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論 文

[2006] Simulation of Discrete Cracking in a Concrete Gravity Dam

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1. INTRODUCTION

The test performed by Carpinteri et al.[1] on the scaled-down 1:40 model of a concrete gravity dam with an initial notch, subjected to equivalent hydraulic and self-weight loadings, is considered. A two-domain BEM approach, including a formulation for body forces, is applied to analyze crack propagation, based on the linear elastic fracture mechanics (LEFM) criteria. Results obtained from the criteria of the maximum tangential stress, the minimum strain energy density function, the maximum tangential strain, and the maximum energy release rate are compared. The numerical simulations are compared with experimental results and also with the analytical results obtained by Carpinteri et al.[1] using a cohesive fracture model through a finite element code.

2. EXPERIMENTAL STUDY

Carpinteri et al.[1] tested a scaled-down 1:40 model specimen of a concrete gravity dam. The specimen contained a horizontal notch on the upstream side located at a quarter of the dam height, with a notch to depth ratio of 0.1. The scheme of the testing set-up is shown in Fig. 1, where all dimensions are in the unit of cm. The hydraulic load was generated by means of a servo-controlled actuator and was applied onto the upstream side of the model specimen. This force was distributed into four concentrated loads, their intensity increasing with depth and acting directly onto the upstream wall as shown in Fig. 1. Two tests were carried out. The first test (TEST 1) involved simulation of the effect of self-weight of the prototype on the fracture process by introducing an artificial system of vertical forces into the model. In this test, loading was stopped at the peak load when unstable failure occurred at the weakest section of the dam foundation. TEST 2 refers to the second test where the same dam specimen, glued and reinforced around the failure, was subjected to stable testing without self-weight simulation.

The mechanical properties of the concrete were as follows: Young's modulus E of 35.7 GPa, Poisson's ratio ν of 0.1, tensile strength σ_t of 3.6 MPa, and fracture energy

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G_F of 184 N/m. The value of G_F was obtained from standard three point bend tests performed according to the RILEM recommendation at the time the model specimen was loaded to failure. Fracture toughness of concrete was computed from the given value of G_F, based on LEFM, for the case of plane strain as $K_{IC} = \sqrt{EG_F/(1-\nu^2)} = 2.58\ MNm^{-3/2}$.

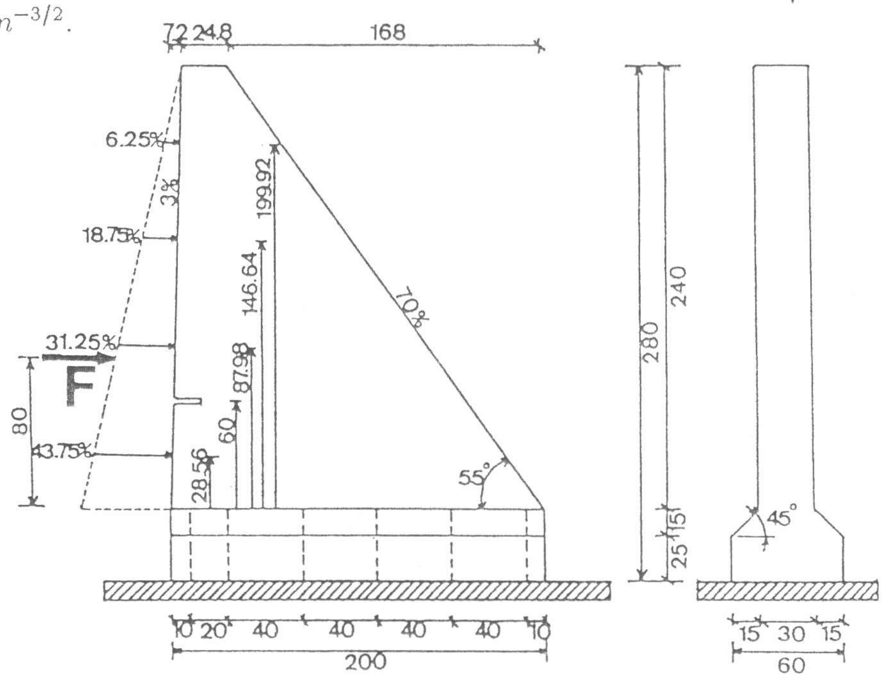

Fig. 1 Experimental set-up of gravity dam model[1]

3. MIXED-MODE FRACTURE THEORIES

3.1 MAXIMUM TANGENTIAL STRESS "$\sigma_{\theta max}$"

This theory states that the crack extension starts in the radial direction, perpendicular to the direction of greatest tension $\sigma_{\theta max}$. The angle of crack growth θ is obtained by solving the following equation[2]:

$$K_I \sin\theta + K_{II}(3\cos\theta - 1) = 0, \tag{1}$$

and the crack propagates when

$$\cos\frac{\theta}{2}\left[\frac{K_I}{K_{IC}}\cos^2\frac{\theta}{2} - \frac{3}{2}\frac{K_{II}}{K_{IC}}\sin\theta\right] \geq 1, \tag{2}$$

where K_I and K_{II} are the mode I and mode II stress intensity factors, respectively.

