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# [2005] Shear Analysis of Reinforced Concrete Beams by Fictitious Crack Model Using Orthogonal Rod Elements

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## 1. INTRODUCTION

The performance of shear analysis for reinforced concrete members is an important area of structural engineering research. Shear failure of concrete structures can be sudden, without warning, and can lead to catastrophic results. To capture the effects of shear has followed generally an empirically-based approach as considered in codes.

This paper will present the research work performed by the authors into the application of fracture mechanics, in conjunction with finite element analysis, to shear behavior and will illustrate the size effects and its consequences in shear design. Numerous experimental and analytical studies have indicated that the traditional approach utilizing ACI[1], whereby a constant value of shear capacity, $V_c$, is computed for any size member, may not be keeping with the actual behavior exhibited. Many studies have shown that the shear capacity of reinforced concrete beams without stirrups is significantly influenced by the effect of member size. Recognizing this size effect, the JSCE[2] in Japan and CEB[3] in Europe incorporated this effect in shear design equations and obtained reasonable predictions. Therefore, the shear strength obtained from the JSCE and CEB will be used for comparisons and to verify the numerical results from the finite element analysis.

In this paper the application of fracture mechanics in the form of fictitious crack model to the shear problem is executed, In this model orthogonal rod elements are provided to predict the shear behavior and formation of cracks. Here some improvements in the model[4] have been considered.

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## 2. FICTITIOUS CRACK MODEL WITH ORTHOGONAL ROD ELEMENTS

The fictitious crack model was introduced by Hillerborg, et al. [5] and it utilizes an energy-based approach to predict the formation of cracks in concrete. In this approach, the fracture energy, $G_F$, is associated with the stress-crack width curve. If the tensile stress reaches the concrete cracking stress, crack will occur and the stress will decrease as the crack opens.

In the previous model[4], the stiffness only perpendicular to the crack path was considered. The shear stiffness along the crack path was neglected. As a consequence, the deformation diagram of the beam showed that there is a big slip along the crack path since the beginning (from 1st step of calculation) as shown in Fig 1(a). This is due to the assumption that the shear stiffness along the predefined crack surface was neglected; thus, the provision of shear stiffness along the crack path is necessary. Another link element should be provided in the shear direction. Since it is very difficult to get the information about the Mode II fracture energy, many researchers are using the Mode I fracture energy both for tension and shear. For simplicity the elastic shear modulus is provided in this analysis. Shear modulus can be calculated as follows:

$$G_c = \frac{E_c}{2(1+\nu)} \tag{1}$$

These link elements parallel to crack plane are assumed to have the stiffness during the elastic stage, but once crack occurs, it is assumed to lose the stiffness at all. Fig.1(b) shows the deformation shape of the beam with shear stiffness. The idea of orthogonal rod elements is shown in Fig.2.

