論文 Analytical Study on Pure Torsion Behavior of Concrete Colmuns Using 3D-Lattice Model

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ABSTRACT: The physical understanding and the numerical simulation of the characteristics of R.C. columns have been the focus of intense efforts specially after the severe damage in the Hanshin Earthquake. Structural systems which are subjected to any kind of loads become quite difficult to analyze rigorously in their original physical forms especially when the degree of freedom becomes large. So in this work, to simulate column behavior in 3D stress state, the authors seek the possibility of simplifying the analysis of the calculations of the behavior of concrete columns under pure torsion using the 3D-lattice model. The applicability of the lattice model is investigated in comparison with many existing experimental data.
KEYWORDS: 3D-lattice model, constitutive equation, failure mode.

1. INTRODUCTION

To capture the change of the shear resisting mechanism of reinforced concrete columns, such as initiation of diagonal cracking, yielding of reinforcement and crushing of web concrete, and also to establish a practical macroscopic torsion resisting model, the lattice model has been developed in three dimensions. The calculation procedures using 3D-lattice model are rather simple and worthwhile to be investigated if it gives results in an allowable limit of accuracy. It is true that the lattice model still involves several problems, for example, the implementation of that model in three dimensions and the fixation for the value of the area of each member. In the 3D-lattice model, in which concrete beams are modeled into an assembly of truss components, is investigated in this research work. Since the lattice model assumes the compatibility condition and the equilibrium condition, it gives one of the lower bound solution within the limit of the constitutive model for materials.

In this research, based on the analytical results obtained from the 3D-lattice model, the variation of the contribution for the cross-section area of different member of the model into the original value of the cross section of the reinforced concrete column is studied.

2. LATTICE MODEL

2.1 THE OUTLINE OF THE MODIFIED LATTICE MODEL

Fig.1 shows the schematic diagram of a concrete column after the initiation of diagonal cracking. Neglecting the shear stresses on the crack surfaces, triaxial compression-tension stresses exist in an infinitely small element parallel to the diagonal crack direction . Considering the existence of triaxial stress state in web concrete, we assume the lattice model as shown in Fig.2. In the 3D-lattice model, a reinforced concrete column, that is originally a continuum, is assumed to be as assembly of four truss components representing the four sides of the column and four arch elements. If only the resistance of a truss mechanism with the diagonal strut of 45° is taken into account, each truss is considered as a plane truss with diagonals and an arch member using the lattice model [1] by modeling each member as the section below.

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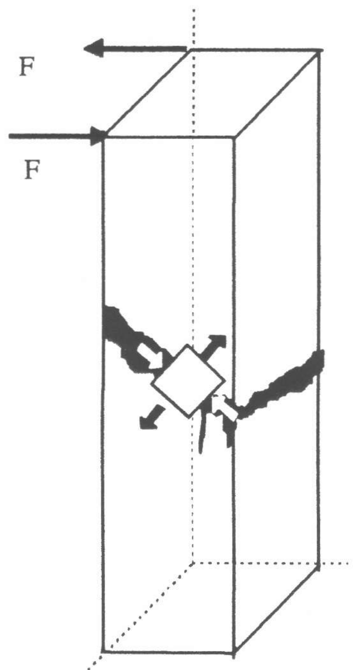

