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論　文

# [2172] Loss of Tendon's Eccentricity in Externally Prestressed Concrete Beam

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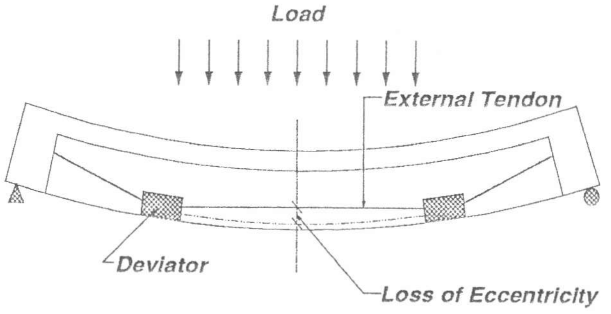
## 1. INTRODUCTION

In external prestressing system, tendons are arranged outside the cross-section of a prestressed concrete beam. The externally prestressed concrete (PC) beam, in previous studies[1,2], can be treated as an unbonded prestressing member because no bonding between concrete and tendons exists. However, unlike internal unbonded tendons, external tendons are formed the profile by deviating at the deviators. When the externally PC beam subjects to bending, the tendon would not follow the concrete deflection, except at deviator point. The large deflection of PC beam, particularly when the beam is close to failure, could induce the secondary effect on flexural strength of PC beam, namely *"loss of tendon's eccentricity"*. The loss of eccentricity would be influenced by the location and arrangement of the deviators. Fig.1 shows a typical deflection and loss of tendon's eccentricity in the externally PC beam.

In this study, to investigate the effect of loss of tendon's eccentricity on ultimate flexural strength of externally PC beams, 1)tests of externally PC beams having different arrangement of deviator points, and internally unbonded PC beam were carried out, 2)an analytical program considering the effect of loss of tendon's eccentricity was developed, 3)by using the analytical program, the parametric study changing the variables on the influenced parameters were investigated.



* Large deflection of beam induces a secondary effect, "Loss of Tendon's Eccentricity"

**Fig.1 Typical deflection and loss of tendon's eccentricity in externally PC beam**

## 2. OUTLINE OF TESTS

### 2.1 Test specimens

From Fig.1, the factors which mainly induce the loss of tendon's eccentricity in externally PC beam are *the distance between deviators (Sd)* and *the number of deviators (two or more)*. To investigate the effect of loss of tendon's eccentricity on ultimate flexural strength caused by the mentioned factors, tests of;(1)two

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Table 1 : Test variables and materials

| Specimen No. | Span length L, (m) | Distance between deviators Sd, (m) | Concrete strength f'c, (MPa) | Tendon |  | Introduced prestress |  | Reinforcement |  |
|---|---|---|---|---|---|---|---|---|---|
|  |  |  |  | Material | Aps (cm$^2$) | Force (kN) | %of $f_{pu}$ | Material | As (cm$^2$) |
| 1. External PC, w/2-Dev | 5.20 | 1.80 | 39.50 | SWPR7B: 2-ϕ15.2 mm | 2.774 | 268 | 51% | Deformed bars<br>Grade SD35 :<br>Top: 4-ϕ6mm<br>Bot: 3-ϕ10mm | 1.24<br>2.14 |
| 2. External PC, w/2-Dev |  | 3.00 |  |  |  | 278 | 53% |  |  |
| 3. External PC, w/3-Dev |  | 1.50 |  |  |  | 278 | 53% |  |  |
| 4. Internal unbonded PC |  | 3.00<br>*(btw deflected pt.)* |  | SWPR19: 1-ϕ19.3 mm<br>*(unbonded Type)* | 2.437 | 273 | 60% |  |  |