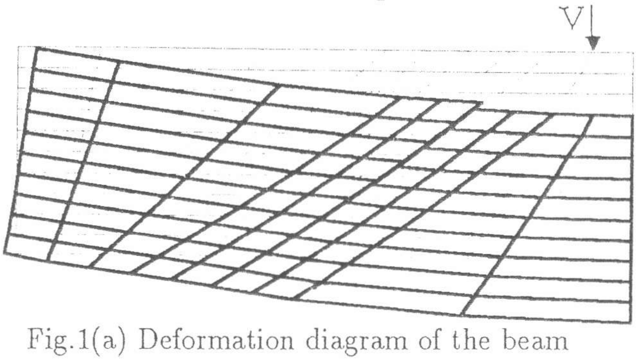



Fig.1(a) Deformation diagram of the beam without shear stiffness

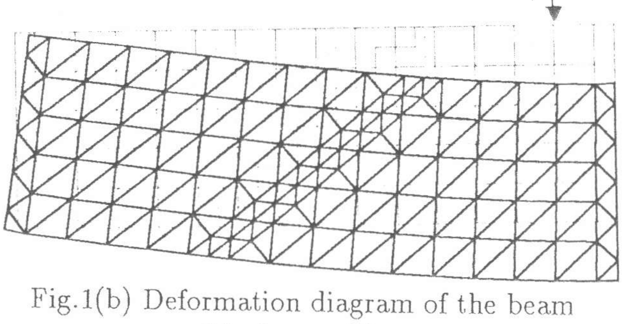



Fig.1(b) Deformation diagram of the beam with shear stiffness

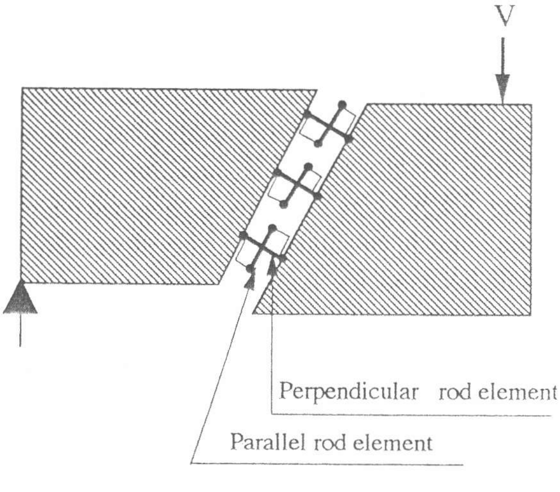



Fig.2. Fictitious crack model with orthogonal rod elements

## 3. ASSUMPTIONS AND APPROXIMATION

In a real beam there are many cracks before failure, such as, bending cracks as well as shear cracks. As a simplification it is assumed that only one crack exists in the beam, and this shear crack will lead to shear failure.

The development of cracks in a beam is to a certain extent of a stochastic nature. There will also be a certain distance between the cracks, because there must be a certain bond length to develop the concrete stresses causing a new crack. For these reasons the shear crack leading to failure may appear at slightly different position in different beams. For real beams this leads to a scatter in test results.

In a numerical analysis, it is necessary to take into account that the critical crack may form in any position. Therefore different crack positions have to be analyzed. In the analysis, the critical crack is taken as the crack which gives the lowest failure load.

Typical stress versus crack width curve for concrete is shown in Fig.3(a), and equivalent 1/4 Model for this relationship is shown in Fig.3(b). $\sigma$-w relationship can be changed into $\sigma$-$\epsilon$ relationship using the rod elements of unit length (Fig.4). This procedure is as follows:

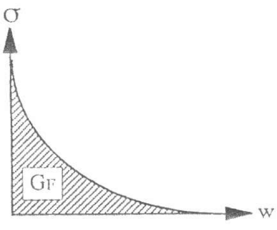


Fig.3(a). Tension softening curve for concrete

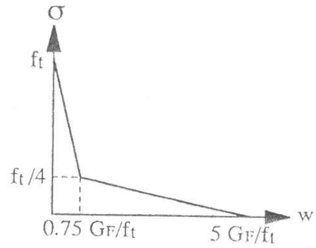


Fig.3(b). 1/4 Model for tension softening curve

$$G_F = \frac{1}{2} f_t w_o \quad (2)$$

$$w_o = \frac{2G_F}{f_t} \quad (3)$$

$$\epsilon_o = \frac{w_o}{l} \quad (4)$$

$$\epsilon_o = \frac{2G_F}{f_t l} \quad (5)$$

$$\sigma = f_t - \frac{f_t^2 l \epsilon}{2G_F} \quad (6)$$

$$E_{ROD} = -\frac{f_t^2 l}{2G_F} \quad (7)$$

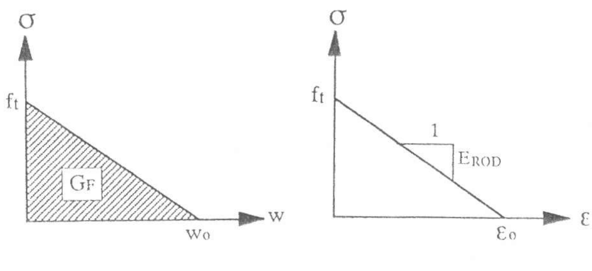


Fig.4. Change of $\sigma$-w curve into $\sigma$-$\epsilon$ curve

Where, $\sigma$ is the tensile stress of concrete, $w$ is the crack width, $\epsilon$ is the strain of the rod element. $l$ is the length of rod element. In this analysis, $l$ is assumed to be equal to the unit length ($l$=1cm) and $G_F$ is assumed to be $100N/m$. This stiffness of the rod element, $E_{ROD}$, is incorporated in the total stiffness matrix.

## 4. ANALYTICAL RESULTS

Table 1 shows the variation of nominal shear strength with crack inclination '$\theta$' and 'x/d' value.

Table 1. Variation of nominal shear strength with crack inclination '$\theta$' and 'x/d' value

| Angle $\theta$ for x/d=1 | shear strength (MPa) |
|---|---|
| 35° | 1.976 |
| 38° | 1.700 |
| 40° | 1.617 |
| 43° | 1.860 |
| 45° | 2.540 |

| Value of x/d for $\theta$ = 40° | shear strength (MPa) |
|---|---|
| 0.50 | 1.780 |
| 0.80 | 1.750 |
| 1.00 | 1.620 |
| 1.30 | 1.800 |
| 1.50 | 2.180 |