3.2 MINIMUM STRAIN ENERGY DENSITY FUNCTION "$S_{\theta min}$"

The strain energy density function is defined as follows:

$$\begin{aligned} S(\theta) = {} & \left\{[(1+\cos\theta)(\kappa - \cos\theta)]\,K_I^2 + 2\sin\theta\,[2\cos\theta - (\kappa-1)]\,K_I K_{II} \right. \\ & \left. + [(\kappa+1)(1-\cos\theta) + (1+\cos\theta)(3\cos\theta - 1)]\,K_{II}^2\right\}/16G, \end{aligned} \tag{3}$$

where G is the shear modulus equal to $E/2(1+\nu)$, and κ is equal to $(3-4\nu)$ for plane strain and $(3-\nu)/(1+\nu)$ for plane stress. In this theory[3], crack extension takes place in the direction along which the strain energy density function possesses a minimum value, i.e., when

$$\frac{dS}{d\theta}=0 \qquad \text{and} \qquad \frac{d^2S}{d\theta^2}>0. \tag{4}$$

An iterative minimization procedure is utilized to solve eq. 4 and compute the values of θ and $S_{\theta min}$. The crack propagation would be initiated when $S_{\theta min}$ reaches the critical value $S_{cr}=(1-2\nu)(1+\nu)K_{IC}^2/2E$, i.e., when

$$\frac{S_{\theta min}}{S_{cr}}\geq 1. \tag{5}$$

3.3 MAXIMUM TANGENTIAL STRAIN "$\varepsilon_{\theta max}$"

The tangential strain ε_θ is given by,

$$\begin{aligned}\varepsilon_\theta &= \frac{K_I}{E\sqrt{2\pi r}}\left[\cos\frac{\theta}{2}\left(\frac{3}{4}-\frac{5}{4}\nu\right)+\frac{1}{4}\cos\frac{3\theta}{2}(1+\nu)\right]\\ &+\frac{K_{II}}{E\sqrt{2\pi r}}\left[\sin\frac{\theta}{2}\left(-\frac{3}{4}+\frac{5}{4}\nu\right)-\frac{3}{4}\sin\frac{3\theta}{2}(1+\nu)\right].\end{aligned} \tag{6}$$

Crack extension direction θ is determined by maximizing ε_θ through an iterative maximization procedure[4]. The crack propagates when $\varepsilon_{\theta max}$ reaches the critical value $\varepsilon_{cr}=K_{IC}(1-\nu)/E\sqrt{2\pi r}$, that is,

$$\frac{\varepsilon_{\theta max}}{\varepsilon_{cr}}\geq 1. \tag{7}$$

3.4 MAXIMUM ENERGY RELEASE RATE "$G_{\theta max}$"

The angle of crack propagation θ is obtained by maximization of the energy release rate function G_θ by iterations[5], where

$$\begin{aligned}G_\theta &= \frac{4}{E(3+\cos^2\theta)^2}\left(\frac{1-\theta/\pi}{1+\theta/\pi}\right)^{\theta/\pi}\Big[(1+3\cos^2\theta)K_I^2\\ &\qquad -8\sin\theta\cos\theta K_IK_{II}+(9-5\cos^2\theta)K_{II}^2\Big].\end{aligned} \tag{8}$$

The crack propagates when $G_{\theta max}$ exceeds the critical strain energy release rate $G_{IC}=K_{IC}^2/E$, hence

$$\frac{G_{\theta max}}{G_{IC}}\geq 1. \tag{9}$$

4. ANALYTICAL MODEL AND PROCEDURE

The analytical procedure is exactly the same as that employed in the analysis of mixed-mode fracture in center-notched beams[6]. One additional characteristic of the present analysis is the inclusion of the body force term in the BEM formulation[7]. This feature is introduced to study the effect of the dam self-weight on the fracture process.