Fig.1 Concrete Column Including Torque Crack

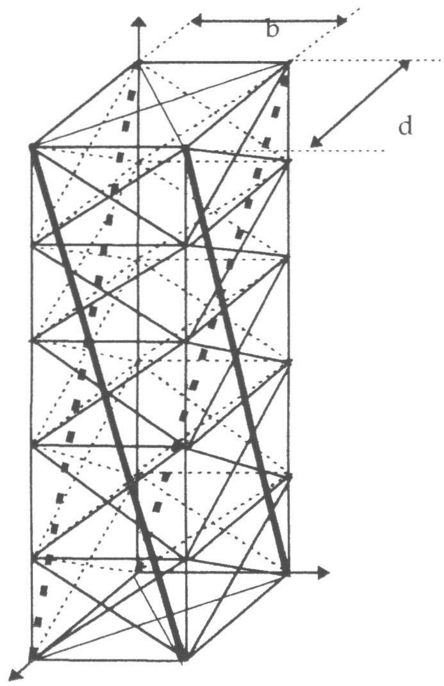

Fig.2 R.C. Column using 3D-Lattice Model

2.2 THE MODELING OF EACH MEMBER

Fig.3 shows the diagram of cross section of a concrete column modeled into the 3D-lattice model. The web concrete is divided into vertical members, diagonal compression and tension members and four arch elements. Each area has been calculated as shown in Fig.3. The ratios of the width and depth of the arch member are assumed to be "t_1" and "t_2" to the breadth and the depth of the original cross section, respectively. Reinforcements are modeled as vertical members which represent the main reinforcement of the column, and as horizontal members which are equivalent to the area of the cross section of the stirrups. Also to make the cross section of the column stable there are two cross diagonal steel members between each of two opposite corner nodes with area equivalent to 10% of the area of the stirrups.

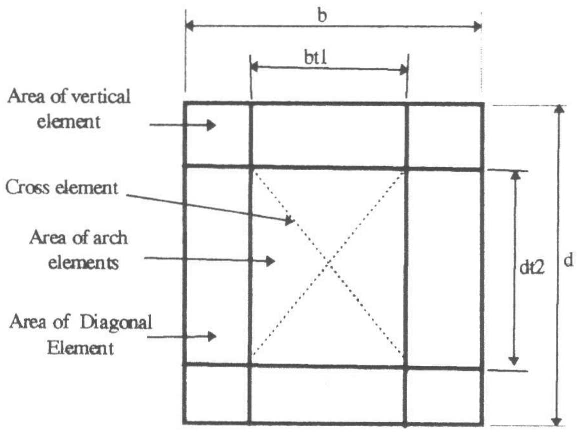

Fig.3 Cross Section at any Node of the Column in the 3D-Lattice Model

The modeling of the arch member of web concrete is one of the most important aspects in the lattice model of this investigation. The thick solid line in Fig.2 represents the arch member corresponding to the web concrete. Although the inclination angle of diagonal members is fixed at 45 degrees in the direction of the depth, but it does not matter if we change it to keep the position of the nodes in the direction of the width in case of column with rectangular cross section. The values of "t_1" and "t_2" are determined as follows. Assuming that a unit torsion moment is applied to a concrete column having the specified t_1 and t_2 values ($0<t_1<1$) and ($0<t_2<1$), the potential energy can be calculated based on the elastic analysis. The potential energy can be obtained from the strain energy of each element and the external work done due to the unit of torsion moment. Actually the calculation of the potential energy is changed with increase of nonlinearity of the concrete column. But this method is just only to get as a first approximation. Fig.4 shows the example of the change of the potential energy with the t_1 and t_2 values.

3. THE CONSTITUTIVE EQUATION OF EACH ELEMENT

3.1 TENSION CONCRETE ELEMENT

The stress-strain relation of tension member of concrete has been taken as linear in the elastic stage and non-linear after cracking as expressed in Eq.(1) and Eq.(2) and is shown in Fig.5 [2].

For ascending branch ($\varepsilon_r \le \varepsilon_{cr}$)

$$\sigma_r = E_C \varepsilon_{Cr} \tag{1}$$

For descending branch ($\varepsilon_r > \varepsilon_{cr}$)

$$\sigma_r = f_{cr}\left(\frac{\varepsilon_{cr}}{\varepsilon_r}\right)^{0.4} \tag{2}$$

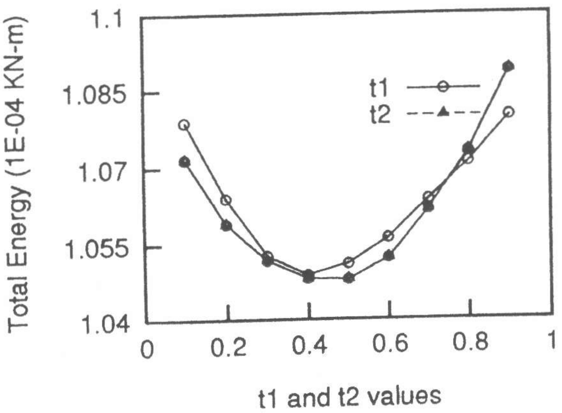

Fig.4 The change of Potential Energy due to $t_1 and t_2$ values

3.2 COMPRESSION CONCRETE ELEMENT

The diagonal compression member of concrete and arch members resist the diagonal compression caused by torsion. The stress-strain relationship is assumed in the equations (3) and (4) and the corresponding stress-strain relationship is shown in Fig.6 [2].