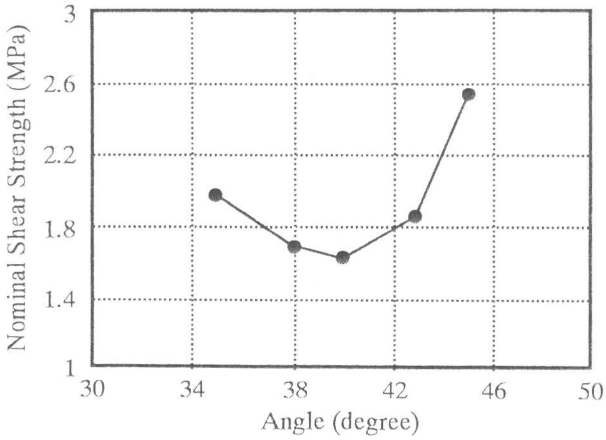



Fig.5. Variation of nominal shear strength with inclination of diagonal crack

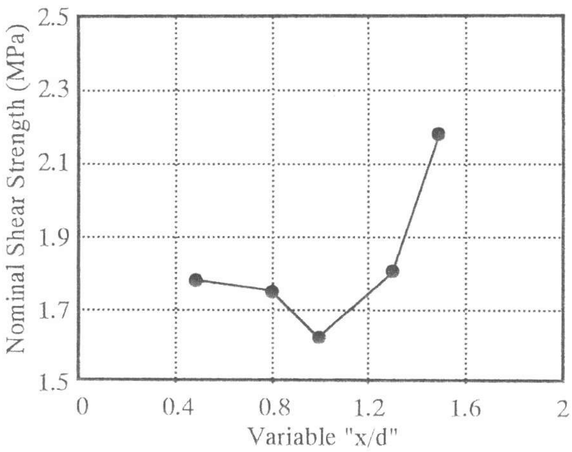



Fig.6. Variation of nominal shear strength with "x/d"

Depending upon the results shown in Figs.5 and 6, the crack inclination at an angle of 40° and the distance $d$ from the support were selected to proceed the analyses. The finite element mesh with the crack angle and orientation is shown in Fig.7. A typical result is presented in terms of load-displacement curve, which is shown in Fig.8.

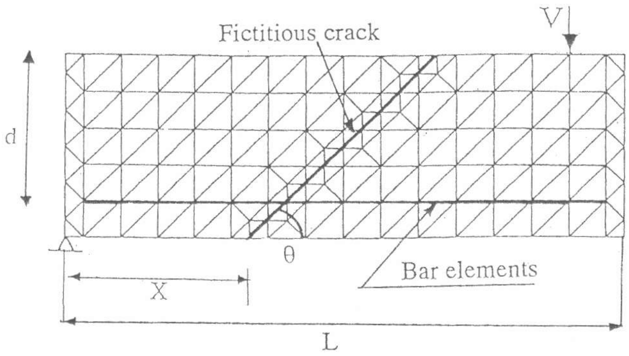



Fig.7. Finite element mesh for reinforced concrete beam

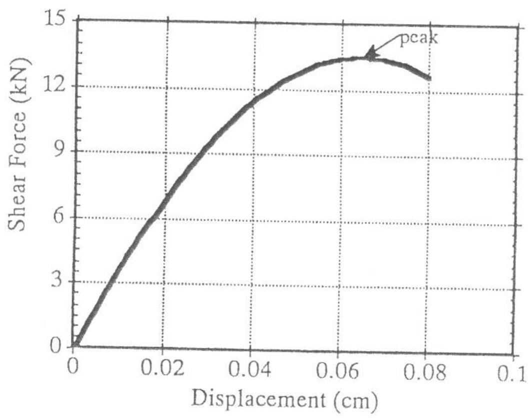
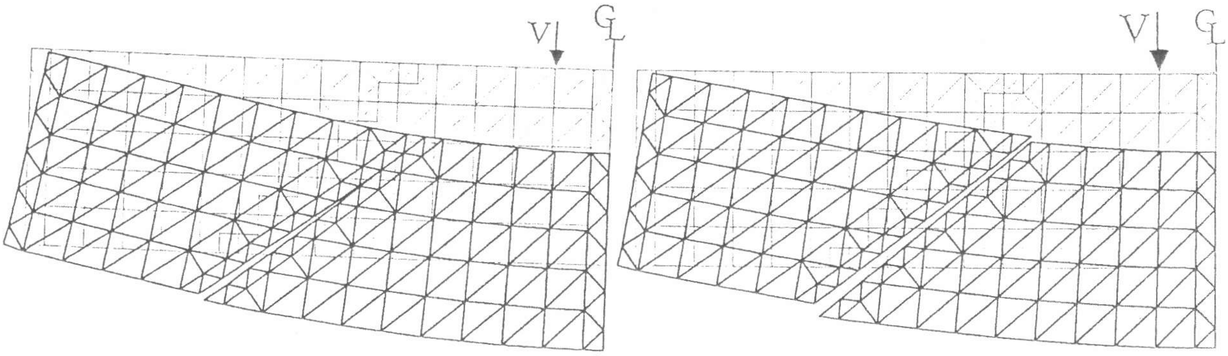