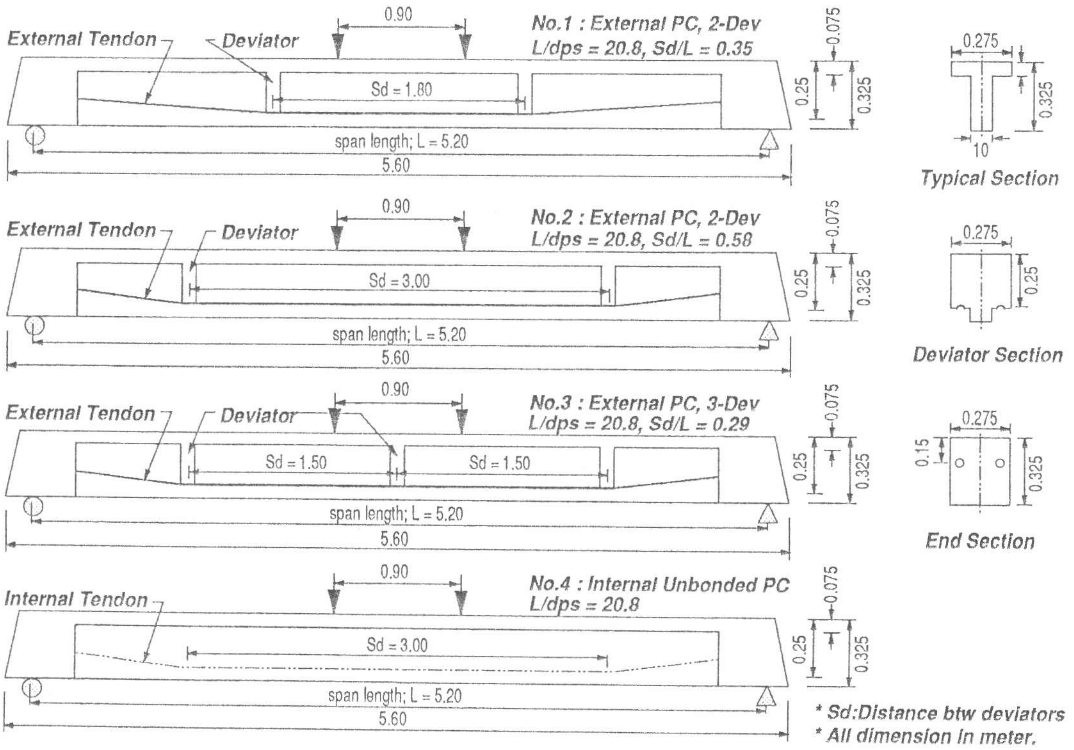


Fig.2 Detail of test specimens

externally PC beams No.1 and No.2, with distance between deviators *(Sd)* of 1.80 and 3.00 m, respectively; and **(2)**one externally PC beam with three deviators (No.3), one of which was provided at midspan to eliminate such a loss of eccentricity (see Fig.2), were carried out in order to compare with; **(3)**an internal unbonded PC beam (No.4), which has no loss of tendon's eccentricity. The test variables and detail of the test specimens are shown in Table 1 and Fig.2, respectively.

All test specimens were simply supported T-section beams with a span length of 5.20 m. Two prestressing tendons of type SWPR7B with a diameter of 15.2 mm were symmetrically arranged on the both sides of the section of externally PC beam (No.1, No.2 and No.3). For the unbonded PC beam (No.4), a single prestressing tendon of type unbonded-SWPR19 (the tendon is greased inside the plastic tube to reduce friction) with a diameter of 19.3 mm, was placed inside the concrete section through the aluminum sheath to protect bonding from concrete. Three deformed bars grade SD35 with a diameter of 10 mm, were placed near bottom surface of the section to prevent a severely single crack due to tension.