For ascending branch ($\varepsilon_d / \xi\varepsilon_o \le 1$)

$$\sigma_d = \xi f_c' \left[2\left(\frac{\varepsilon_d}{\xi\varepsilon_o}\right) - \left(\frac{\varepsilon_d}{\xi\varepsilon_o}\right)^2 \right] \tag{3}$$

For descending branch ($\varepsilon_d / \xi\varepsilon_o > 1$)

$$\sigma_d = \xi f_c' \left[1 - \left(\frac{\frac{\varepsilon_d}{\xi\varepsilon_o} - 1}{\frac{2}{\xi} - 1} \right)^2 \right] \tag{4}$$

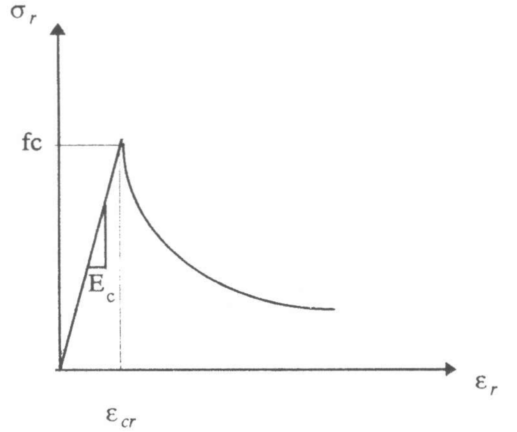

Fig.5 Tensile Stress -Strain Curve of Concrete

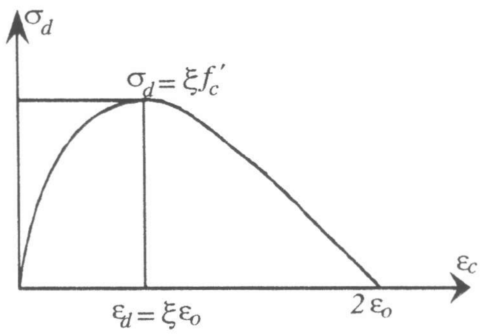

Fig.6 Compression Stress-Strain Curve of Concrete

3.3 STEEL ELEMENT

The stress-strain relation for reinforcing bars is assumed to be elasto-plastic for both of tension and compression members as shown in Fig.7 [2].

For ascending branch ($|\varepsilon| \leq \varepsilon_o$) $\sigma = E_s \varepsilon_o$ (5)

For plastic part ($|\varepsilon| > \varepsilon_o$) $\sigma = E_S \varepsilon_O + 0.01 E_S (\varepsilon - \varepsilon_o)$ (6)

A finite element computer program has been suggested to solve this space truss in three dimensions using the lattice model. Nonlinear incremental analysis is performed by displacement control. The convergence technique employed is Newton-Raphson's Method [3].

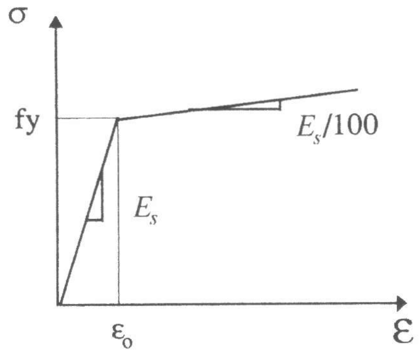

Fig.7 Stress-Strain Curve for Steel

4. EXAMINATION ON THE APPLICABILITY OF THE 3D-LATTICE MODEL

To examine the applicability of 3D-lattice model on the torsion behavior of reinforced concrete columns, several experimental data, in which the applied torsion moment-angle of twist relationship is represented, are picked up. Fig.8 to Fig.13 show the comparisons of calculated results by 3D-lattice model with Thomas Hsu's experiment results [4]. The details of main steel, lateral stirrups and strength properties are indicated in Table 1 [5]. The calculated and experimental results are compared until the ultimate torque. Failure angles of twist by analysis are almost similar and close to the experimental data.

