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論 文

[2098] Effects of the Position of Inflexure Points on the Ultimate Shear Capacity of Reinforced Concrete Beams

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1. BACKGROUND OF STUDY

In most of the design codes, shear capacity of reinforced concrete linear members without web reinforcements is calculated by an empirical formula which is principally based on the experimental results of simply supported beams. However, in such structures as frames or continuous beams, inflexure points located within shear span influence the shear capacity. Y. Aoyagi and T. Endo [1],[2] conducted an experimental research, in which they simulated the structural conditions prevalent in statically indeterminate reinforced concrete structures. Referring their experimental data, they proposed a method to estimate shear capacity of the specific members based on the provisions specified in JSCE Code [3]. However, in the JSCE code no clear descriptions are made regarding the definition of shear span "a" for the cases of statically indeterminate linear members with an inflexure point in the shear span. The authors assumed a fictitious support at the inflexure point and divided the shear span "a" into two portions, i.e. the equivalent shear spans "a_1" an "a_2" as shown in Fig. 1. Except for the definition of the shear span, the JSCE formula for ultimate shear capacities of a simple beams were applied.

The design procedure is reproduced below:

- In case of shallow beams ($a/d \geq 2$), the following equation is to be used :

$$V_{cd,s} = f_{vcd,s} \cdot b_w \cdot d \tag{1}$$

where $V_{cd,s}$ is the shear capacity provided by shallow beam in the diagonal tensile shear failure mode,

$f_{vcd,s} = 0.9\,\beta_d\,\beta_p\,\beta_a\,f_{cd}^{\ 1/3}$

$\beta_d = (100/d)^{1/4}$ (d in cm), if $\beta_d > 1.5$ then $\beta_d = 1.5$

$\beta_p = (100\rho_w)^{1/3}$, $\rho_w = A_s/(b_w d)$, if $\beta_p > 1.5$ then $\beta_p = 1.5$

$\beta_a = (0.75 + \dfrac{1.4}{a/d})$ (for $a = a_1$ and a_2)

d : effective depth (cm)

b_w : web width (cm)

A_s : area of tensile reinforcement (cm^2)

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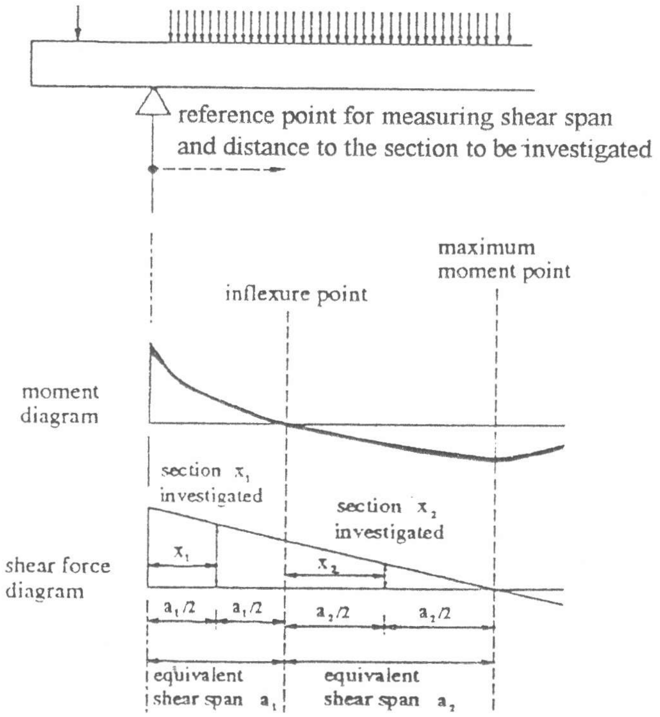


Fig. 1 Definition for Equivalent Shear Span and Position of Section to be Investigated

a : equivalent span (cm), i.e., a_1 : distance from the nearest support to inflexure point,
a_2 : distance from the maximum moment point to inflexure point.
f_{cd} : design compressive strength of concrete (kgf/cm^2)

- In case of deep beams ($a/d<2$), the following equation is to be used :

$$V_{cd,d} = f_{vcd,d} \cdot b_w \cdot d \tag{2}$$

where $V_{cd,d}$ is the shear capacity provided by deep beam in the shear failure mode,

$$f_{vcd,d} = 0\cdot 6\ \beta_p\ \beta_d\ \beta_a \sqrt{f_{cd}}$$

$$\beta_a = \frac{5}{1+(a/d)^2}$$, β_d and β_p are the same as above.