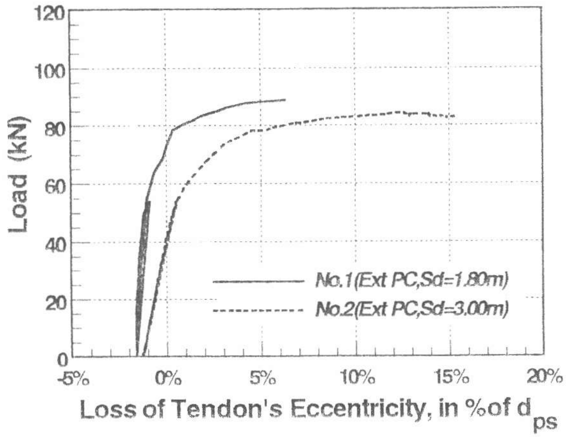
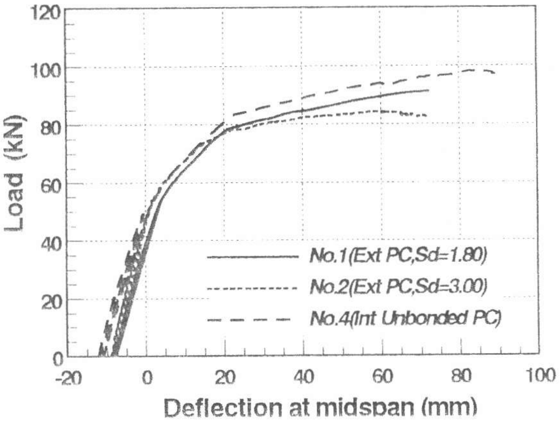


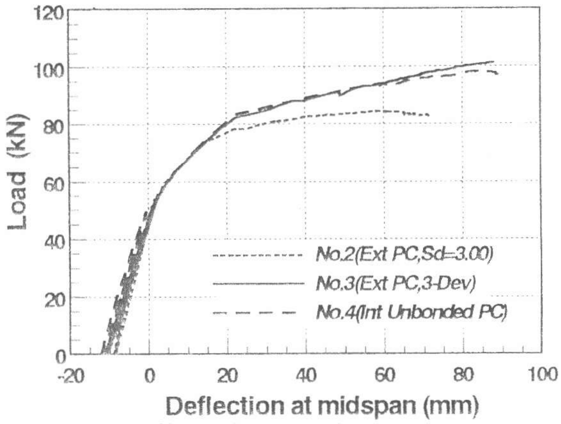
Fig.3 Load-loss of eccentricity curves

## 2.2) Test Procedure

Before prestressing, electrical strain gages were attached on tendons, reinforcement, and concrete surface along the depth of section at midspan to measure strains in each components. Load cells were fixed at the ends of tendons to monitor forces in each tendon. Prestress force of 275 kN, in approximate, corresponding to 50%-60% of nominal tensile strength of tendon was introduced to the specimens.



*(a) Effect of distance btw deviators, Sd*



*(b)Effect of deviator's arrangement*

Fig.4 Load-deflection curves

Two-point loading with a distance of 0.90 m. between loads was symmetrically applied to the specimen in two cycles. The crack load was recorded when the first visible crack was observed in first cycle. Afterwards, load was rebounded to zero, and then the specimen was re-loaded until failure occurred. The displacement transducers were fixed at the midspan and the deviator points to measure deflections so that the loss of tendon's eccentricity can be monitored.

## 3. TEST RESULTS AND DISCUSSIONS

### 3.1 Effect of distance between deviators, *Sd*

For specimen No.1 and No.2, the comparison of loss of tendon's eccentricity at midspan section, which is critical for flexure, was illustrated in Fig.3 (relation between load and loss of eccentricity). Specimen No.2 which possessed large distance between deviator (*Sd*) showed more loss of eccentricity than that of No.1 (15.3% and 6.3% of $d_{ps}$). To make clear the effect of loss of tendon's eccentricity on flexural behavior induced by *Sd*, load-deflection curves of externally PC No.1 and No.2 were compared with that of internal unbonded PC No.4 which has no loss of tendon's eccentricity (see Fig.4(a)). Before cracking of concrete, they showed almost the same elastic behavior and stiffness. However, after cracking, specimen No.1 and No.2 gave smaller flexural strength than that of No.4 ,(83.40, 91.25 and 98.10 kN, respectively). It is obviously found that the flexural strength of externally PC beam must be reduced due to the effect of loss of eccentricity induced by *Sd* (15% of ultimate load of No.2 was lowered when compared to that of No.4). The summary of test and analytical results are, in comparison, shown in Table 2.