The four LEFM criteria of Section 3 are implemented in a two-domain boundary element program module to simulate crack propagation in the gravity dam model for the plane strain condition. The two-domain boundary element analytical model of the concrete dam is given in Fig. 2. Initially, the stitching interface is taken as a straight line that connects the notch tip to point C on the downstream side of the dam. The choice of point C is based on the fact that, for the given dam model, the crack is expected to initiate from the notch tip in a pure mode I type of fracture. Therefore, point C was taken as shown in Fig. 2, because it results in the smallest K_{II}/K_I ratio at the notch tip which represents the closest approximation to the pure mode I state of fracture. The same element length of 50 mm is considered for the external boundaries of domains 1 and 2, the stitching interface between the two domains, and for the crack increment.

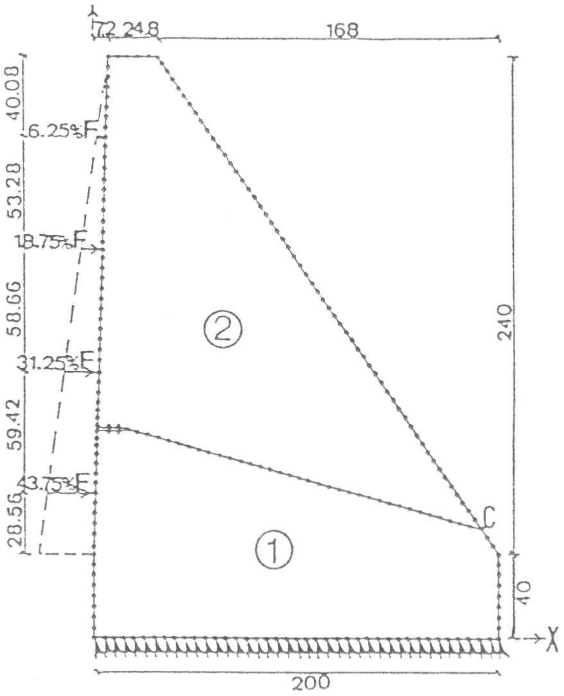

Fig. 2 Two-domain BEM model

5. RESULTS AND DISCUSSION

Figs. 3 (a), (b), (c), and (d) show the actual and simulated crack trajectories corresponding to the $\sigma_{\theta max}$, $S_{\theta min}$, $\varepsilon_{\theta max}$, and $G_{\theta max}$ criteria, respectively. The shaded area represents the crack front across the dam width for TEST 2, which was obtained by Carpinteri et al.[1]. Analytical crack trajectories determined by Carpinteri et al.[1] based on the cohesive crack model for mixed-mode condition implemented in a finite element code are also included for the sake of comparison. Reasonable agreement between the simulated BEM crack trajectories, the actual ones, and Carpinteri's ones is observed. It is also noted that the four criteria of LEFM give almost the same results. Furthermore, throughout the process of crack propagation, the normalized mode I stress intensity factor is almost a unity, with the corresponding normalized mode II stress intensity factor being very small. This suggests that the mechanism of crack propagation is governed by pure tensile mode I fracture. This is observed in all the cases of the four criteria of

LEFM.