Fig.8. Shear force-displacement curve

## 5. CRACK PROPAGATION

The deformation diagram of a beam is shown in Fig.9. When the load reaches the peak resistance, the crack opens and propagates towards the compression zone and after the peak when the stress of rod element enters in the softening region the crack completely opens and stresses reach to zero.



a. At peak resistance

b. After the peak resistance

Fig.9. Crack propagation along crack path

## 6. SIZE EFFECT

The behavior of a reinforced concrete beam is significantly affected by the beam size, and the failure changes from ductile to brittle as the beam size increases. Moreover, the shear strength capacity decreases as the beam size increases. This behavior is known as the size effect.

Analytical results were compared with the JSCE [2] and CEB[3] equations. These equations are as follows:

JSCE:

$$V_c = \left[0.19\ (100\rho_w f'_c)^{1/3} d^{-1/4}\right] b_w d \tag{8}$$

CEB:

$$V_c = \left[0.15\ (100\rho_w f'_c)^{1/3}\ \left[(1+\sqrt{(0.2/d)}\right]\ (3d/a_v)^{1/3}\right] b_w d \tag{9}$$

where $V_c$ is the shear capacity, $f'_c$ is the concrete strength, $b_w$ is the member width, $d$ is the effective depth, $\rho_w$ is the reinforcement ratio. $f'_c$ is in MPa and $d$ is in m. In the above equations the size effect is proportional to $d^{-1/4}$ and $[1+\sqrt{(0.2/d)}]$, respectively.

The results of this study with the design equation of JSCE and CEB are shown in Fig.10. According to this figure, the tendency of shear strength to decrease with an increase in beam size has been obtained and shows the good agreement with JSCE and

CEB design equations in which the size effect has been incorporated. Beams showed the significant decrease in shear strength with the increase in size.

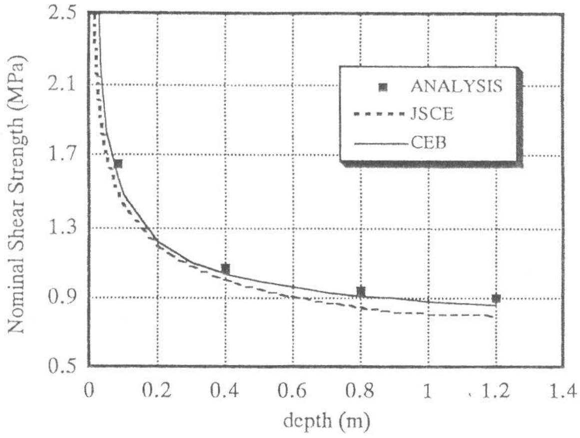



Fig.10. Comparison of analysis with JSCE and CEB equations

h = 10, 50, 100, 150 cm

d = 0.8 h $\rho_w = 2.0$ %

$\theta$ = 40 degree fc' = 37 MPa

x/d = 1.0 fy = 400 MPa

$b_w$ = 10 cm

## 7. CONCLUSION

Shear analysis can be improved using orthogonal rod elements to prevent the slip along the crack path. From this study, it was concluded that the shear behavior of reinforced concrete beam without reinforcement is much affected by the inclination and location of the diagonal crack. It is observed that by increasing the value of inclination angle of crack, shear strength starts decreasing and gives the minimum value of shear capacity. So we fixed the angle for the diagonal crack at this minimum value. Similarly the nominal shear strength is also affected by the location of crack. We obtained the minimum value of the shear strength at the distance, *d* from the support, so we fixed this value for our analysis.

The shear capacity predicted by the finite element analysis shows good agreements with the JSCE and CEB shear equations. All three showed that the shear strength of reinforced concrete beams is significantly affected by the beam size.

## REFERENCES


1. ACI Committee 318,ACI Building Code Requirements for Reinforced Concrete, American Concrete Institute, Detroit, MI.,1989.
2. JSCE, Standard Specification for Design and Construction of Concrete Structure,partI (Design), 1st ed.,Tokyo, 1986.
3. CEB, CEB-FIP Model Code 1990., Bulletin d'Information No.213/214,1993.
4. Zareen,N. and Niwa,J.,"Nonlinear Finite Element Analysis for Shear Behavior of Concrete Beams Based on Fracture Mechanics Approach,"Proc. of JCI, June 1993.
5. Hillerborg, A., Modeer,M. and Peterson, P.E."Analysis of Crack Formation and Crack Growth in Concrete by Means of Fracture Mechanics and Finite Element," Cement and Concrete Research, 1976.