### 3.2 Effect of deviator's arrangement

As described earlier, flexural strength of externally PC beam could be reduced due to presenting of loss of eccentricity at midspan. To eliminate such a loss, an additional deviator at midspan was provided in specimen No.3. In Fig.4(b), the flexural behavior of specimen No.3 was compared with those of No.2 and No.4, which possessed the same tendon profile (see Fig.2). It is found that specimen No.3 showed almost the same flexural behavior as that of No.4, but stronger than that of No.2. It, therefore, could be indicated that a proper arrangement of deviator in externally PC beam which eliminated the loss of tendon's eccentricity should be considered in design of externally PC beam.

**Table 2 : Summary of Test Results in Comparison to Analytical Results**

| Specimen No. | Load at first Crack (kN) | | Applied Load at Ultimate (kN) | | Force in Tendon at Ultimate (kN) | |
|---|---|---|---|---|---|---|
| | Test | Analysis | Test | Analysis | Test | Analysis |
| 1. Ext PC with 2-dev (Sd=1.80 m) | 53.75 | 55.33<br>*(1.029)* | 91.25 | 91.43<br>*(1.001)* | 367.20 | 370.72<br>*(1.009)* |
| 2. Ext PC with 2-dev (Sd=3.00 m) | 54.00 | 54.94<br>*(1.017)* | 83.40 | 86.03<br>*(1.031)* | 378.00 | 372.98<br>*(0.987)* |
| 3. Ext PC with 3-dev (Sd=1.50 m) | 53.75 | 55.33<br>*(1.029)* | 101.25 | 100.35<br>*(0.991)* | 409.85 | 398.68<br>*(0.973)* |
| 4. Internal Unbonded PC | 49.00 | 55.72<br>*(1.137)* | 98.10 | 100.06<br>*(1.020)* | 358.95 | 385.34<br>*(1.074)* |

*Note : The values in ( ) represent correlation of Analysis/Test*

## 4. ANALYSIS AND COMPARISON TO TEST RESULTS

### 4.1 Analytical approach

The analysis of beams prestressed with unbonded tendons cannot be determined from the analysis of **beam section** (section-dependent). But it must be determined from the analysis of **deformation of the entire structure** (member-dependent)[2]. In this study, to obtain an accurate flexural strength of unbonded member (including externally PC beam), an analytical approach based on; 1)*compatibility of deformation*, that is the total deformation of tendon equals that of concrete located at tendon level[3] and; 2)the use of *bond reduction coefficient*, $\Omega$,[1,2] to estimate the stress in unbonded tendon, $f_{ps}$, was established.

$$\Omega = \frac{(\Delta \varepsilon_{psu})_{av}}{(\Delta \varepsilon_{cps})_m} \quad \text{.....} (1)$$

$$f_{ps} = E_{ps}(\varepsilon_{pe} + \Omega(\varepsilon_{ce} + \varepsilon_c(\frac{d_{ps}}{c} - 1))) \quad ..(2)$$

where $(\Delta \varepsilon_{psu})_{av}$ is strain increase in unbonded tendon, $(\Delta \varepsilon_{cps})_m$ is the strain increase in concrete, $\varepsilon_c$ is concrete strain at compression fiber beyond the effective prestress, $\varepsilon_{ce}$ and $\varepsilon_{pe}$ is strain of concrete and tendon at effective prestress, $f_{ps}$ is stress in unbonded tendon, $d_{ps}$ is depth from compression fiber to tendon level; all are taken at section of maximum moment.

By discrete element method[4], load-displacement relationship of such an unbonded or externally PC beam can be obtained. For a value of concrete strain ($\varepsilon_c$) at section of maximum moment, the $\Omega$, in Eq.(2), is trial to obtain the stress and force in tendon until the equilibrium of force is satisfied. At this stage, beam are divided into small segments, and then moment distribution, strain distribution of concrete at tendon level, and strain distribution of tendon of the entire span of PC beam can be obtained. Finally, the deflection of the beam can be computed. In case of externally PC beam, change of tendon level (depth of tendon, $d_{ps}$), which implied the loss of tendon's eccentricity, has to be computed prior the next step of calculation. This procedure would be repeated until the ultimate limit state is reached ($\varepsilon_{cu} = 0.0035$).