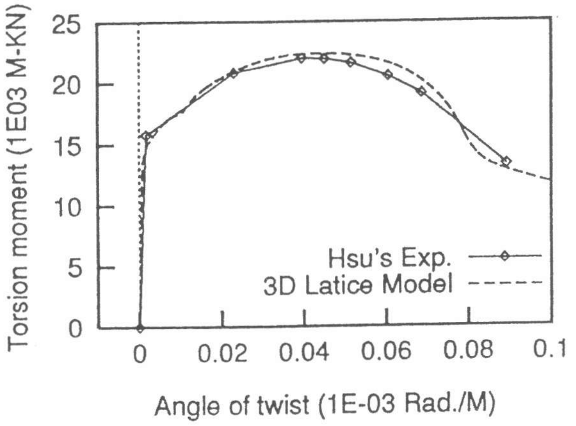

Fig. 8: Torque Moment-Twist Curve of Beam B1

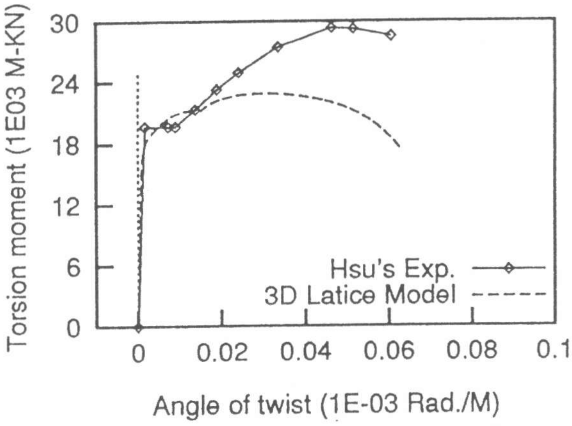

Fig. 9: Torque Moment-Twist Curve of Beam B2

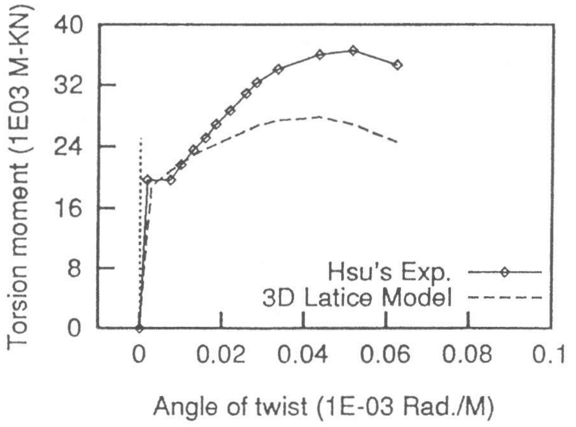

Fig. 10: Torque Moment-Twist Curve of Beam B3

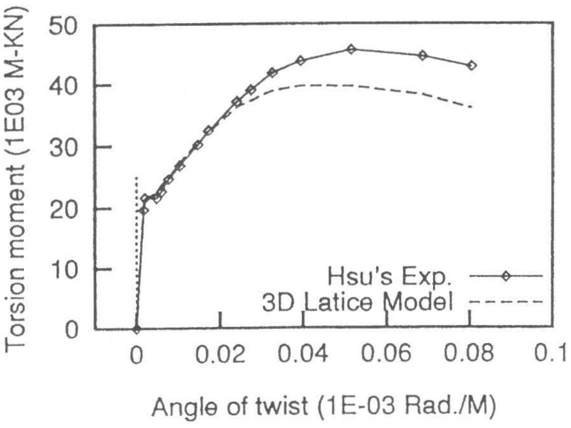

Fig. 11: Torque Moment-Twist Curve of Beam B4

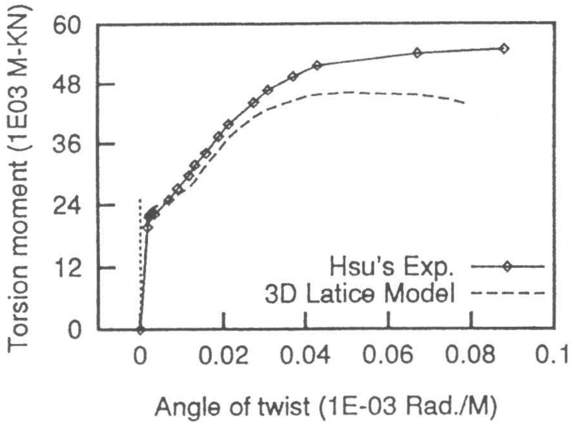

Fig. 12: Torque Moment-Twist Curve of Beam B5

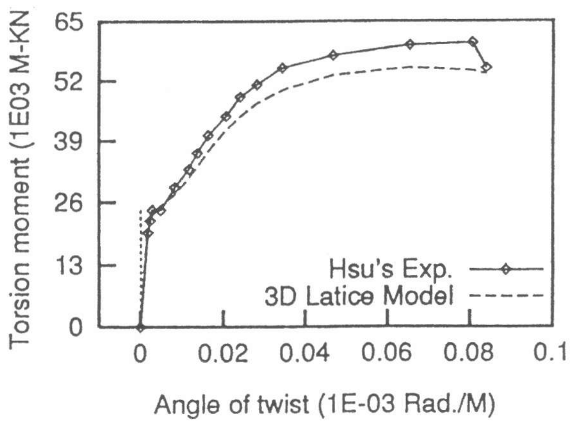

Fig. 13: Torque Moment-Twist Curve of Beam B6

Table:1 Properties of Concrete and Reinforcement for the Studied Beams [4]

Beam	f_{ly} MPa.	f_{sy} MPa.	Area of steel cm^2	Stirrups cm^2, cm	f_c' MPa.	f_{sp} MPa.
B1	313.830	341.430	5.160	0.5 at 15.24	27.590	3.256
B2	316.695	320.040	8.000	1.29 at 18.1	28.620	3.131
B3	327.630	320.040	11.352	1.29 at 12.7	28.070	2.820
B4	320.040	323.490	15.480	1.29 at 9.2	30.556	3.145
B5	332.460	321.420	18.660	1.29 at 6.9	29.040	2.990
B6	331.770	322.800	22.980	1.29 at 5.72	28.830	3.500

Where

f_{ly} is yield strength of longitudinal bars

f_{sy} is yield strength of stirrups

f_c' is the cylinder compressive strength of concrete

f_{sp} is the split-cylinder tensile strength of c

5. MECHANISM OF TORSION FAILURE FOR PLAIN CONCRETE COLUMN