For each equivalent shear span, whichever the higher of shear capacity as calculated by th equations for shallow and deep beam is adopted and finally the lowest one, either shea capacity corresponding to a_1 or a_2 is considered for design. The sections to be considered i design are at the distance x_1 from support and at the distance x_2 from inflexure point which ar the half of equivalent spans of a_1 and a_2, respectively, but should satisfy the followin conditions:
$h/2 \le x_1 \le 1.5d$ and $h/2 \le x_2 \le 1.5d$ (Fig. 1).
where h and d are the height and the effective depth of the beam, respectively.

Y. Aoyagi and T. Endo[1],[2] performed the tests in which reinforced concrete members were subjected to distributed loading. In this paper, the effect of the position of inflexure point on shear capacity of the beams subjected to concentrated load is discussed.

2. OUTLINE OF THE EXPERIMENT

A total of nine specimens were constructed and tested in this study. Seven RC beams were subjected to four-point loading (Fig. 2) and two beams subjected to two-point loading in the span only which are used as the control beams for comparison. The principle configuration of the specimens were beams of rectangular cross section of constant size 15 cm*(b)* x 25 cm*(h)* and length of 340 cm. The ratio of the load in the overhanging portion to the single load in the span was kept constant of 1.60. a_1/d ratio of 0.50, 1, 2, 3 and 4 were taken as a study parameter, where a_1 is the distance of inflexure point from the nearest support and d is the effective depth of the beam. The maximum size of coarse aggregate was 25 mm. Slump of 7-8 cm was tried to be maintained to have similar workability as well as strength. The compressive strength at the time of test ranged from 180 to 230 kgf/cm². The main reinforcement ratio was kept constant (2%). Two 20 mm diameter deformed bar with a yielding strength of 4000 kgf/cm² was used at top and bottom throughout the beam with a concrete cover of 2 cm. In order to localize the shear failure in the left-hand span, the right-hand span and overhanging portions were reinforced with web reinforcement. Details are shown in Fig. 3.

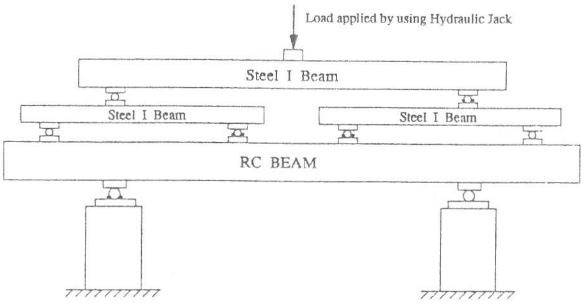


Fig. 2 Loading Scheme

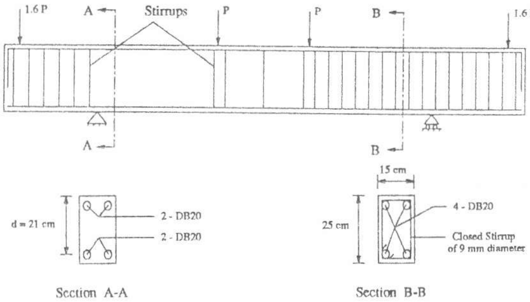


Fig. 3 Details of Reinforcement

3. METHOD OF TEST

The supporting condition for beam was simple support at both ends on 220 cm center to center distance. The concentrated loads were applied by using hydraulic jacks and transferred to different points by means of steel I beams. During tests, loads were measured by load cells and strains by strain gauges. Two cycles of loading were applied as described below.

First Cycle : Up to the load at which extreme fiber stress of the concrete beam reaches tensile strength of concrete, i.e. first flexural cracking which was observed by visual inspection, and then the load was fully released. Alignment of loading and checking of data acquisition systems were performed during and after this cycle.

Second Cycle : Up to the load at which final failure of the beam occurred.

4. EXPERIMENTAL RESULTS AND OBSERVATIONS

4.1 FAILURE MODES OF THE BEAMS

The experimental results showed that all the beams failed in shear. When load was gradually applied, the first flexural crack occurred at the point of maximum moment either at support or at mid span depending on the position of inflexure point. The flexure cracks gradually increased in number as well as in length. The flexure cracks in the left shear span originated the inclined cracks. As the load increased, the applied shear force was taken by aggregate interlock, dowel action, uncracked concrete, etc. Finally, the width of the diagonal cracks increased and suddenly the beam failed in shear. Crack patterns of a typical beam are shown in Fig. 4.