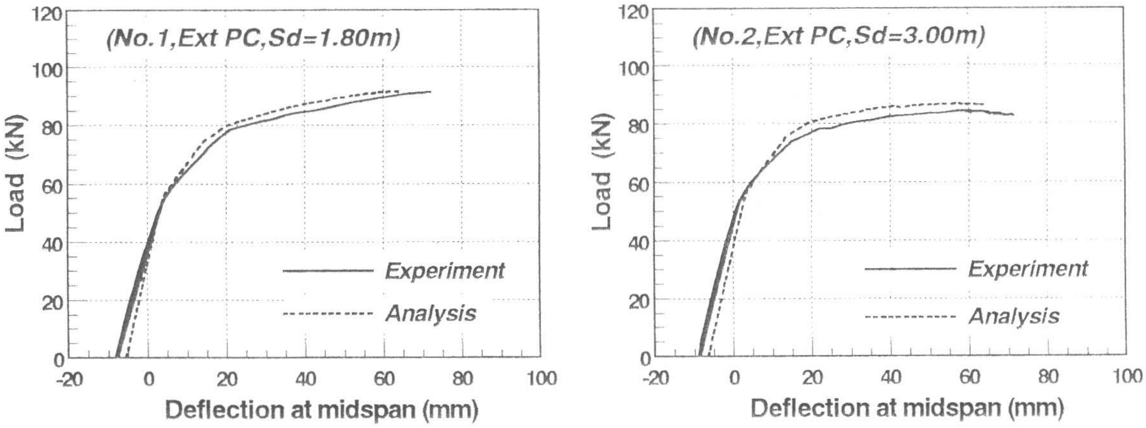


Fig.5 Comparison of Load-deflection curves

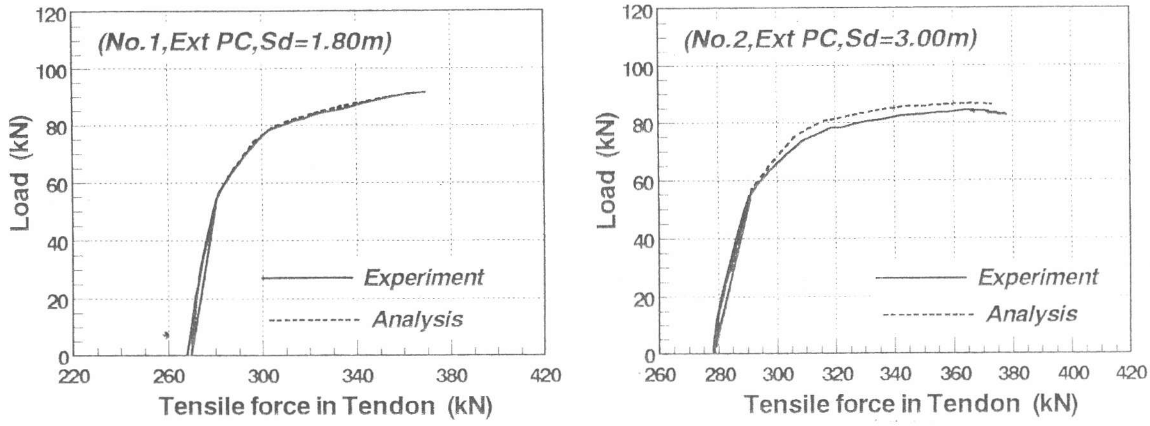


Fig.6 Comparison of Load-tensile force in tendon Curves

### 4.2 Comparison of Test with Analytical results

The load-deflection and load-tensile force in tendon curves obtained from the above analytical approach are compared to those obtained from test results of specimen No.1 and No.2, as illustrated in Fig.5 and Fig.6, respectively. It was found that an acceptable agreement between test and analytical results were obtained. In addition, the comparison of flexural strength between test and analytical results are also done as shown in Table 2. As results, it is believed that the use of compatibility of deformation and bond reduction coefficient concepts could be efficiently applied to predict the flexural strength of externally PC beams. Consequently, the attempt to propose an equation for design purpose would be done in the further study.