A number of plain concrete columns with rectangular cross-section have been investigated under torsion to control the mechanism of torsion failure. It is found that the failure of plain concrete occurs very suddenly. The failure of the column occurred by the development of a tension cracks in the diagonal members which are inclined at 45° with the axis of twist. It looks that the failure has been caused by bending about an axis parallel to the wider face of the section and inclined with 45° to the axis of the column. This failure mode is quite similar to the actual failure mode of experiments [6].

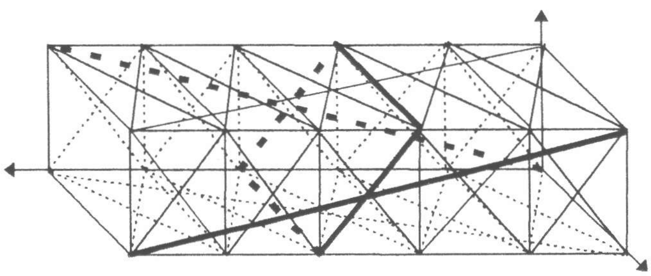

Fig: 14 Cracks of plain concrete section under pure torsion

6. CONCLUSION

Using this 3D lattice model, a nonlinear analysis is performed. Although the 3D-lattice model is considered as simplified analytical method in which the total degree of freedom is quite small compared with normal finite element analysis using solid element and it includes many assumptions during calculation, such as the ratio of the width of the arch members and the diagonal members of concrete. It has found that the prediction for the torsion resisting mechanism of concrete columns by the 3D lattice model may be possible after the detailed investigation of proper dimensioning of each truss member which will be carried out from now on. It is seen that the simulation of failure mechanism of a plain concrete column using 3D lattice model under pure torsion may also be possible. The failure of the column is occurred suddenly by the development of a tension crack in a plane inclined at 45° to the axis of twist.

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