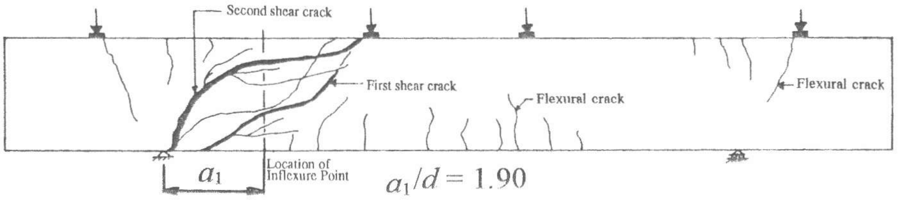
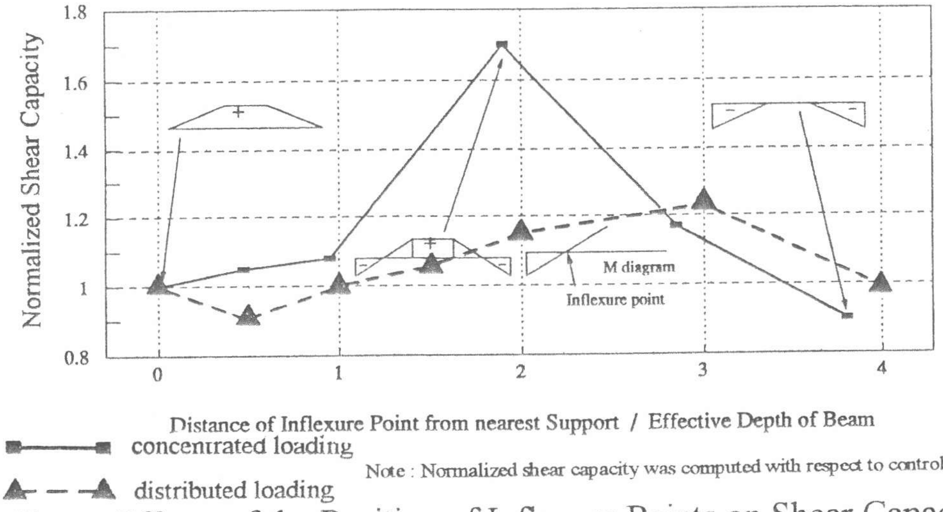


Fig. 4 Crack Pattern of Typical Beams at Failure Condition

4.2 SHEAR CAPACITY OF THE BEAMS

In order to visualize the effect of the position of inflexure points, shear capacities normalized by that of the control beam were plotted against the equivalent shear span/efffective depth ratios (a_1/d) in Fig. 5. Previous results for the distributed loading tests are also shown. It can be seen that relative positions of inflexure points affect conspicuously the shear capacity especially for the case of concentrated point loading.



Note : Normalized shear capacity was computed with respect to control beam

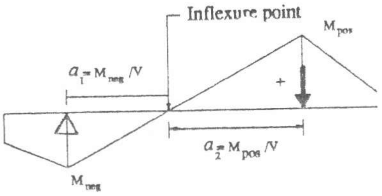
Fig. 5 Effects of the Position of Inflexure Points on Shear Capacity

The effect of inflexure point can be explained by considering the formation of imaginary hinge at the position of inflexure point in the span. Due to this hinge formation, total unsupported shear span can be divided into two spans, i.e., a_1 and a_2 (as defined in Fig. 1). Since the beam with shorter shear span can resist higher shear force, shear failure will occur in the larger of the shear spans of a_1 and a_2. As tabulated in Table 1, in case of $a_1/d = 3.80$, the larger shear span is $a_1 = 80$ cm which is the highest among all cases. Hence, the beam in this case carried the minimum shear force. On the other hand, as can be seen in Table 1, in case of beams of a_1/d ratio of 1.90, the total shear span is divided into two parts by forming artificial hinge at the point of inflexure point just at the center of the support and the span load point, and hence the beam carried the maximum loads.