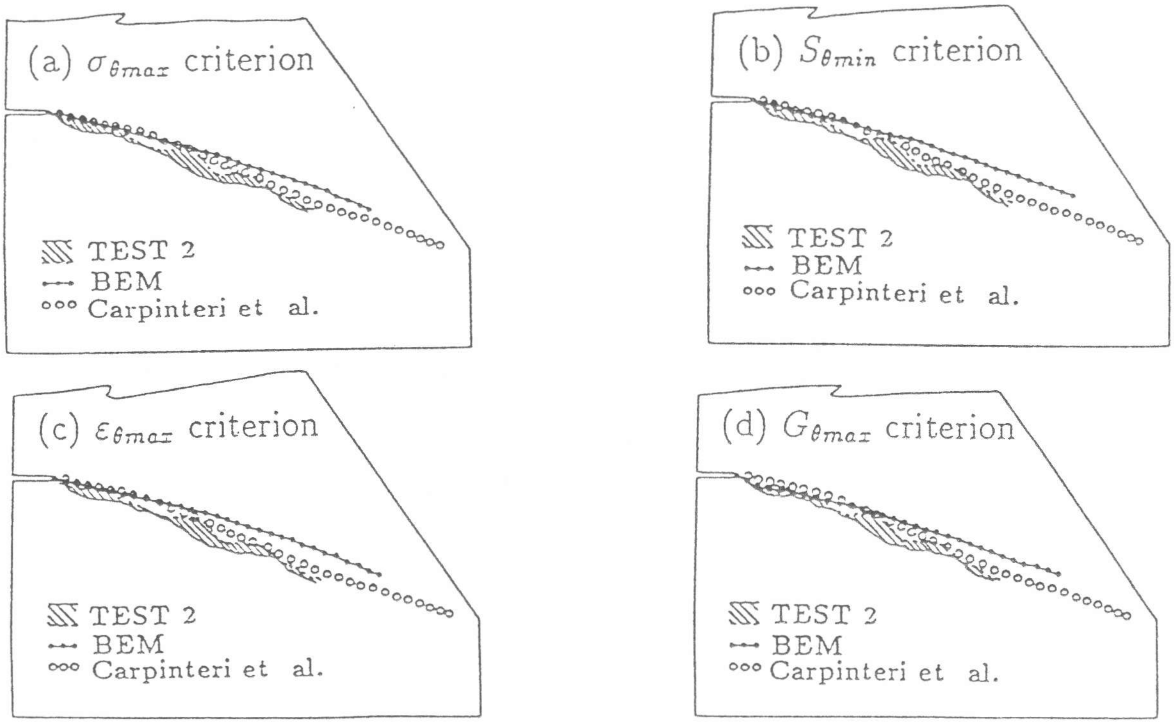

Fig. 3 Actual and simulated crack trajectories

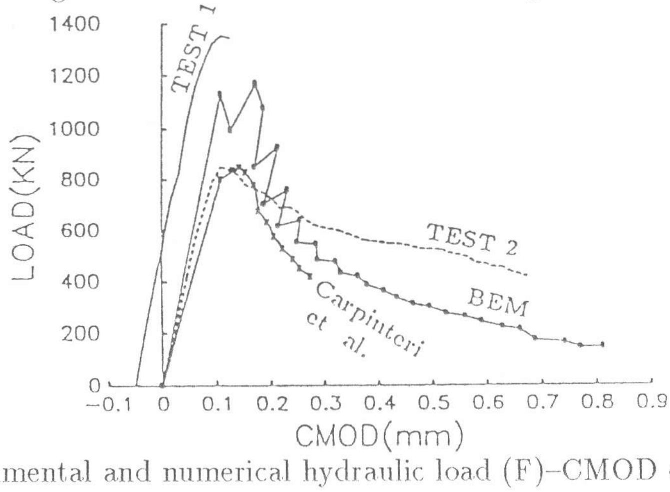

Fig. 4 Experimental and numerical hydraulic load (F)–CMOD curve ($\sigma_{\theta max}$)

The BEM simulated hydraulic load (F)–CMOD curve based on the $\sigma_{\theta max}$ criterion is compared to the experimental results (TEST 1 & TEST 2), and to the analytical result obtained by Carpinteri et al.[1] in Fig. 4. Even though a snapback in the BEM analytical load in the early stages of crack propagation is observed, the BEM analytical curve reproduces the essential feature of the experimental results manifested in the long descending post-peak tail. Fig. 5 shows the BEM simulated load–CMOD curves for the four LEFM criteria. The obtained results for the $\sigma_{\theta max}$, $S_{\theta min}$, and $\varepsilon_{\theta max}$ criteria are almost indistinguishable, with that of $G_{\theta max}$ giving slightly lower values of the load in the final stages of crack propagation. Consequently, the four criteria perform equally well in simulating mixed-mode crack propagation. Because the four investigated criteria are all based on LEFM, the peak loads simulated by BEM are determined at the same CMOD values.

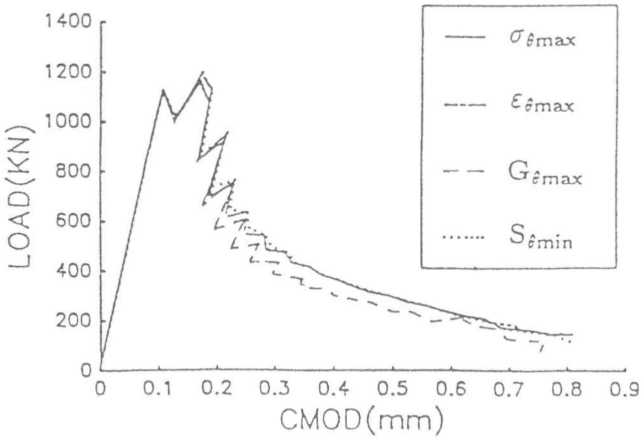

Fig. 5 BEM simulated hydraulic load (F)–CMOD curves

6. CONCLUSIONS

The investigated four LEFM criteria of the maximum tangential stress, the minimum strain energy density function, the maximum tangential strain, and the maximum strain energy release rate perform equally well in the simulation of mixed-mode cracking in the dam model by two-domain BEM. It is also observed that, for the given loading and boundary conditions of the dam model, the crack is observed to propagate in a pure mode I type of fracture.

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