eq.(9) was solved to obtain the response of the composite system using a sampling time of 0.05 s. The calculated responses for m_1 and m_2 were used in the identification, where the calculated z_2 and $\dot{z}_2$ were used as input data, and the calculated responses of z_1 or $\dot{z}_1$ were used as the observed data.

In implementing the EK-WGI procedure, the initial values for displacement and velocity were set as 0. The initial error covariance matrix is set as 0.1 for response and 100 for unknown parameters. The noise covariance matrix for the observation response was taken as 1.0. A weighted value of 100 was used in the global iteration. The displacement or the velocity time histories can be used as the observation. In this paper, the first 200 values of the velocity time history of m_1 were used as the observed data. Shown in Table 2 are the results obtained from the velocity time history. It can be seen that the parameters were reasonably estimated. It must be noted that the convergence of parameters is affected by many factors such as the observed data, sampling time, sampling length and the initial values. These factors must be taken into account for in-depth study.

Table 2. Results of identification of parameters of secondary system

Parameter	True Value	Velocity	
		Initial Value	Final Value
$\mathbf{C}^*_{ss}$	2.50	1.0	2.499
$\mathbf{K}^*_{ss}$	625.0	1.0	625.0
$\mathbf{K}^*_{sb}$	−225.0	1.0	−225.0
$\mathbf{C}^*_{sb}$	−0.90	1.0	−0.899

Table 3. Estimated parameters of the shear building (first and second stories)

Parameter	c_1	k_1	c_2	k_2
True Value	32.0	8000.0	18.0	4500.0
Estimated Value	31.992	8000.0	17.988	4500.0

Multiplying the estimated parameters by $\mathbf{M}_{ss}$ which is assumed to be known, we will obtain the estimates of $\mathbf{C}_{ss}$, $\mathbf{K}_{ss}$, $\mathbf{K}_{sb}$ and $\mathbf{C}_{sb}$. Using the relationship between the parameters of the secondary system and the actual stiffness and the damping parameters of the shear building, the estimates of the first story parameters can be obtained as c_1 and k_1. Incidentally, the second story parameters, c_2 and k_2 were also estimated as shown in Table 3. It was shown that the parameters of the first and the second stories of the shear building were reasonably estimated.

6. CONCLUSION

A procedure for localized identification of structural parameters using the extended Kalman filter was presented. In the procedure, a structure was decomposed into two substructures which were attached at a common boundary, and three systems, primary, boundary and secondary systems, were formed and the identification of parameters was concentrated on the secondary system. The localized identification procedure was illustrated by analyzing a simple shear building where the identification was concentrated on the first story. The application to the shear building was useful and practical especially for highrise buildings since the structural parameters at the lower levels can be estimated without considering the response at the higher levels. Although it was not shown in this paper, the proposed method can also be used to identify the total structure by dividing the structure into a number of substructures and applying the method to each substructure. To verify the capability and usefulness of the localized identification procedure, applications to more complicated structures must be conducted.

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