Table 1 Effects of Inflexure Points on Shear Capacity

Ratio a_1/d	a_1 (cm)	a_2 (cm)	Max. of a_1 and a_2 (cm)	Shear capacity (kgf)
3.80	80	0	80	3173
3.80	80	0	80	3269
2.85	60	20	60	4152
1.90	40	40	40	5973
1.90	40	40	40	6167
0.95	20	60	60	3850
0.47	10	70	70	3748

Note : a_1 is the distance of inflexure point-from the nearest support, a_2 is the distance between inflexure point and applied load in the span and d is effective depth.



5. DISCUSSIONS OF THE TEST RESULTS

5.1 EFFECT OF LOADING TYPES

The results for concentrated loads were compared with those for distributed loads which were obtained by Y. Aoyagi et.al [1],[2]. The present test results show that in case of concentrated load, the position of inflexure points influenced the shear capacity as much as 70% in reference to the control beam of two-point loading (Fig. 5). On the other hand, as reported in [1],[2], the increase of shear capacity was 25% for distributed load. In case of distributed load, maximum shear was found to be taken by the beam of condition at which the ratio a_1/d is 3 [1],[2] rather than 2 for concentrated load in the present study (Fig. 5). The main reason may be due to the fact that in case of concentrated load the shear force is constant throughout the shear span, and hence any point to be investigated has the same shear force. On the other hand in case of distributed load the shear force varies linearly throughout the beam, and hence every point to be investigated has different shear force.

5.2 COMPARISON OF EXPERIMENTAL RESULTS WITH SHEAR CAPACITIES OBTAINED BY PROPOSED ESTIMATION

The experimental results were compared with the existing different equations for the estimation of shear capacity in Fig. 6. From this comparison, it is clear that the deviation of the results as computed by the equations proposed by Y. Aoyagi et.al (Eqs. (1) and (2)) is the lowest.

6. CONCLUSIONS

1) The positions of inflexure points have greater effects on the ultimate shear capacity of reinforced concrete beams. It can be assumed that at the position of inflexure point in the span

an imaginary hinge is generated and divide the shear span into two equivalent ones. Hence, the section can take more shear capacity as much as 70% for the case of concentrated load obtained in the present study and 25% for that of distributed load [1],[2]. Therefore, it is very important to consider this effect in practical design.

2) By using the equation proposed by Y. Aoyagi et.al. [1],[2] considering the effects of inflexure points, the experimental results, for all cases of the ratio of distance of inflexure point from the nearest support and effective depth of beam = 0.5~4, are most closer to the calculated ones compared with other proposed equations.

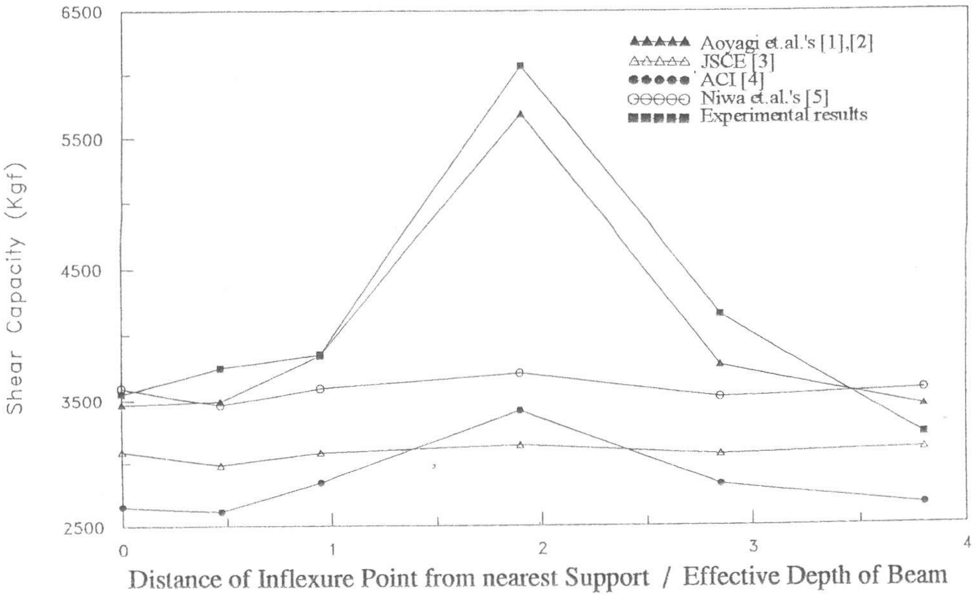


Fig. 6 Comparisons of Various Shear Capacity Estimation Methods

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