## 5. DISCUSSION OF LOSS OF TENDON'S ECCENTRICITY AND BOND REDUCTION COEFFICIENT

The effect of some parameters such as distance between deviators-to-span ratio $(Sd/L)$ and span-to-depth ratio $(L/d_{ps})$[2] on flexural strength of externally PC beams were investigated. The externally PC beam with the same sectional properties as that of the test specimens, under a typical loading pattern of third-point loading (two-point loads with distance of one-third of span length), was considered. By using the established analytical program, the loss of tendon's eccentricity and bond reduction coefficient, $\Omega$ (at ultimate limit stage), of externally PC beams having various values of $Sd/L$ (from 0.333 to 0.80) and $L/d_{ps}$ (from 10 to 30) were calculated. The relations of span-to-depth

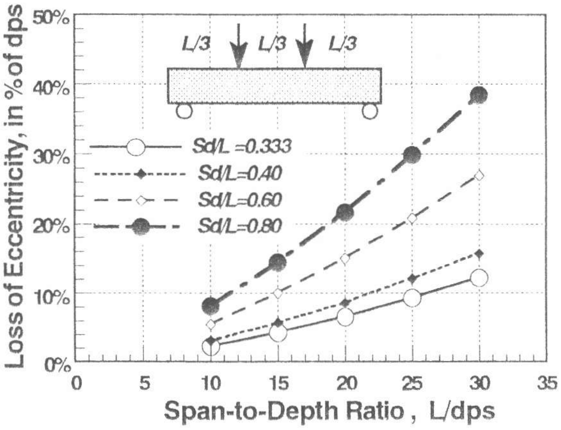


Fig.7 Loss of eccentricity versus span-to-depth ratio

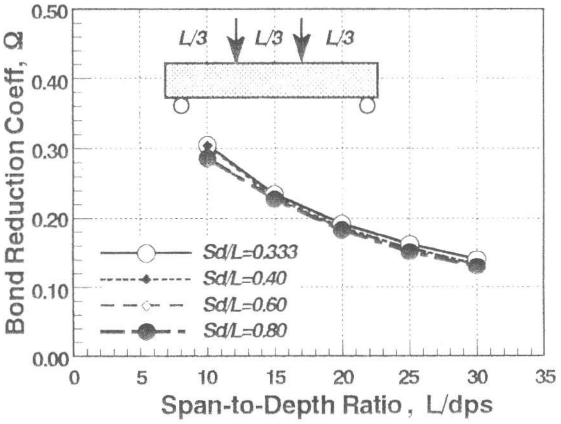


Fig.8 Span-to-depth ratio versus bond reduction coefficient

ratio, $L/d_{ps}$ versus (1)loss of tendon's eccentricity, and (2)bond reduction coefficient, $\Omega$ were illustrated in Fig.7 and Fig.8, respectively. It is found, from fig.7, that the loss of tendon's eccentricity in externally PC beam is increased with both $L/d_{ps}$ and $Sd/L$. Meanwhile, the bond reduction coefficient, $\Omega$, is seemed to be influenced by only $L/d_{ps}$ because all of $Sd/L$ curves show the same tendency. (see Fig.8). By this similar way, it is possible, in the further study, to propose the equations to predict the flexural strength of externally PC beam through the uses of some parameters such as loss of eccentricity and bond reduction coefficient for design purposes.

## 6. CONCLUSIONS

The experimental investigations of externally PC beam and internal unbonded PC beams were comparatively made to study the effect of loss of tendon's eccentricity on ultimate flexural strength. To accurately predict such a flexural behavior of externally PC beam considering the effect of loss of eccentricity, the analytical approach was developed, and compared with the test results. In this study, the conclusions can be drawn as the followings;

- In comparison to an internal unbonded PC beam which has no loss of eccentricity , the externally PC beam with large distance between deviators (No.2) gives a remarkable lower flexural strength than that of internal unbonded PC beam.
- It is found that the externally PC beam possessing a proper arrangement of deviator to eliminate the loss of tendon's eccentricity (No.3) shows the same flexural behavior as that of the internal unbonded PC beam.
- The analytical approach using the concept of deformation compatibility and bond reduction coefficient gives an acceptable agreement with the test results.
- By using the analytical program, parametric study on the influenced parameters was carried out as a guidance to propose the equation for design purposes of externally PC beam in the further study.

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